



Alan S. Morris
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Measurement and Instrumentation

THEORY AND APPLICATION **THIRD EDITION**



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Preface

The foundations of this book lie in the highly successful text, *Principles of Measurement and Instrumentation*, by Alan S. Morris. The first edition of this was published in 1988, and a second, revised and extended edition appeared in 1993. This was followed in 2001 by a text with further revisions to the content and a new title, *Measurement and Instrumentation Principles*.

The first edition of this current text then followed in 2011. In developing this, the opportunity was taken to strengthen the book by bringing in a second author, Professor Reza Langari of Texas A&M University, who has made significant contributions especially in the areas of data acquisition and signal processing and the implementation of these using industry-standard LabView software. As well as this new contribution by Professor Langari, this edition covered many new developments in the field of measurement. In particular, it covered the significant recent advances that there have been in smart sensors, intelligent instruments, microsensors, data acquisition, digital signal processing, digital recorders, digital fieldbuses, and new methods of signal transmission. The rapid growth of digital components within measurement systems also created a need to establish procedures in the book for measuring and improving the reliability of the software that is used within such components. Formal standards governing instrument calibration procedures and measurement system performance were extended beyond the traditional area of quality assurance systems (International Organization for Standardization [ISO]9000) into new areas such as environmental protection systems (ISO14000). Thus, when published in 2011, the book was reasonably up-to-date with all of the recent developments in measurement systems up to that time.

One notable development since the first edition has been a large increase in the amount of material on measurement uncertainty. This includes an extended discussion on induced measurement noise and the various sources of this, such as inductive coupling, capacitive (electrostatic) coupling, noise due to multiple earths, noise in the form of voltage transients, thermoelectric potentials, shot noise, and electrochemical potentials. There is also a significant increase in the number of worked examples. As a consequence, this expansion necessitated splitting the previous single chapter on this topic into two chapters.

The past 4 years have also seen continual developments of new sensors, and especially microscale (MEMS) and nanoscale (NEMS) ones. These developments are covered in the chapter on sensor technologies and also in later chapters devoted to sensors for measuring

particular physical variables. At the same time, the continued usage of devices built on older technologies has been reviewed, resulting in the exclusion of those that are now uncommon. The number of end-of-chapter student problems has also been expanded significantly, because the process of solving such problems is felt to be a valuable aid in the learning process for students.

The overall aim of the book continues to be to present the topics of sensors and instrumentation, and their use within measurement systems, as an integrated and coherent subject. Measurement systems, and the instruments and sensors used within them, are of immense importance in a wide variety of domestic and industrial activities. The growth in the sophistication of instruments used in industry has been particularly significant as advanced automation schemes have been developed. Similar developments have also been evident in military and medical applications.

Unfortunately, the crucial part that measurement has in all of these systems tends to get overlooked, and measurement is therefore rarely given the importance that it deserves. For example, much effort goes into designing sophisticated automatic control systems, but little regard is given to the accuracy and quality of the raw measurement data that such systems use as their inputs. This disregard of measurement system quality and performance means that such control systems will never achieve their full potential, because it is difficult to increase their performance beyond the quality of the raw measurement data on which they depend.

Ideally, the principles of good measurement and instrumentation practice should be taught throughout the duration of engineering courses, starting at an elementary level and moving on to more advanced topics as the course progresses. With this in mind, the material contained in this book is designed both to support introductory courses in measurement and instrumentation and to provide in-depth coverage of advanced topics for higher-level courses. In addition, besides its role as a student course text, it is anticipated that the book will be useful to practicing engineers, both to update their knowledge of the latest developments in measurement theory and practice, and to serve as a guide to the typical characteristics and capabilities of the range of sensors and instruments that are currently in use.

Following the usual pattern with measurement textbooks, the early chapters deal with the principles and theory of measurement, and then subsequent chapters cover the ranges of instruments and sensors that are available for measuring various physical quantities. This order of coverage has been chosen so that the general characteristics of measuring instruments, and their behavior in different operating environments, are well established before the reader is introduced to the procedures involved in choosing a measurement device for a particular application. This ensures that the reader will be properly equipped to appreciate and critically appraise the various merits and characteristics of different instruments when faced with the task of choosing a suitable instrument in any given situation.

It should be noted that, although measurement theory inevitably involves some mathematics, the mathematical content of the book has deliberately been kept to the

minimum necessary for the reader to be able to design and build measurement systems that perform to a level commensurate with the needs of the automatic control scheme or other system that they support. Where mathematical procedures are necessary, worked examples are provided as necessary throughout the book to illustrate the principles involved. Self-assessment questions are also provided in critical chapters to enable readers to test their level of understanding.

The early chapters are organized such that all of the elements in a typical measurement system are presented in a logical order, starting with the capture of a measurement signal by a sensor and then proceeding through the stages of signal processing, sensor output transducing, signal transmission, and signal display or recording. Ancillary issues, such as calibration and measurement system reliability, are also covered. Discussion starts with a review of the different classes of instrument and sensor available, and the sort of applications in which these different types are typically used. This opening discussion includes analysis of the static and dynamic characteristics of instruments and an exploration of how these affect instrument usage. A comprehensive discussion of measurement system errors then follows, with appropriate procedures for quantifying, analyzing, and reducing errors being presented across two chapters. The importance of calibration procedures in all aspects of measurement systems, and particularly to satisfy the requirements of standards such as ISO9000 and ISO14000, is recognized by devoting a full chapter to the issues involved. This is followed by a chapter devoted to presenting the range of variable conversion elements (transducers) and techniques that are used to convert nonelectrical sensor outputs into electrical signals, with particular emphasis on electrical bridge circuits. The problems of signal transmission are considered in the next chapter, and various means of improving the quality of transmitted signals are presented. This is followed by a chapter explaining data acquisition techniques and discussing the various analogue and digital signal-processing procedures that are used to attenuate noise and improve the quality of signals. The next chapter provides a comprehensive introduction to the features of the LabVIEW software package. Various examples are provided in this chapter that explain the application of LabVIEW to implement the data acquisition and digital signal processing techniques covered earlier in Chapter 8. The following chapter then discusses the various indicating and test instruments that are used to display and record electrical measurement signals. This chapter also covers data presentation methods and related issues such as least-squares curve fitting, confidence tests, and correlation tests. The next consideration is the subject of intelligent devices and the related issues of digital computation techniques, input–output interfaces, data buses, data networks, and fieldbus technologies. The following chapter then discusses the issue of measurement system reliability, and the effect of unreliability on plant safety systems. This discussion includes the subject of software reliability, because computational elements are embedded in many measurement systems. Finally, this initial set of chapters covering measurement theory concludes with a chapter on the various sensor technologies that are in use. This coverage includes discussion on recently developed technologies, and particularly the advances in micro devices (MEMS and NEMS).

Subsequent chapters then provide comprehensive coverage of the main types of sensor and instrument that exist for measuring all of the physical quantities that a practicing engineer is likely to meet in normal situations. However, although the coverage is as comprehensive as possible, the distinction is emphasized among (a) instruments that are current and in common use; (b) instruments that are current but not widely used except in special applications, for reasons of cost or limited capabilities; and (c) instruments that are largely obsolete as regards new industrial implementations, but which are still encountered on older plant that was installed some years ago. As well as emphasizing this distinction, some guidance is given about how to go about choosing an instrument for a particular measurement application and how to implement appropriate calibration techniques. It should be noted that the reader familiar with previous editions will notice the exclusion of some devices previously included, as a result of their use being largely discontinued.

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Fundamentals of measurement systems

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1.1 Introduction

Measurement techniques have been of immense importance ever since the start of human civilization, when measurements were first needed to regulate the transfer of goods in barter trade in order to ensure that exchanges were fair. The industrial revolution during the 19th century brought about a rapid development of new instruments and measurement techniques to satisfy the needs of industrialized production techniques. Since that time, there has been a large and rapid growth in new industrial technology. This has been particularly evident since the middle of the 20th century, because of the many developments in electronics in general and computers in particular. In turn, this has required a parallel growth in new instruments and measurement techniques.

The massive growth in the application of computers to industrial process control and monitoring tasks has greatly expanded the requirement for instruments to measure, record, and control process variables. As modern production techniques dictate working to ever tighter accuracy limits, and as economic forces to reduce production costs become more severe, so the requirement for instruments to be both accurate and cheap becomes ever harder to satisfy. This latter problem is at the focal point of the research and development efforts of all instrument manufacturers. In the past few years, the most cost-effective means of improving instrument accuracy has been found in many cases to be the inclusion of digital computing power within instruments themselves. These intelligent instruments therefore feature prominently in current instrument manufacturers' catalogs.

This opening chapter will cover some fundamental aspects of measurement. Firstly, we will look at how standard measurement units have evolved from the early units used in barter trade to the more exact units belonging to the Imperial and metric measurement systems. We will then go on to study the major considerations in designing a measurement system. Finally, we will look at some of the main applications of measurement systems.

1.2 Measurement units

The very first measurement units were those used in barter trade to quantify the amounts being exchanged and to establish clear rules about the relative values of different commodities. Such early systems of measurement were based on whatever was available as a measuring unit. For purposes of measuring length, the human torso was a convenient tool, and gave us units of the hand, the foot, and the cubit. Although generally adequate for barter trade systems, such measurement units are of course imprecise, varying as they do from one person to the next. Therefore, there has been a progressive movement toward measurement units that are defined much more accurately.

The first improved measurement unit was a unit of length (the meter) defined as 10^{-7} times the polar quadrant of the earth. A platinum bar made to this length was established as a standard of length in the early part of the 19th century. This was superseded by a superior quality standard bar in 1889, manufactured from a platinum-iridium alloy. Since that time, technological research has enabled further improvements to be made in the standard used for defining length. Firstly, in 1960, a standard meter was redefined in terms of 1.65076373×10^6 wavelengths of the radiation from krypton-86 in vacuum. More recently, in 1983, the meter was redefined yet again as the length of path traveled by light in an interval of $1/299,792,458$ seconds. In a similar fashion, standard units for the measurement of other physical quantities have been defined and progressively improved over the years.

The early establishment of standards for the measurement of physical quantities proceeded in several countries at broadly parallel times, and in consequence, several sets of units emerged for measuring the same physical variable. For instance, length can be measured in yards, meters, or several other units. Apart from the major units of length, subdivisions of standard units exist such as feet, inches, centimeters, and millimeters, with a fixed relationship between each fundamental unit and its subdivisions.

Yards, feet, and inches belong to the Imperial system of units, which is characterized by having varying and cumbersome multiplication factors relating fundamental units to subdivisions such as 1760 (miles to yards), 3 (yards to feet), and 12 (feet to inches). The metric system is an alternative set of units, which includes for instance the unit of the meter and its centimeter and millimeter subdivisions for measuring length. All multiples and subdivisions of basic metric units are related to the base by factors of 10, and such

units are therefore much easier to use than Imperial units. However, in the case of derived units such as velocity, the number of alternative ways in which these can be expressed in the metric system can lead to confusion.

As a result of this, an internationally agreed set of standard units (SI units or Systèmes Internationales d'Unités) has been defined, and strong efforts are being made to encourage the adoption of this system throughout the world. In support of this effort, the SI system of units will be used exclusively in this book. However, it should be noted that the Imperial system is still widely used in the engineering industry, particularly in the United States.

The full range of fundamental SI measuring units and the further set of units derived from them are given in [Tables 1.1 and 1.2](#). Conversion tables relating common Imperial and metric units to their equivalent SI units can also be found in Appendix 1.

1.3 Measurement system design

In this section, we will look at the main considerations in designing a measurement system. Firstly, we will learn that a measurement system usually consists of several separate components, although only one component might be involved for some very simple measurement tasks. We will then go on to look at how measuring instruments and systems are chosen to satisfy the requirements of particular measurement situations.

1.3.1 Elements of a measurement system

A *measuring system* exists to provide information about the physical value of some variable being measured. In simple cases, the system can consist of only a single unit that gives an output reading or signal according to the magnitude of the unknown variable applied to it. However, in more complex measurement situations, a measuring system

Table 1.1: Fundamental SI units.

Quantity	Standard unit	Symbol
(a) Fundamental units		
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Luminous intensity	candela	cd
Matter	mole	mol
(b) Supplementary fundamental units		
Plane angle	radian	rad
Solid angle	steradian	sr

Table 1.2: Derived SI units.

Quantity	Standard unit	Symbol	Derivation formula
Area	square meter	m ²	
Volume	cubic meter	m ³	
Velocity	meter per second	m/s	
Acceleration	meter per second squared	m/s ²	
Angular velocity	radian per second	rad/s	
Angular acceleration	radian per second squared	rad/s ²	
Density	kilogram per cubic meter	kg/m ³	
Specific volume	cubic meter per kilogram	m ³ /kg	
Mass flow rate	kilogram per second	kg/s	
Volume flow rate	cubic meter per second	m ³ /s	
Force	newton	N	kg-m/s ²
Pressure	pascal	Pa	N/m ²
Torque	newton meter	N-m	
Momentum	kilogram meter per second	kg-m/s	
Moment of inertia	kilogram meter squared	kg-m ²	
Kinematic viscosity	square meter per second	m ² /s	
Dynamic viscosity	newton second per sq meter	N-s/m ²	
Work, energy, heat	joule	J	N-m
Specific energy	joule per cubic meter	J/m ³	
Power	watt	W	J/s
Thermal conductivity	watt per meter kelvin	W/m-K	
Electric charge	coulomb	C	A-s
Voltage, e.m.f., pot diff	volt	V	W/A
Electric field strength	volt per meter	V/m	
Electric resistance	ohm	Ω	V/A
Electric capacitance	farad	F	A-s/V
Electric inductance	henry	H	V-s/A
Electric conductance	siemen	S	A/V
Resistivity	ohm meter	Ω-m	
Permittivity	farad per meter	F/m	
Permeability	henry per meter	H/m	
Current density	ampere per square meter	A/m ²	
Magnetic flux	weber	Wb	V-s
Magnetic flux density	tesla	T	Wb/m ²
Magnetic field strength	ampere per meter	A/m	
Frequency	hertz	Hz	s ⁻¹
Luminous flux	lumen	lm	cd-sr
Luminance	candela per square meter	cd/m ²	
Illumination	lux	lx	lm/m ²
Molar volume	cubic meter per mole	m ³ /mol	
Molarity	mole per kilogram	mol/kg	
Molar energy	joule per mole	J/mol	

consists of several separate elements as shown in Fig. 1.1. These components might be contained within one or more boxes, and the boxes holding individual measurement elements might be either close together or physically separate. The term *measuring*

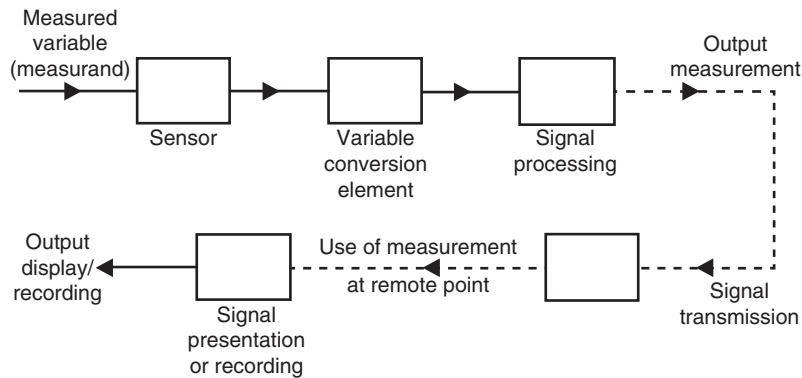


Figure 1.1
Elements of a measuring system.

instrument is commonly used to describe a measurement system, whether it contains only one or many elements, and this term will be widely used throughout this text.

The first element in any measuring system is the *primary sensor*; this gives an output that is a function of the measurand (the input applied to it). For most but not all sensors, this function is at least approximately linear. Some examples of primary sensors are a liquid-in-glass thermometer, a thermocouple, and a strain gauge. In the case of the mercury-in-glass thermometer, the output reading is given in terms of the level of the mercury, and so this particular primary sensor is also a complete measurement system in itself. However, in general, the primary sensor is only part of a measurement system. The types of primary sensors available for measuring a wide range of physical quantities are presented in Chapters 14–21 of this book.

One or more *secondary sensors* are needed if the characteristics of the primary sensor are affected by the environmental conditions in which the primary sensor is operating. These measure the environmental conditions and allow the appropriate correction to be made to the measurements provided by the primary sensor. Such correction is done automatically in intelligent sensors (see Chapter 11).

Conversion elements are needed where the output variable of a primary transducer is in an inconvenient form and has to be converted to a more convenient form. For instance, the displacement-measuring strain gauge has an output in the form of a varying resistance. The resistance change cannot be easily measured and so it is converted to a change in voltage by a *bridge circuit*, which is a typical example of a conversion element. In some cases, the primary sensor and conversion element are combined, and the combination is known as a *transducer*¹.

¹ In some cases, the word *sensor* is used generically to refer to both transducers and transmitters.

Signal processing elements exist to improve the quality of the output of a measurement system in some way. A very common type of signal processing element is the electronic amplifier, which amplifies the output of the primary transducer or conversion element, thus improving the sensitivity and resolution of measurement. This element of a measuring system is particularly important where the primary transducer has a low output. For example, thermocouples have a typical output of only a few millivolts. Other types of signal processing element are those that filter out induced noise and remove mean levels, etc. In some devices, signal processing is incorporated into a transducer, which is then known as a *transmitter*.¹

In addition to these three components just mentioned, some measurement systems have one or two other components, firstly to transmit the signal to some remote point and secondly to display or record the signal if it is not fed automatically into a feedback control system. Signal transmission is needed when the observation or application point of the output of a measurement system is some distance away from the site of the primary transducer. Sometimes, this separation is made solely for purposes of convenience, but more often, it follows from the physical inaccessibility or environmental unsuitability of the site of the primary transducer for mounting the signal presentation/recording unit. The signal transmission element has traditionally consisted of single or multicore cable, which is often screened to minimize signal corruption by induced electrical noise. However, fiber-optic cables are being used in ever-increasing numbers in modern installations, in part because of their low transmission loss and imperviousness to the effects of electrical and magnetic fields.

The final optional element in a measurement system is the point where the measured signal is utilized. In some cases, this element is omitted altogether because the measurement is used as part of an automatic control scheme, and the transmitted signal is fed directly into the control system. In other cases, this element in the measurement system takes the form either of a signal presentation unit or of a signal-recording unit. These take many forms according to the requirements of the particular measurement application, and the range of possible units is discussed more fully in Chapter 10.

1.3.2 Choosing appropriate measuring instruments

The starting point in choosing the most suitable instrument to use for measurement of a particular quantity in a manufacturing plant or other system is the specification of the instrument characteristics required, especially parameters like the desired measurement accuracy, resolution, sensitivity, and dynamic performance (see next chapter for definitions of these). It is also essential to know the environmental conditions that the instrument will

be subjected to, as some conditions will immediately either eliminate the possibility of using certain types of instruments or else will create a requirement for expensive protection of the instruments. It should also be noted that protection reduces the performance of some instruments, especially in terms of their dynamic characteristics (for example, sheaths protecting thermocouples and resistance thermometers reduce their speed of response). Provision of this type of information usually requires the expert knowledge of personnel who are intimately acquainted with the operation of the manufacturing plant or system in question. Then, a skilled instrument engineer, having knowledge of all the instruments that are available for measuring the quantity in question, will be able to evaluate the possible list of instruments in terms of their accuracy, cost, and suitability for the environmental conditions and thus choose the most appropriate instrument. As far as possible, measurement systems and instruments should be chosen that are as insensitive as possible to the operating environment, although this requirement is often difficult to meet because of cost and other performance considerations. The extent to which the measured system will be disturbed during the measuring process is another important factor in instrument choice. For example, significant pressure loss can be caused to the measured system in some techniques of flow measurement.

Published literature is of considerable help in the choice of a suitable instrument for a particular measurement situation. Many books are available that give valuable assistance in the necessary evaluation by providing lists and data about all the instruments available for measuring a range of physical quantities (see Chapters 14–21). However, new techniques and instruments are being developed all the time, and therefore a good instrumentation engineer must keep abreast of the latest developments by reading the appropriate technical journals regularly.

The instrument characteristics discussed in the next chapter are the features that form the technical basis for a comparison between the relative merits of different instruments. Generally, the better the characteristics, the higher the cost. However, in comparing the cost and relative suitability of different instruments for a particular measurement situation, considerations of durability, maintainability, and constancy of performance are also very important because the instrument chosen will often have to be capable of operating for long periods without performance degradation and a requirement for costly maintenance. In consequence of this, the initial cost of an instrument often has a low weighting in the evaluation exercise.

Cost is very strongly correlated with the performance of an instrument, as measured by its static characteristics. Increasing the accuracy or resolution of an instrument, for example, can only be done at a penalty of increasing its manufacturing cost. Instrument choice therefore proceeds by specifying the minimum characteristics required by a measurement situation and then searching manufacturers' catalogs to find an instrument whose

characteristics match those required. To select an instrument with characteristics superior to those required would only mean paying more than necessary for a level of performance greater than that needed.

As well as purchase cost, other important factors in the assessment exercise are instrument durability and the maintenance requirements. Assuming that one had \$20,000 to spend, one would not spend \$15,000 on a new motor car whose projected life was 5 years if a car of equivalent specifications with a projected life of 10 years was available for \$20,000. Likewise, durability is an important consideration in the choice of instruments. The projected life of instruments often depends on the conditions in which the instrument will have to operate. Maintenance requirements must also be taken into account, as they also have cost implications.

As a general rule, a good assessment criterion is obtained if the total purchase cost and estimated maintenance costs of an instrument over its life are divided by the period of its expected life. The figure obtained is thus a cost per year. However, this rule becomes modified where instruments are being installed on a process whose life is expected to be limited, perhaps in the manufacture of a particular model of car. Then, the total costs can only be divided by the period of time that an instrument is expected to be used for, unless an alternative use for the instrument is envisaged at the end of this period.

To summarize, instrument choice is a compromise between performance characteristics, ruggedness and durability, maintenance requirements, and purchase cost. To carry out such an evaluation properly, the instrument engineer must have a wide knowledge of the range of instruments available for measuring particular physical quantities, and they must also have a deep understanding of how instrument characteristics are affected by particular measurement situations and operating conditions.

1.4 Measurement system applications

Today the techniques of measurement are of immense importance in most facets of human civilization. Present-day applications of measuring instruments can be classified into three major areas. The first of these is their use in regulating trade, applying instruments that measure physical quantities such as length, volume, and mass in terms of standard units. The particular instruments and transducers employed in such applications are included in the general description of instruments presented in the later chapters of this book.

The second application area of measuring instruments is in monitoring functions. These provide information that enables human beings to take some prescribed action accordingly. The gardener uses a thermometer to determine whether he should turn the heat on in his greenhouse or open the windows if it is too hot. Regular study of a barometer allows us to decide whether we should take our umbrellas if we are planning

to go out for a few hours. While there are thus many uses of instrumentation in our normal domestic lives, the majority of monitoring functions exist to provide the information necessary to allow a human being to control some industrial operation or process. In a chemical process for instance, the progress of chemical reactions is indicated by the measurement of temperatures and pressures at various points, and such measurements allow the operator to take correct decisions regarding the electrical supply to heaters, cooling water flows, valve positions, etc. One other important use of monitoring instruments is in calibrating the instruments used in the automatic process control systems described below.

Use as part of automatic feedback control systems forms the third application area of measurement systems. Fig. 1.2 shows a functional block diagram of a simple temperature control system in which the temperature T_a of a room is maintained at a reference value T_d . The value of the controlled variable T_a , as determined by a temperature-measuring device, is compared with the reference value T_d , and the difference e is applied as an error signal to the heater. The heater then modifies the room temperature until $T_a = T_d$. The characteristics of the measuring instruments used in any feedback control system are of fundamental importance to the quality of control achieved. The accuracy and resolution with which an output variable of a process is controlled can never be better than the accuracy and resolution of the measuring instruments used. This is a very important principle, but one that is often inadequately discussed in many texts on automatic control systems. Such texts explore the theoretical aspects of control system design in considerable depth, but fail to give sufficient emphasis to the fact that all gain and phase margin performance calculations etc. are entirely dependent on the quality of the process measurements obtained.

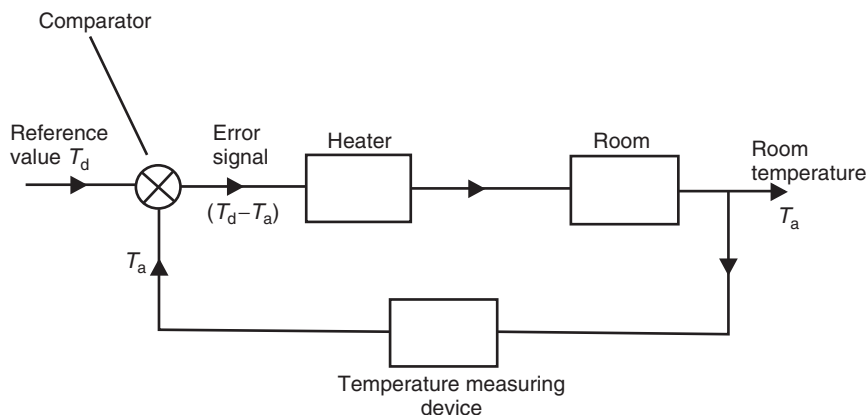


Figure 1.2
Elements of a simple closed-loop control system.

1.5 Summary

This opening chapter has covered some fundamental aspects of measurement systems. Firstly, we looked at the importance of having standard measurement units and how these have evolved into the Imperial and metric systems of units. We then went on to look at the main aspects of measurement system design and, in particular, what the main components in a measurement system are and how these are chosen for particular measurement requirements. Finally, we have had a brief look at the range of applications of measurement systems.

1.6 Problems

- 1.1 How have systems of measurement units evolved over the years?
- 1.2 What are the main elements in a measurement system and what are their functions? Which elements are not needed in some measurement systems and why are they not needed?
- 1.3 What are the main factors governing the choice of a measuring instrument for a given application?
- 1.4 Name and discuss three application areas for measurement systems.
- 1.5 What are the main considerations in designing a measurement system?
- 1.6 Explain the following and give some examples: (a) Imperial units, (b) SI units, (c) primary sensor, (d) secondary sensor, (e) signal processing

Instrument types and performance characteristics

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2.1 Introduction

Two important aspects of measurement that were covered in the opening chapter concerned how to choose appropriate instruments for a particular application and a review of the main applications of measurement. Both of these activities require knowledge of the characteristics of different classes of instruments and, in particular, how these different classes of instrument perform in different applications and operating environments. Therefore, we start this chapter by reviewing the various classes of instruments that exist. All instruments can be divided into active and passive ones according to whether they have an energy source contained within them. The next distinction is between null-type instruments that require adjustment until a datum level is reached and deflection-type instruments that give an output measurement in the form of either a deflection of a pointer against a scale or a numerical display. The third distinction covered is between analog and digital instruments, which differ according to whether the output varies continuously

(analog instrument) or in discrete steps (digital instrument). Fourth, we shall look at the distinction between instruments that are merely indicators and those that have a signal output. Indicators give some visual or audio indication of the magnitude of the measured quantity and are commonly found in the process industries. Instruments with a signal output are commonly found as part of automatic control systems. The final distinction we will consider is between smart and nonsmart instruments. Smart, often known as intelligent, instruments are very important today and predominate in most measurement applications. Because of their importance, they are given more detailed consideration later in Chapter 11.

The second part of this chapter looks at the various attributes of instruments that determine their performance and suitability for different measurement requirements and applications. We will first look at the static characteristics of instruments. These are their steady-state attributes (when the output measurement value has settled to a constant reading after any initial varying output) such as accuracy, measurement sensitivity, and resistance to errors caused by variations in their operating environment. We will then examine the dynamic characteristics of instruments. These describe their behavior following the time that the measured quantity changes value up until the time when the output reading attains a steady value. Various kinds of dynamic behavior can be observed in different instruments ranging from an output that varies slowly until it reaches a final constant value to an output that oscillates about the final value until a steady reading is obtained. The dynamic characteristics are an important factor in deciding on the suitability of an instrument for a particular measurement application. Finally, at the end of the chapter, we will briefly consider the issue of instrument calibration, although this is considered in much greater detail later in Chapter 5.

2.2 Review of instrument types

Instruments can be subdivided into separate classes according to several criteria. These subclassifications are useful for broadly establishing several attributes of particular instruments such as accuracy, cost, and general relevance for different applications.

2.2.1 Active and passive instruments

Instruments are divided into active or passive ones according to whether the instrument output is entirely produced by the quantity being measured or whether the quantity being measured simply modulates the magnitude of some external power source. This is illustrated by examples.

An example of a passive instrument is the pressure-measuring device shown in [Fig. 2.1](#). The pressure of the fluid is translated into a movement of a pointer against a scale.

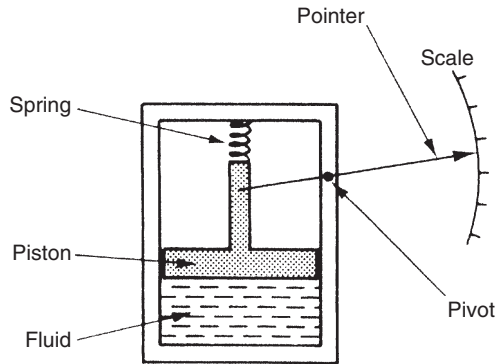


Figure 2.1

Passive pressure gauge.

The energy expended in moving the pointer is derived entirely from the change in pressure measured: there are no other energy inputs to the system.

An example of an active instrument is a float-type fuel tank level indicator as shown in [Fig. 2.2](#). Here, the change in fuel level moves a potentiometer arm, and the output signal consists of a proportion of the external voltage source applied across the two ends of the potentiometer. The energy in the output signal comes from the external power source: the primary transducer float system is merely modulating the value of the voltage from this external power source.

In active instruments, the external power source is usually in electrical form, but in some cases, it can be other forms of energy such as a pneumatic or hydraulic one.

One important difference between active and passive instruments is the level of measurement resolution that can be obtained. With the simple pressure gauge shown, the amount of movement made by the pointer for a particular pressure change is closely

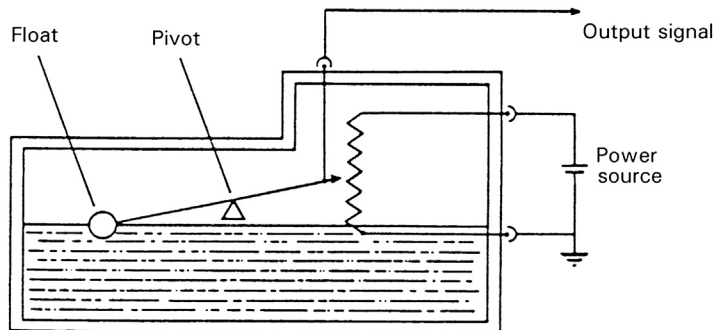


Figure 2.2

Fuel tank level indicator.

defined by the nature of the instrument. Although it is possible to increase measurement resolution by making the pointer longer, such that the pointer tip moves through a longer arc, the scope for such improvement is clearly restricted by the practical limit of how long the pointer can conveniently be. In an active instrument, however, adjustment of the magnitude of the external energy input allows much greater control over measurement resolution. Although the scope for improving measurement resolution is much greater incidentally, it is not infinite because of limitations placed on the magnitude of the external energy input, in consideration of heating effects and for safety reasons.

In terms of cost, passive instruments are normally of a simpler construction than active ones and are therefore cheaper to manufacture. Therefore, the choice between active and passive instruments for a particular application involves carefully balancing the measurement resolution requirements against cost.

2.2.2 Null-type and deflection-type instruments

The pressure gauge just mentioned is a good example of a deflection type of instrument, in which the value of the quantity being measured is displayed in terms of the amount of movement of a pointer. An alternative type of pressure gauge is the dead-weight gauge shown in Fig. 2.3, which is a null-type instrument. Here, weights are put on top of the piston until the downward force balances the fluid pressure. Weights are added until the piston reaches a datum level, known as the null point. Pressure measurement is made in terms of the value of the weights needed to reach this null position.

The accuracy of these two instruments depends on different things. The accuracy of the first type depends on the linearity and calibration of the spring, while the accuracy of the second type relies on the calibration of the weights. Because calibration of weights is much easier than careful choice and calibration of a linear-characteristic spring, the second type of instrument will normally be more accurate. This is in accordance with the general rule that null-type instruments are more accurate than deflection types.

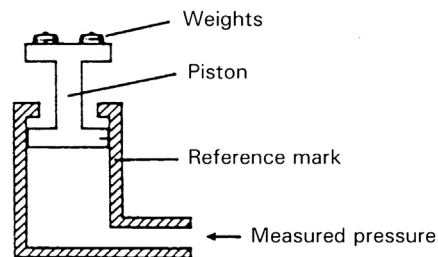


Figure 2.3
Dead-weight pressure gauge.

In terms of use, the deflection-type instrument is clearly more convenient. It is far simpler to read the position of a pointer against a scale than to add and subtract weights until a null point is reached. A deflection-type instrument is therefore the one that would normally be used in the workplace. However, for calibration duties, the null-type instrument is preferable because of its superior accuracy. The extra effort required to use such an instrument is perfectly acceptable in this case because of the infrequent nature of calibration operations.

2.2.3 Analog and digital instruments

An analog instrument gives an output that varies continuously as the quantity being measured changes. The output can have an infinite number of values within the range that the instrument is designed to measure. The deflection-type of pressure gauge described earlier in this chapter (Fig. 2.1) is an example of an analog instrument. As the input value changes, the pointer moves with a smooth continuous motion. Although the pointer can therefore be in an infinite number of positions within its range of movement, the number of different positions that the eye can discriminate between is strictly limited. This discrimination depends on how large the scale is and how finely it is divided.

A digital instrument has an output that varies in discrete steps and so can have only a finite number of values. The rev-counter in Fig. 2.4 is an example of a digital instrument. A cam is attached to the revolving body whose motion is being measured, and on each revolution the cam opens and closes a switch. The switching operations are counted by an electronic counter. This system can count only whole revolutions and cannot discriminate any motion that is less than a full revolution.

The distinction between analog and digital instruments has become particularly important with the rapid growth in the application of microcomputers to automatic control systems. Any digital computer system, of which the microcomputer is but one example, performs

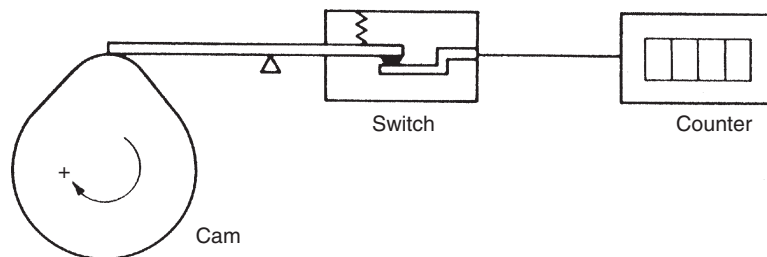


Figure 2.4
Rev-counter.

its computations in digital form. An instrument whose output is in digital form is therefore particularly advantageous in such applications, because it can be interfaced directly to the control computer. Analog instruments must be interfaced to the microcomputer by an analog-to-digital (A/D) converter, which converts the analog output signal from the instrument into an equivalent digital quantity that can be read into the computer. This conversion has several disadvantages. First, the A/D converter adds a significant cost to the system. Second, a finite time is involved in the process of converting an analog signal to a digital quantity, and this time can be critical in the control of fast processes in which the accuracy of control depends on the speed of the controlling computer. Degrading the speed of operation of the control computer by imposing a requirement for A/D conversion thus impairs the accuracy by which the process is controlled.

2.2.4 Indicating instruments and instruments with a signal output

The final way in which instruments can be divided is between those that merely give an audio or visual indication of the magnitude of the physical quantity measured and those that give an output in the form of a measurement signal whose magnitude is proportional to the measured quantity.

The class of indicating instruments normally includes all null-type instruments and most passive ones. Indicators can also be further divided into those that have an analog output and those that have a digital display. A common analogue indicator is the liquid-in-glass thermometer. Another common indicating device, which exists in both analogue and digital forms, is the bathroom scale. The older mechanical form of this is an analogue type of instrument that gives an output consisting of a rotating pointer moving against a scale (or sometimes a rotating scale moving against a pointer). More recent electronic forms of bathroom scale have a digital output consisting of numbers presented on an electronic display. One major drawback with indicating devices is that human intervention is required to read and record a measurement. This process is particularly prone to error in the case of analog output displays, although digital displays are not prone to error unless the human reader is careless.

Instruments that have a signal-type output are commonly used as part of automatic control systems. In other circumstances, they can also be found in measurement systems in which the output measurement signal is recorded in some way for later use. This subject is covered in later chapters. Usually, the measurement signal involved is an electrical voltage, but it can take other forms in some systems such as an electrical current, an optical signal, or a pneumatic signal.

2.2.5 Smart and nonsmart instruments

The advent of the microprocessor has created a new division in instruments between those that incorporate a microprocessor (smart) and those that do not. Smart devices are considered in detail in Chapter 11.

2.3 Static characteristics of instruments

If we have a thermometer in a room and its reading shows a temperature of 20°C, it does not really matter whether the true temperature of the room is 19.5°C or 20.5°C. Such small variations around 20°C are too small to affect whether we feel warm enough or not. Our bodies cannot discriminate between such close levels of temperature, and therefore a thermometer with an inaccuracy of $\pm 0.5^\circ\text{C}$ is perfectly adequate. If we had to measure the temperature of certain chemical processes, however, a variation of 0.5°C might have a significant effect on the rate of reaction or even the products of a process. A measurement inaccuracy much less than $\pm 0.5^\circ\text{C}$ is therefore clearly required.

Accuracy of measurement is thus one consideration in the choice of instrument for a particular application. Other parameters such as sensitivity, linearity, and the reaction to ambient temperature changes are further considerations. These attributes are collectively known as the static characteristics of instruments and are given in the data sheet for a particular instrument. Values quoted for instrument characteristics in such a data sheet apply only when the instrument is used under specified standard calibration conditions. Due allowance must be made for variations in the characteristics when the instrument is used under other conditions.

The various static characteristics are defined in the following paragraphs.

Accuracy and inaccuracy (measurement uncertainty): The *accuracy* of an instrument is a measure of how close the output reading of the instrument is to the correct value. In practice, it is more usual to quote the *inaccuracy* or *measurement uncertainty* value rather than the accuracy value for an instrument. Inaccuracy or measurement uncertainty is the extent to which a reading might be wrong and is often quoted as a percentage of the full-scale reading of an instrument.

■ Example 2.1

A pressure gauge with a measurement range of 0–10 bar has a quoted inaccuracy of $\pm 1.0\%$ of the full-scale reading.

- (a) What is the maximum measurement error expected for this instrument?
- (b) What is the likely measurement error expressed as a percentage of the output reading if this pressure gauge is measuring a pressure of 1 bar?



■ Solution

(a) The maximum error expected in any measurement reading is 1.0% of the full scale reading, which is 10 bar for this particular instrument. Hence, the maximum likely error is $1.0\% \times 10 \text{ bar} = 0.1 \text{ bar}$.

(b) The maximum measurement error is a constant value related to the full-scale reading of the instrument, irrespective of the magnitude of the quantity that the instrument is actually measuring. In this case, as worked out earlier, the magnitude of the error is 0.1 bar. Thus, when measuring a pressure of 1 bar, the maximum possible error of 0.1 bar is 10% of the measurement value.

This example carries a very important message. Because the maximum measurement error in an instrument is usually related to the full-scale reading of the instrument, measuring quantities that are substantially less than the full-scale reading means that the possible measurement error is amplified. For this reason, it is an important system design rule that instruments are chosen such that their range is appropriate to the spread of values being measured, so that the best possible accuracy is maintained in instrument readings. Clearly, if we are measuring pressures with expected values between 0 and 1 bar, we would not use an instrument with a measurement range of 0–10 bar.

Precision/repeatability/reproducibility: *Precision* is a term that describes an instrument's degree of freedom from random errors. If a large number of readings are taken of the same quantity by a high-precision instrument, the spread of readings will be very small. Precision is often, although incorrectly, confused with accuracy. High precision does not imply anything about measurement accuracy. A high-precision instrument may have a low accuracy. Low-accuracy measurements from a high-precision instrument are normally caused by a bias in the measurements, which is removable by recalibration.

The terms “repeatability” and “reproducibility” mean approximately the same but are applied in different contexts as discussed later. *Repeatability* describes the closeness of output readings when the same input is applied repetitively over a short time, with the same measurement conditions, same instrument and observer, same location, and same conditions of use maintained throughout. *Reproducibility* describes the closeness of output readings for the same input when there are changes in the method of measurement, observer, measuring instrument, location, conditions of use, and time of measurement. Both terms thus describe the spread of output readings for the same input. This spread is referred to as repeatability if the measurement conditions are constant and as reproducibility if the measurement conditions vary.

The degree of repeatability or reproducibility in measurements from an instrument is an alternative way to express its precision. Fig. 2.5 illustrates this more clearly. The figure shows the results of tests on three industrial robots that were programmed to place components at a particular point on a table. The target point was at the center of the concentric circles shown, and the black dots represent the points where each robot actually deposited components at each attempt. Both the accuracy and precision of Robot 1 is shown to be low in this trial. Robot 2 consistently puts the component down at approximately the same place, but this is the wrong place. Therefore, it has high precision but low accuracy. Finally, Robot 3 has both high precision and high accuracy because it consistently places the component at the correct target position.

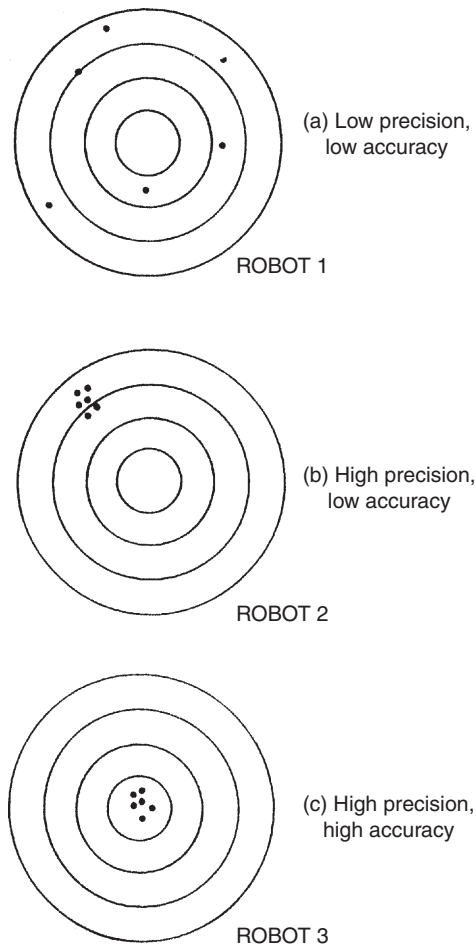


Figure 2.5
Comparison of accuracy and precision.

■ Example 2.2

The width of a room is measured 10 times by an ultrasonic rule and the following measurements are obtained (in meters): 5.381, 5.379, 5.378, 5.382, 5.380, 5.383, 5.379, 5.377, 5.380, and 5.381.

The width of the same room is then measured by a calibrated steel tape that gives a reading of 5.374 m, which can be taken as the correct value for the width of the room.

- (a) What is the measurement precision of the ultrasonic rule?
- (b) What is the maximum measurement inaccuracy of the ultrasonic rule?

■ Solution

(a) The mean (average) value of the 10 measurements made with the ultrasonic rule is 5.380 m.

The maximum deviation below this mean value is -0.003 m and the maximum deviation above the mean value is $+0.003$ m. Thus, the precision of the ultrasonic rule can be expressed as ± 0.003 m (± 3 mm).

(b) The correct value of the room width has been measured as 5.374 m by the calibrated steel rule. All ultrasonic rule measurements are above this, with the largest value being 5.383 m. This last measurement is the one that exhibits the largest measurement error. This maximum measurement error can be calculated as $5.383 - 5.374 = 0.009$ m (9 mm). Thus, the maximum measurement inaccuracy can be expressed as $+9$ mm.

This example illustrates nicely that although the ultrasonic rule has fairly high precision, its actual measurement inaccuracy is substantially inferior.

Tolerance: Tolerance is a term closely related to accuracy and defines the maximum error that is to be expected in some value. Although, strictly speaking, it is not a static characteristic of measuring instruments, it is mentioned here because the accuracy of some instruments is sometimes quoted as a tolerance value. When used correctly, tolerance describes the maximum deviation of a manufactured component from some specified value. For instance, crankshafts are machined with a diameter tolerance quoted as so many microns (10^{-6} m), and electric circuit components such as resistors have tolerances of perhaps 5%.

■ Example 2.3

A packet of resistors bought in an electronics component shop gives the nominal resistance value as $1000\ \Omega$ and the manufacturing tolerance as $\pm 5\%$. If one resistor is chosen at random from the packet, what is the minimum and maximum resistance value that this particular resistor is likely to have?

■ Solution

The minimum likely value is $1000\ \Omega - 5\% = 950\ \Omega$.

The maximum likely value is $1000\ \Omega + 5\% = 1050\ \Omega$.

Range or span: The range or span of an instrument defines the minimum and maximum values of a quantity that the instrument is designed to measure.

■ Example 2.4

A particular micrometer is designed to measure dimensions between 50 and 75 mm. What is its measurement range?

■ Solution

The measurement range is simply the difference between the maximum and minimum measurements. Thus, in this case the range is $75 - 50 = 25\ \text{mm}$.

Threshold: If the input to an instrument is gradually increased from zero, the input will have to reach a certain minimum level before the change in the instrument output reading is of a large enough magnitude to be detectable. This minimum level of input is known as the threshold of the instrument. Manufacturers vary in the way that they specify threshold for instruments. Some quote absolute values, whereas others quote threshold as a percentage of full-scale readings. As an illustration, a car speedometer typically has a threshold of about 15 miles/h. This means that if the vehicle starts from rest and accelerates, no output reading is observed on the speedometer until the speed reaches 15 miles/h.

Resolution: When an instrument is showing a particular output reading, there is a lower limit on the magnitude of the change in the input measured quantity that produces an observable change in the instrument output. Like threshold, resolution is sometimes specified as an absolute value and sometimes as a percentage of the full-scale deflection. A major factor influencing the resolution of an instrument is how finely its output scale is divided into subdivisions. Using a car speedometer as an example again, this has subdivisions of typically 10 miles/h. This means that when the needle is between the scale markings, we cannot estimate speed more accurately than to the nearest 5 miles/h. This value of 5 miles/h thus represents the resolution of the instrument.

Linearity: It is normally desirable for the output reading of an instrument to be linearly proportional to the quantity being measured. The X's marked on Fig. 2.6 show a plot of the typical output readings of an instrument when a sequence of input quantities are applied to it. Normal procedure is to draw a good fit straight line through the X's, as

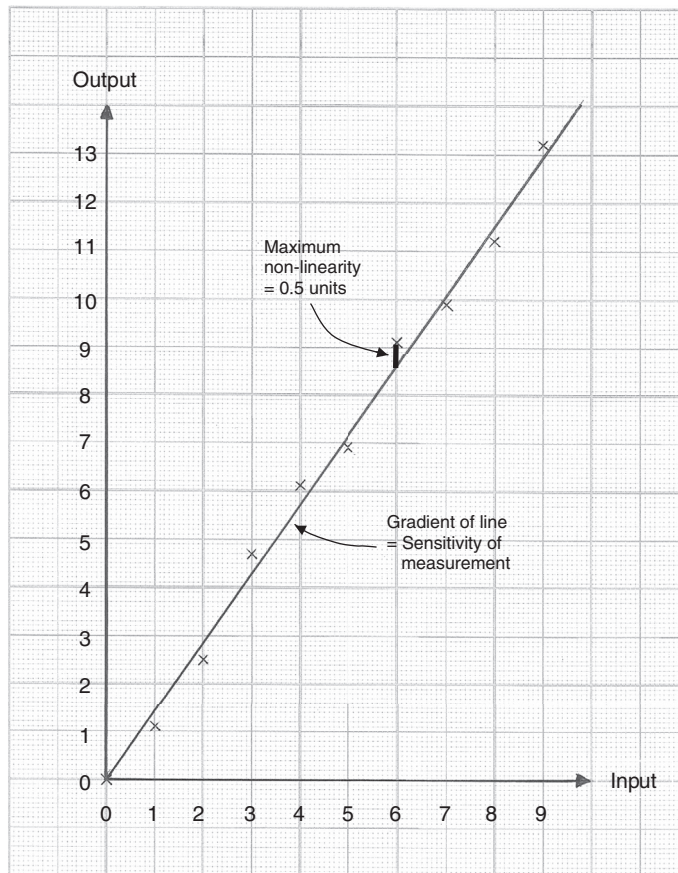


Figure 2.6
Instrument output characteristic.

shown in Fig. 2.6. (Although this can often be done with reasonable accuracy by eye, it is always preferable to apply a mathematical least-squares line-fitting technique, as described in Chapter 10.) The nonlinearity is then defined as the maximum deviation of any output reading marked X from this straight line. Nonlinearity is usually expressed as a percentage of full-scale reading.

■ Example 2.5

Suppose the instrument characteristic shown in Fig. 2.6 is that of a pressure sensor, in which the input units are expressed in bars from 1 to 9 bars and the output units are expressed in volts from 1 to 13 V.

- (a) What is the maximum nonlinearity expressed as a percentage of the full-scale deflection?
- (b) What is the resolution of the sensor as determined by the instrument characteristic given?

■ Solution

(a) The maximum nonlinearity is the maximum deviation of any data point on Fig. 2.6 away from a straight line drawn through the data points. This is shown by the thick vertical line drawn in Fig. 2.6. The length of this line is 0.5 units, which translates to 0.5 V. The full-scale deflection (calculated for the fitted straight line) is 13.0 units, which translates to 13.0 V.

The maximum nonlinearity can therefore be expressed as $\frac{0.5}{13} \times 100 = 3.8\%$ of the full-scale deflection.

(b) The resolution of the sensor as determined from the graph in Fig. 2.6 is the smallest change in input that is detectable. For the graph paper illustrated, the naked eye cannot determine anything smaller than one small square, which is one-tenth of a unit or 0.1 bar. This figure of 0.1 bar pressure is therefore the resolution of the sensor as determined from the graph.

Sensitivity of measurement: The sensitivity of measurement is a measure of the change in instrument output that occurs when the quantity being measured changes by a given amount. Thus, sensitivity is the ratio:

$$\frac{\text{scale deflection}}{\text{value of measurand producing deflection}}$$

The sensitivity of measurement is therefore the slope of the straight line drawn in Fig. 2.6. If, for example, a pressure of 2 bar produces a deflection of 10 degrees in a pressure transducer, the sensitivity of the instrument is 5 degrees/bar (assuming that the deflection is zero with zero pressure applied).

■ Example 2.6

The following resistance values of a platinum resistance thermometer were measured at a range of temperatures. Determine the measurement sensitivity of the instrument in ohms/°C.

Resistance (Ω)	Temperature ($^{\circ}\text{C}$)
307	200
314	230
321	260
328	290

■ Solution

If these values are plotted on a graph, the straight-line relationship between resistance change and temperature change is obvious.

For a change in temperature of 30°C, the change in resistance is 7 Ω . Hence the measurement sensitivity = $7/30 = 0.233 \Omega/^{\circ}\text{C}$.

Sensitivity to disturbance: All calibrations and specifications of an instrument are valid only under controlled conditions of temperature, pressure, and so on. These standard ambient conditions are usually defined in the instrument specification. As variations occur in the ambient temperature, and so forth, certain static instrument characteristics change, and the sensitivity to disturbance is a measure of the magnitude of this change. Such environmental changes affect instruments in two main ways, known as *zero drift* and *sensitivity drift*. Zero drift is sometimes known by the alternative term, *bias*.

Zero drift or bias describes the effect in which the zero reading of an instrument is modified by a change in ambient conditions. This causes a constant error that exists over the full range of measurement of the instrument. The mechanical form of bathroom scale

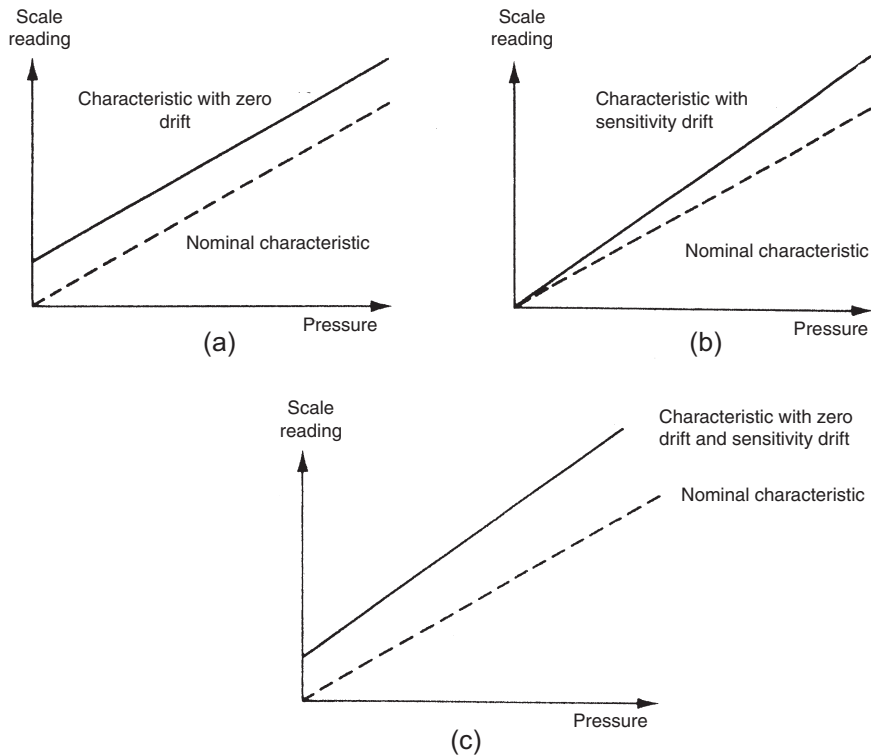


Figure 2.7

Effects of disturbance: (A) Zero drift. (B) Sensitivity drift. (C) Zero drift plus sensitivity drift.

is a common example of an instrument that is prone to zero drift. It is usual to find that there is a reading of perhaps 1 kg with no one standing on the scale. If someone known to weigh 70 kg were to get on the scale, the reading would be 71 kg, and if someone known to weigh 100 kg were to get on the scale, the reading would be 101 kg. Zero drift is normally removable by calibration. In the case of the bathroom scale just described, a thumbwheel is usually provided that can be turned until the reading is zero with the scales unloaded, thus removing the zero drift.

The typical unit by which such zero drift is measured is volts/ $^{\circ}$ C. This is often called the *zero drift coefficient* related to temperature changes. If the characteristic of an instrument is sensitive to several environmental parameters, it will have several zero drift coefficients, one for each environmental parameter. A typical change in the output characteristic of a pressure gauge subject to zero drift is shown in [Fig. 2.7A](#).

■ Example 2.7

This table shows the output measurements of a voltmeter under two sets of conditions:

- (a) use in an environment kept at 20°C, which is the temperature at which it was calibrated; and
- (b) use in an environment at 50°C.

Voltage readings at calibration temperature of 20°C (assumed correct)	Voltage readings at temperature of 50°C
10.2	10.5
20.3	20.6
30.7	31.0
40.8	41.1

Determine the zero drift when it is used in the 50°C environment, assuming that the measurement values when it was used in the 20°C environment are correct. Also calculate the zero drift coefficient.

■ Solution

The zero drift at 50°C is the constant difference between the pairs of output readings (i.e., 0.3 V).

The zero drift coefficient is the magnitude of drift (0.3 V) divided by the magnitude of the temperature change causing the drift (30°C). Thus, the zero drift coefficient is $0.3/30 = 0.01 \text{ V}/^\circ\text{C}$.

Sensitivity drift (also known as *scale factor drift*) defines the amount by which an instrument's sensitivity of measurement varies as ambient conditions change. It is quantified by sensitivity drift coefficients that define how much drift there is for a unit change in each environmental parameter to which the instrument characteristics are sensitive. Many components within an instrument are affected by environmental fluctuations, such as temperature changes: for instance, the modulus of elasticity of a spring is temperature-dependent. Fig. 2.7B shows what effect sensitivity drift can have on the output characteristic of an instrument. Sensitivity drift is measured in units of the form (angular degree/bar)/°C. If an instrument suffers both zero drift and sensitivity drift at the same time, the typical modification of the output characteristic is as shown in Fig. 2.7C.

■ Example 2.8

A spring balance is calibrated in an environment at 20°C and has the following deflection/load characteristic.

Load (kg)	0	1	2	3
Deflection (degrees)	0	20	40	60

It is then used in an environment at 30°C and the following deflection/load characteristic is measured.

Load (kg)	0	1	2	3
Deflection (degrees)	5	27	49	71

Determine the zero drift and sensitivity drift per degrees Celsius change in ambient temperature.

■ Solution

At 20°C, the deflection/load characteristic is a straight line. Sensitivity = 20 degrees/kg.

At 30°C, the deflection/load characteristic is still a straight line. Sensitivity = 22 degrees/kg.

Zero drift (bias) = 5 degrees (the no-load deflection).

Sensitivity drift = 2 degrees/kg.

Zero drift/°C = 5/10 = 0.5 degrees/°C.

Sensitivity drift/°C = 2/10 = 0.2 (degrees per kg)/°C.

Hysteresis effects: Fig. 2.8 illustrates the output characteristic of an instrument that exhibits *hysteresis*. If the input-measured quantity to the instrument is steadily increased from a negative value, the output reading varies in the manner shown in curve (a). If the input variable is then steadily decreased, the output varies in the manner shown in curve (b). The noncoincidence between these loading and unloading curves is known as *hysteresis*. Two quantities are defined, *maximum input hysteresis* and *maximum output hysteresis*, as shown in Fig. 2.8. These are normally expressed as a percentage of the full-scale input or output reading, respectively.

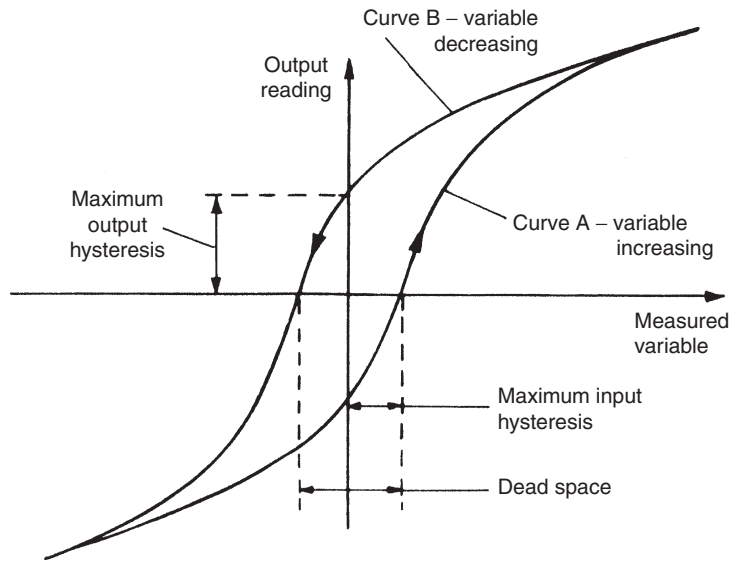


Figure 2.8
Instrument characteristic with hysteresis.

Hysteresis is most commonly found in instruments that contain springs, such as the passive pressure gauge (Fig. 2.1). It is also evident when friction forces in a system have different magnitudes depending on the direction of movement, such as in the pendulum-scale¹ mass-measuring device. Devices such as the mechanical flyball¹ (a device for measuring rotational velocity) suffer hysteresis from both of these sources because they have friction in moving parts and contain a spring. Hysteresis can also occur in instruments that contain electrical windings formed round an iron core, because of magnetic hysteresis in the iron. This occurs in devices such as the variable inductance displacement transducer, the linear variable differential transformer (LVDT) and the rotary differential transformer.

Dead space: Dead space is defined as the range of different input values over which there is no change in output value. Any instrument that exhibits hysteresis also displays dead space, as marked in Fig. 2.8. Some instruments that do not suffer from any significant hysteresis can still exhibit a dead space in their output characteristics, however. Backlash in gears is a typical cause of dead space and results in the sort of instrument output characteristic shown in Fig. 2.9. Backlash is commonly experienced in gear sets used to convert between translational and rotational motion (which is a common technique used to measure translational velocity).

¹ Both the pendulum scale and the mechanical flyball are old devices that are now rarely used.

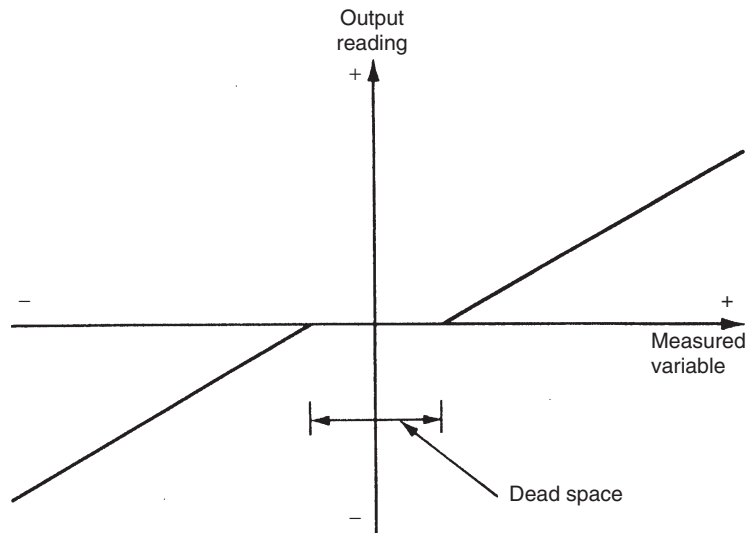


Figure 2.9
Instrument characteristic with dead space.

2.4 Dynamic characteristics of instruments

The static characteristics of measuring instruments are concerned only with the steady-state reading to which the instrument settles, such as the accuracy of the reading.

The dynamic characteristics of a measuring instrument describe its behavior between the time a measured quantity changes value and when the instrument output attains a steady value in response. As with static characteristics, any values for dynamic characteristics quoted in instrument data sheets apply only when the instrument is used under specified environmental conditions. Outside these calibration conditions, some variation in the dynamic parameters can be expected.

In any linear, time-invariant measuring system, the following general relation can be written between input and output for time (t) > 0:

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \dots + a_1 \frac{dq_o}{dt} + a_0 q_o = b_m \frac{d^m q_i}{dt^m} + b_{m-1} \frac{d^{m-1} q_i}{dt^{m-1}} + \dots + b_1 \frac{dq_i}{dt} + b_0 q_i \quad (2.1)$$

where q_i is the measured quantity, q_o is the output reading, and $a_o \dots a_n, b_o \dots b_m$ are constants.

The reader whose mathematical background is such that this equation appears daunting should not worry unduly, because only certain special, simplified cases of it are applicable under normal measurement situations. The major point of importance is to have a practical

appreciation of the manner in which various different types of instrument respond when the measurand applied to them varies.

If we limit consideration only to that of step changes in the measured quantity, Eq. (2.1) reduces to:

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \dots + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \quad (2.2)$$

Further simplification can be made by taking certain special cases of Eq. (2.2), which collectively apply to nearly all measurement systems.

2.4.1 Zero-order instrument

If all coefficients $a_1 \dots a_n$ other than a_0 in Eq. (2.2) are assumed to be zero:

$$a_0 q_o = b_0 q_i \quad \text{or} \quad q_o = b_0 q_i / a_0 = K q_i \quad (2.3)$$

where K is a constant known as the instrument sensitivity, as defined earlier.

Any instrument that behaves according to Eq. (2.3) is said to be of zero-order type. After a step change in the measured quantity at time t , the instrument output moves immediately to a new value at the same time instant t , as shown in Fig. 2.10. A potentiometer, which measures motion, is a good example of such an instrument, in which the output voltage changes instantaneously as the slider is displaced along the potentiometer track.

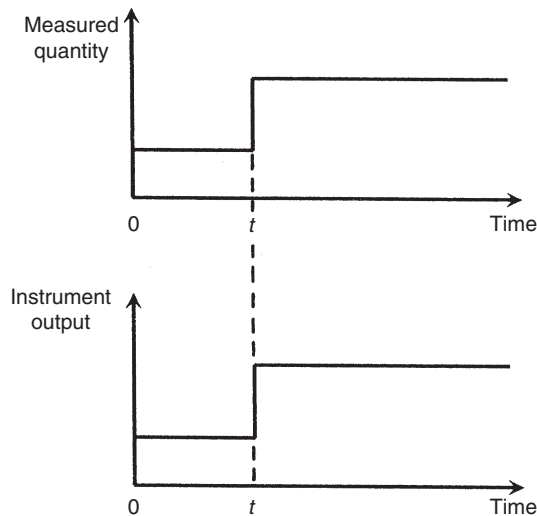


Figure 2.10
Zero-order instrument characteristic.

2.4.2 First-order instrument

If all coefficients $a_2 \dots a_n$ except for a_0 and a_1 are assumed to be zero in Eq. (2.2) then:

$$a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \quad (2.4)$$

Any instrument that behaves according to Eq. (2.4) is known as a first-order instrument. If d/dt is replaced by the D operator in Eq. (2.4), we get:

$$a_1 D q_o + a_0 q_o = b_0 q_i$$

and rearranging this then gives:

$$q_o = \frac{(b_0/a_0)q_i}{[1 + (a_1/a_0)D]} \quad (2.5)$$

Defining $K = b_0/a_0$ as the static sensitivity and $\tau = a_1/a_0$ as the time constant of the system, Eq. (2.5) becomes:

$$q_o = \frac{Kq_i}{1 + \tau D} \quad (2.6)$$

If Eq. (2.6) is solved analytically, the output quantity q_o in response to a step change in q_i at time t varies with time in the manner shown in Fig. 2.11. The time constant τ of the step response is the time taken for the output quantity q_o to reach 63% of its final value.

The thermocouple (see Chapter 14) is a good example of a first-order instrument. It is well-known that if a thermocouple at room temperature is plunged into boiling water, the output voltage does not rise instantaneously to a level indicating 100°C but instead approaches a reading indicating 100°C in a manner similar to that shown in Fig. 2.11.

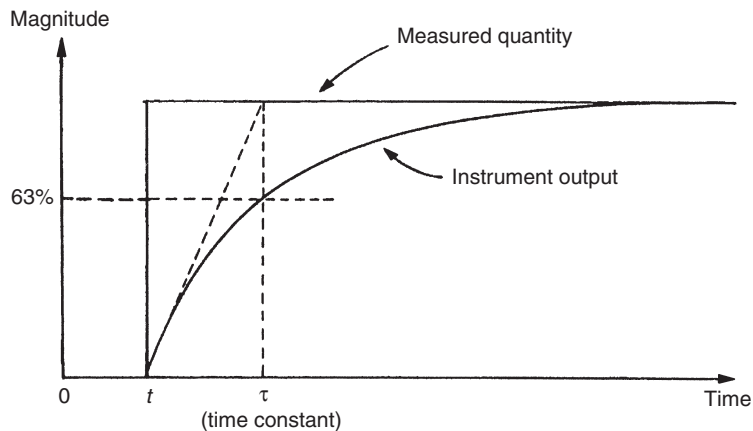


Figure 2.11
First-order instrument characteristic.

A large number of other instruments also belong to this first-order class. This is particularly important in control systems, in which it is necessary to consider the time lag that occurs between a measured quantity changing in value and the measuring instrument indicating the change. Fortunately, the time constant of many first-order instruments is small relative to the dynamics of the process being measured, and so no serious problems are created.

■ Example 2.9

A balloon is equipped with temperature- and altitude-measuring instruments and has radio equipment that can transmit the output readings of these instruments back to ground. The balloon is initially anchored to the ground with the instrument output readings in steady-state. The altitude-measuring instrument is approximately zero-order and the temperature transducer first-order with a time constant of 15 seconds. The temperature on the ground, T_0 , is 10°C and the temperature T_x at an altitude of x meters is given by the relation: $T_x = T_0 - 0.01x$

(a) If the balloon is released at time 0, and thereafter rises upward at a velocity of 5 m/s, draw a table showing the temperature and altitude measurements reported at intervals of 10 seconds over the first 50 seconds of travel. Show also in the table the error in each temperature reading.

(b) What temperature does the balloon report at an altitude of 5000 m?

■ Solution

To answer this question, it is assumed that the solution of a first-order differential equation has been presented to the reader in a mathematics course. If the reader is not so equipped, the solution presented next will be difficult to follow.

Let the temperature reported by the balloon at some general time t be T_r . Then, T_x is related to T_r by the relation:

$$T_r = \frac{T_x}{1 + \tau D} = \frac{T_0 - 0.01x}{1 + \tau D} = \frac{10 - 0.01x}{1 + 15D}$$

The balloon rises at 5 m/s; hence, $x = 5t$, and thus

$$T_r = \frac{10 - 0.05t}{1 + 15D}$$

The transient or complementary function part of the solution ($T_x = 0$) is given by:

$$T_{r_{cf}} = Ce^{-t/15}.$$

The particular integral part of the solution is given by: $T_{r_{pi}} = 10 - 0.05(t - 15)$.

Thus, the whole solution is given by: $T_r = T_{r_{cf}} + T_{r_{pi}} = Ce^{-t/15} + 10 - 0.05(t - 15)$.

Applying initial conditions: At $t = 0$, $T_r = 10$, i.e., $10 = Ce^{-0} + 10 - 0.05(-15)$.

Thus $C = -0.75$ and the solution can be written as:

$$T_r = 10 - 0.75e^{-t/15} - 0.05(t - 15).$$

Using this expression to calculate T_r for various values of t , this table can be constructed:

Time	Altitude	Temperature reading	Temperature error
0	0	10	0
10	50	9.86	0.36
20	100	9.55	0.55
30	150	9.15	0.65
40	200	8.70	0.70
50	250	8.22	0.72

(b) At 5000 m, $t = 1000$ s. Calculating T_r from this expression:

$$T_r = 10 - 0.75e^{-1000/15} - 0.05(1000 - 15)$$

The exponential term approximates to zero, and so T_r can be written as: $T_r \approx 10 - 0.05(985) = -39.25^\circ\text{C}$.

This result might have been inferred from this table, where it can be seen that the error is converging toward a value of 0.75. For large values of t , the transducer reading lags the true temperature value by a period of time equal to the time constant of 15 seconds. In this time, the balloon travels a distance of 75 m and the temperature falls by 0.75°C . Thus, for large values of t , the output reading is always 0.75°C less than it should be. ■

2.4.3 Second-order instrument

If all coefficients $a_3 \dots a_n$ other than a_0 , a_1 , and a_2 in Eq. (2.2) are assumed to be zero, we get:

$$a_2 \frac{d^2 q_o}{dt^2} + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \quad (2.7)$$

Applying the D operator again: $a_2D^2q_o + a_1Dq_o + a_0q_o = b_0q_i$, and rearranging:

$$q_o = \frac{b_0q_i}{a_0 + a_1D + a_2D^2} \quad (2.8)$$

It is convenient to reexpress the variables a_0 , a_1 , a_2 , and b_0 in Eq. (2.8) in terms of three parameters: K (static sensitivity), ω (undamped natural frequency), and ξ (damping ratio), where:

$$K = b_0/a_0 \quad ; \quad \omega = \sqrt{a_0/a_2} \quad ; \quad \xi = a_1/2\sqrt{a_0a_2}$$

$$\xi \text{ can be written as } \xi = \frac{a_1}{2a_0\sqrt{a_2/a_0}} = \frac{a_1\omega}{2a_0}.$$

If Eq. (2.8) is now divided by a_0 , we get:

$$q_o = \frac{(b_0/a_0)q_i}{1 + (a_1/a_0)D + (a_2/a_0)D^2} \quad (2.9)$$

The terms in Eq. (2.9) can be written in terms of ω and ξ as follows:

$$\frac{b_0}{a_0} = K \quad ; \quad \left(\frac{a_1}{a_0}\right)D = \frac{2\xi D}{\omega} \quad ; \quad \left(\frac{a_2}{a_0}\right)D^2 = \frac{D^2}{\omega^2}$$

Hence, dividing Eq. (2.9) by q_i and substituting for a_0 , a_1 , and a_2 gives:

$$\frac{q_o}{q_i} = \frac{K}{D^2/\omega^2 + 2\xi D/\omega + 1} \quad (2.10)$$

This is the standard equation for a second-order system, and any instrument whose response can be described by it is known as a second-order instrument. If Eq. (2.10) is solved analytically, the shape of the step response obtained depends on the value of the damping ratio parameter ξ . The output responses of a second-order instrument for various values of ξ after a step change in the value of the measured quantity at time t are shown in Fig. 2.12. For case (A) in which $\xi = 0$, there is no damping and the instrument output exhibits constant amplitude oscillations when disturbed by any change in the physical quantity measured. For light damping of $\xi = 0.2$, represented by case (B), the response to a step change in input is still oscillatory, but the oscillations gradually die down. A further increase in the value of ξ reduces oscillations and overshoot still more, as shown by curves (C) and (D), and finally the response becomes overdamped as shown by curve (E), in which the output reading creeps slowly toward the correct reading. Clearly, the extreme response curves (A) and (E) are grossly unsuitable for any measuring instrument. If an instrument were to be only subjected to step inputs, the design strategy would be to aim toward a damping ratio of 0.707, which gives the critically damped response (C).

Unfortunately, most of the physical quantities that instruments are required to measure do not change in the mathematically convenient form of steps, but rather in the form of ramps of varying slopes. As the form of the input variable changes, so the best value for ξ varies

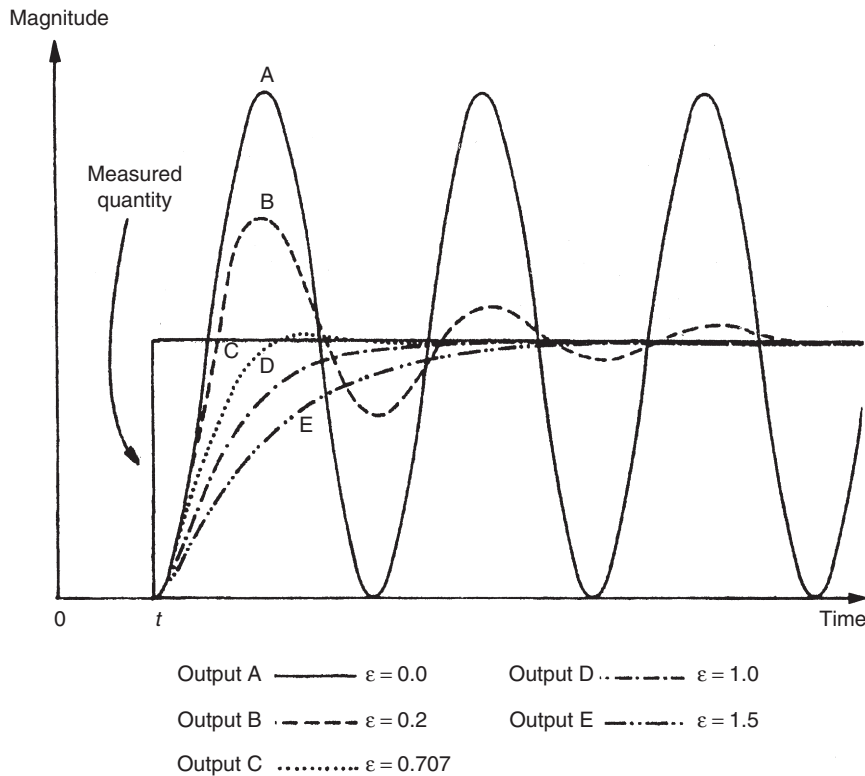


Figure 2.12

Response characteristics of second-order instruments.

and the choice of ξ becomes one of compromise between values that are best for each type of input variable behavior anticipated. Commercial second-order instruments, of which the accelerometer is a common example, are generally designed to have a damping ratio (ξ) somewhere in the range of 0.6–0.8.

2.5 Necessity for calibration

This discussion has described the static and dynamic characteristics of measuring instruments in some detail. However, an important qualification that has been omitted is that an instrument conforms to stated static and dynamic patterns of behavior only after it has been calibrated. It can normally be assumed that a new instrument will have been calibrated when it is obtained from an instrument manufacturer and will therefore initially behave according to the characteristics stated in the specifications. During use, however, its behavior will gradually diverge from the stated specification for a variety of reasons, including mechanical wear and the effects of dirt, dust, fumes, and chemicals in the operating environment. The rate of divergence from standard specifications varies

according to the type of instrument, the frequency of use, and the severity of the operating conditions. However, there will come a time, determined by practical knowledge, when the characteristics of the instrument will have drifted from the standard specification by an unacceptable amount. When this situation is reached, it is necessary to recalibrate the instrument back to the standard specifications. Such recalibration is performed by adjusting the instrument at each point in its output range until its output readings are the same as those of a second standard instrument to which the same inputs are applied. This second instrument is one kept solely for calibration purposes whose specifications are accurately known. Calibration procedures are discussed more fully in Chapter 5.

2.6 Summary

This chapter started by reviewing various different classes of instruments and considering how these differences affect their typical use. We saw, for example, that null-type instruments are favored for calibration duties because of their superior accuracy, whereas deflection-type instruments are easier to use for routine measurements. We also looked at the distinction between active and passive instruments, analog and digital instruments, indicators and signal-output type instruments and, finally, smart and nonsmart instruments. After this, we examined the various static characteristics of instruments. These define the quality of measurements when an instrument output has settled to a steady reading. Several important lessons arose from this coverage. In particular, we saw the important distinction between accuracy and precision, which are often incorrectly taken to mean the same thing. We saw that high precision does not promise anything about measurement accuracy, and in fact a high-precision instrument can sometimes give poor measurement accuracy. The final topic covered in this chapter was the dynamic characteristics of instruments. We saw that there are three kinds of dynamic characteristic, zero order, first order, and second order. Analysis of these showed that both first- and second-order instruments take time to settle to a steady-state reading when the measured quantity changes. It is therefore necessary to wait until the dynamic motion has ended before a reading is recorded. This places a serious limitation on the use of first- and second-order instruments to make repeated measurements. Clearly, the frequency of repeated measurements is limited by the time taken by the instrument to settle to a steady-state reading.

2.7 Problems

- 2.1 Briefly explain four ways in which measuring instruments can be subdivided into different classes according to their mode of operation, giving examples of instruments that fall into each class.

- 2.2 Explain what is meant by:
- (a) active instruments
 - (b) passive instruments
- Give examples of each and discuss the relative merits of these two classes of instruments.
- 2.3 Discuss the advantages and disadvantages of null and deflection types of measuring instrument. What are null types of instrument mainly used for, and why?
- 2.4 Explain the differences between analog and digital instruments and give examples of each type. What advantages do digital instruments have over analog ones?
- 2.5 Explain the difference between static and dynamic characteristics of measuring instruments.
- 2.6 Briefly define and explain all of the static characteristics of measuring instruments.
- 2.7 How is the accuracy of an instrument usually defined? Explain the difference between accuracy and precision, using sketches as appropriate to illustrate the differences.
- 2.8 Draw sketches to illustrate the dynamic characteristics of:
- (a) zero-order instrument
 - (b) first-order instrument
 - (c) second-order instrument
- In the case of a second-order instrument, indicate the effect of different degrees of damping on the time response.
- 2.9 State briefly how the dynamic characteristics of an instrument affect its use.
- 2.10 Define the measurement inaccuracy of a measuring instrument. If a tungsten resistance thermometer with a range of -270°C to $+1100^{\circ}\text{C}$ has a quoted inaccuracy of $\pm 1.5\%$ of full-scale reading, what is the likely measurement error when it is reading a temperature of 950°C ?
- 2.11 What is meant by the term “tolerance” in manufacturing systems? If a batch of steel rods is manufactured to a nominal length of 5 m with a quoted tolerance of $\pm 2\%$, what is the longest and shortest length of rod to be expected in the batch?
- 2.12 A manganin wire pressure sensor has a measurement range of 0–20,000 bar and a quoted inaccuracy of $\pm 1\%$ of full-scale deflection. What is the maximum measurement error when the instrument is reading a pressure of 15,000 bar?
- 2.13 What do you understand by the word “tolerance”? If a batch of ceramic bricks in manufactured with a tolerance of $\pm 1.5\%$, what is the length of the shortest and longest bricks to be expected in the batch?
- 2.14 What is the measurement range for a micrometer designed to measure diameters between 7.5 and 10.0 cm?

- 2.15 A pressure gauge is designed to measure absolute pressures between 0 and 50 bar. If its inaccuracy is quoted as $\pm 0.5\%$ of full-scale reading, calculate (a) its measurement range and (b) the possible measurement error when it is reading a pressure of 10 bar.
- 2.16 A packet of resistors bought in an electronics component shop gives the nominal resistance value as $5000\ \Omega$ and the manufacturing tolerance as $\pm 3\%$. If one resistor is chosen at random from the packet, what is the minimum and maximum resistance value that this particular resistor is likely to have?
- 2.17 A pressure gauge with a measurement range of 0–30 bar has a quoted inaccuracy of $\pm 0.5\%$ of the full-scale reading.
- What is the maximum measurement error expected for this instrument?
 - What is the likely measurement error expressed as a percentage of the output reading if this pressure gauge is measuring a pressure of 5 bar?
 - If the measurement error when reading a pressure of 5 bar is deemed to be too high, what are the two main options for reducing the measurement error? Which of these two options would you recommend, and why would you recommend it?
- 2.18 The width of a room is measured 10 times by an ultrasonic rule and the following measurements are obtained (in meters): 4.292, 4.295, 4.296, 4.293, 4.292, 4.294, 4.293, 4.290, 4.294, and 4.291.
- The width of the same room is then measured by a calibrated steel tape that gives a reading of 4.276 m, which can be taken as the correct value for the width of the room.
- What is the measurement precision of the ultrasonic rule?
 - What is the maximum measurement inaccuracy of the ultrasonic rule?
- 2.19 (a) Explain the difference between precision and accuracy, illustrating your answer with sketches as appropriate.
- (b) A person uses a set of spring-based (analog) bathroom scales to measure his weight. He gets on and off the scales 10 times and obtains the following measurements (in kilograms): 70.1, 69.6, 69.4, 70.4, 69.9, 69.6, 70.2, 70.7, 69.4, and 69.7.
- He then uses a set of calibrated electronic scales to measure his weight that, for the purposes of this exercise, are assumed to have zero error. These scales give a reading of 70.5 kg.
- Calculate (i) the precision of the spring-based scales and (ii) their inaccuracy.
- 2.20 A tungsten/5% rhenium–tungsten/26% rhenium thermocouple has an output electromagnetic field as shown in the following table when its hot (measuring) junction is at the temperatures shown. Determine the sensitivity of measurement for the thermocouple in $\text{mV}/^\circ\text{C}$.

mV	4.37	8.74	13.11	17.48
$^\circ\text{C}$	250	500	750	1000

2.21 A set of known pressures was applied to a pressure gauge and the output voltages were recorded as shown in the following table:

Pressure (bar)	10	30	50	70	100	120	140	160	180	200
Output (volts)	0.45	1.44	2.37	3.25	4.77	5.58	6.49	7.45	8.50	9.46

What is the sensitivity of measurement of the pressure gauge?

2.22 The calibration of a pressure sensor with a range of 0–16 bar is checked by applying known inputs to it in steps of 2 bar. The following measurements were obtained:

Known input pressure (bar)	0	2	4	6	8	10	12	14	16
Output reading (volts)	0.00	0.55	1.25	1.70	2.35	3.15	3.65	4.05	4.85

- (a) Draw a graph by plotting the data points on the graph paper provided and draw a best-fit straight line through the data points. Use this graph to calculate:
- (b) the measurement sensitivity of the pressure sensor.
- (c) the maximum nonlinearity apparent in the measurements as a percentage of the full-scale deflection.

2.23 Define sensitivity drift and zero drift. What factors can cause sensitivity drift and zero drift in instrument characteristics?

2.24 The calibration of a spring balance is checked by applying a set of known masses to it, with the following results:

Known input (grams)	10	20	30	40	50	60	70	80
Output reading (grams)	15.5	25.0	32.0	45.0	52.0	61.0	74.5	81.0

- (a) Draw a graph by plotting the data points on the graph paper provided and draw a best-fit straight line through the data points.
- (b) Which of the following static measurement characteristics are evident in the input–output graph drawn: nonlinearity, dead space, sensitivity drift, threshold, zero drift, hysteresis?
- (c) Calculate the magnitude of the static measurement characteristics that you have just identified.

2.25 Explain the terms “zero drift” and “zero-drift coefficient.”

The following table shows the output measurements of a pressure sensor under two sets of conditions:

- (a) when used in an environment kept at 20°C, which is the temperature at which it was calibrated; and

- (b) when used in an environment at 30°C.

Pressure readings at calibration temperature of 20°C (assumed correct) in units of pascal	Pressure readings at 30°C in units of pascal
1.31	1.50
2.46	2.65
3.63	3.82
4.82	5.01
6.27	6.46

Determine the zero drift when it is used in the 30°C environment, assuming that the measurement values when it was used in the 20°C environment are correct. Also calculate the zero-drift coefficient.

- 2.26 (a) An instrument is calibrated in an environment at 20°C and the following output readings y are obtained for various input values x :

y	13.1	26.2	39.3	52.4	65.5	78.6
x	5	10	15	20	25	30

Determine the measurement sensitivity, expressed as the ratio y/x .

- (b) When the instrument is subsequently used in an environment at 50°C, the input–output characteristic changes to the following:

y	14.7	29.4	44.1	58.8	73.5	88.2
x	5	10	15	20	25	30

Determine the new measurement sensitivity. Then determine the sensitivity drift due to the change in ambient temperature of 30°C.

- 2.27 A spring balance is calibrated in an environment at 20°C and has the following deflection/load characteristic:

Load (kg)	0	0.4	0.8	1.2	1.6	2.0
Deflection (degrees)	0	36	72	108	144	180

It is then used in an environment at 27°C and the following deflection/load characteristic is measured:

Load (kg)	0	0.40	0.8	1.2	1.6	2.0
Deflection (degrees)	6	43	80	117	154	191

Determine the zero drift and sensitivity drift per degrees Celsius change in ambient temperature.

- 2.28 The following temperature measurements were taken with an infrared thermometer that produced biased measurements owing to the instrument being out of calibration. Calculate the bias in the measurements.

Values measured by uncalibrated instrument (°C)	Correct value of temperature (°C)
20	21.5
35	36.5
50	51.5
65	66.5

- 2.29 A spiral Bourdon tube was found to be damaged. When its calibration was checked, the following readings were obtained:

Indicated pressure (bar)	100	200	300	400	500
Correct pressure (bar)	103	203	303	403	503

What is the bias in the readings?

- 2.30 A sensor was calibrated at a temperature of 20°C and the following input (x) and output (y) data were recorded:

Input (x)	0	20	40	60	80	100
Output (y)	0	31	62	93	124	155

It was then recalibrated at a temperature of 40°C and the following input (x) and output (y) data were recorded:

Input (x)	0	20	40	60	80	100
Output (y)	0.5	32.0	63.5	95.0	126.5	158.0

Calculate the zero drift in the sensor caused by the change in temperature from 20°C to 40°C. Also, calculate the measurement sensitivity of the sensor at the two temperatures and the total sensitivity drift.

- 2.31 A load cell is calibrated in an environment at a temperature of 21°C and has the following deflection/load characteristic:

Load (kg)	0	50	100	150	200
Deflection (mm)	0.0	1.0	2.0	3.0	4.0

When used in an environment at 35°C, its characteristic changes to the following:

Load (kg)	0	50	100	150	200
Deflection (mm)	0.2	1.3	2.4	3.5	4.6

- (a) Determine the sensitivity at 21°C and 35°C
- (b) Calculate the total zero drift and sensitivity drift at 35°C.

(c) Then determine the zero drift and sensitivity drift coefficients (in units of $\mu\text{m}/^\circ\text{C}$ and $(\mu\text{m per kg})/^\circ\text{C}$).

2.32 A balloon is equipped with temperature- and altitude-measuring instruments and has radio equipment that can transmit the output readings of these instruments back to ground. The balloon is initially anchored to the ground with the instrument output readings in steady-state. The altitude-measuring instrument is approximately zero order and the temperature transducer is approximately first order with a time constant of 10 seconds. The temperature on the ground, T_0 , is 20°C and the temperature T_h at an altitude of x meters is given by the relation: $T_h = T_0 - 0.012 h$

(a) If the balloon is released at time 0, and thereafter rises upward at a velocity of 6 m/s, draw a table showing the temperature and altitude measurements reported at intervals of 10 seconds over the first 100 seconds of travel. Show also in the table the error in each temperature reading.

(b) What temperature does the balloon report at an altitude of 8000 m?

(c) What is the error in the temperature reading at this altitude of 8000 m?

2.33 The time response of a liquid-in-glass thermometer was tested by plunging it into a container of boiling water and recording the temperature at intervals of 10 seconds over 2 minutes. The thermometer was initially at 0°C and the following readings were recorded:

Time (seconds)	0	10	20	30	40	50	60	70	80	90	100	110	120
Temp ($^\circ\text{C}$)	0	31	52	67	77	84	89	92	95	96	97	98	99

Draw a graph of the output readings against time. Using this graph, estimate the time constant of the thermometer.

2.34 An unmanned submarine is equipped with temperature- and depth-measuring instruments and has radio equipment that can transmit the output readings of these instruments back to the surface. The submarine is initially floating on the surface of the sea with the instrument output readings in steady-state. The depth-measuring instrument is approximately zero order and the temperature transducer first order with a time constant of 50 seconds. The water temperature on the sea surface, T_0 , is 20°C and the temperature T_x at a depth of x meters is given by the relation:

$$T_x = T_0 - 0.01x$$

(a) If the submarine starts diving at time 0 and thereafter goes down at a velocity of 0.5 m/s, draw a table showing the temperature and depth measurements reported at intervals of 100 seconds over the first 500 seconds of travel. Show also in the table the error in each temperature reading.

(b) What temperature does the submarine report at a depth of 1000 m?

2.35 Write the general differential equation describing the dynamic response of a second-order measuring instrument and state the expressions relating the static sensitivity,

undamped natural frequency, and damping ratio to the parameters in this differential equation. Sketch the instrument response for the cases of heavy damping, critical damping, and light damping, and state which of these is the usual target when a second-order instrument is being designed.

- 2.36 A sheathed thermocouple with a time constant of 4.0 seconds is initially at a room temperature of 20°C . It is then plunged into a beaker of boiling water at 100°C . Assuming that the thermocouple has a first-order characteristic, what temperature would it indicate when it had been in the water for 8.0 seconds?

Measurement uncertainty

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3.1 Introduction

We have already been introduced to the subject of measurement uncertainty in the last chapter, in the context of defining the accuracy characteristic of a measuring instrument. The existence of measurement uncertainty means that we would be entirely wrong to assume (though the uninitiated might assume this) that the output of a measuring instrument or larger measurement system gives the exact value of the measured quantity. Measurement errors are impossible to avoid, although we can minimize their magnitude by good measurement system design accompanied by appropriate analysis and processing of measurement data.

We can divide errors in measurement systems into those that arise during the measurement process and those that arise due to later corruption of the measurement signal by induced noise during transfer of the signal from the point of measurement to some other point. [Sections 3.1–3.5](#) cover the errors occurring during the measurement process, followed by discussion of induced noise in [Sections 3.6 and 3.7](#).

It is extremely important in any measurement system to reduce errors to the minimum possible level and then to quantify the maximum remaining error that may exist in any instrument output reading. However, in many cases, there is a further complication that the final output from a measurement system is calculated by combining together two or more measurements of separate physical variables. In this case, special consideration must also be given to determining how the calculated error levels in each separate measurement should be combined together to give the best estimate of the most likely error magnitude in the calculated output quantity. This subject is considered later in Chapter 4.

The starting point in the quest to reduce the incidence of errors arising during the measurement process is to carry out a detailed analysis of all error sources in the system. Each of these error sources can then be considered in turn, looking for ways of eliminating or at least reducing the magnitude of errors. Errors arising during the measurement process can be divided into two groups, known as systematic errors and random errors.

Systematic errors describe errors in the output readings of a measurement system that are consistently on one side of the correct reading, i.e., either all the errors are positive or they are all negative. (N.B. Some books use the alternative name bias errors for systematic errors, although this is not entirely correct since systematic errors include errors like sensitivity drift that are not biases.) Two major sources of systematic errors are system disturbance during measurement and the effect of environmental changes (sometimes known as *modifying inputs*), as discussed in [Sections 3.4.1 and 3.4.2](#). Other sources of

systematic error include bent meter needles, the use of uncalibrated instruments, drift in instrument characteristics, and poor cabling practices. Even when systematic errors due to the above factors have been reduced or eliminated, some errors remain that are inherent in the manufacture of an instrument. These are quantified by the accuracy value quoted in the published specifications contained in the instrument data sheet.

Random errors, which are also called as *precision errors* in some books, are perturbations of the measurement either side of the true value caused by random and unpredictable effects, such that positive errors and negative errors occur in approximately equal numbers for a series of measurements made of the same quantity. Such perturbations are mainly small, but large perturbations occur from time to time, again unpredictably. Random errors often arise when measurements are taken by human observation of an analog meter, especially where this involves interpolation between scale points. Electrical noise can also be a source of random errors. To a large extent, random errors can be overcome by taking the same measurement a number of times and extracting a value by averaging or other statistical techniques, as discussed later in Chapter 4. However, any quantification of the measurement value and statement of error bounds remains a statistical quantity. Because of the nature of random errors and the fact that large perturbations in the measured quantity occur from time to time, the best that we can do is to express measurements in probabilistic terms; we may be able to assign a 95% or even 99% confidence level that the measurement is a certain value within error bounds of say $\pm 1\%$, but we can never attach a 100% probability to measurement values that are subject to random errors. In other words, even if we say that the maximum error is $\leq \pm 0.5\%$ of the measurement reading, there is still a 1% chance that the error is greater than $\pm 0.5\%$.

Finally, a word must be said about the distinction between systematic and random errors. Error sources in the measurement system must be examined carefully to determine what type of error is present, systematic or random, and to apply the appropriate treatment. In the case of manual data measurements, a human observer may make a different observation at each attempt, but it is often reasonable to assume that the errors are random and that the mean of these readings is likely to be close to the correct value. However, this is only true as long as the human observer is not introducing a parallax-induced systematic error as well by persistently reading the position of a needle against the scale of an analog meter from one side rather than from directly above. A human-induced systematic error is also introduced if an instrument with a first-order characteristic is read before it has settled to its final reading. Wherever a systematic error exists alongside random errors, correction has to be made for the systematic error in the measurements before statistical techniques are applied to reduce the effect of random errors.

3.2 Sources of systematic error

The main sources of systematic error in the output of measuring instruments can be summarized as:

- (i) effect of environmental disturbances, often called modifying inputs
- (ii) disturbance of the measured system by the act of measurement
- (iii) changes in characteristics due to wear in instrument components over a period of time
- (iv) resistance of connecting leads

These various sources of systematic error, and ways in which the magnitude of the errors can be reduced, are discussed next.

3.2.1 System disturbance due to measurement

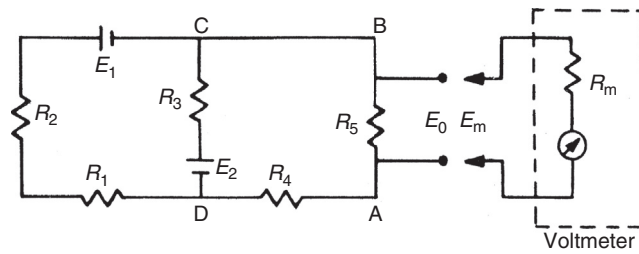
Disturbance of the measured system by the act of measurement is a common source of systematic error. If we were to start with a beaker of hot water and wished to measure its temperature with a mercury-in-glass thermometer, then we would take the thermometer, which would initially be at room temperature, and plunge it into the water. In so doing, we would be introducing a relatively cold mass (the thermometer) into the hot water and a heat transfer would take place between the water and the thermometer. This heat transfer would lower the temperature of the water. While the reduction in temperature in this case would be so small as to be undetectable by the limited measurement resolution of such a thermometer, the effect is finite and clearly establishes the principle that, in nearly all measurement situations, the process of measurement disturbs the system and alters the values of the physical quantities being measured.

A particularly important example of this occurs with the orifice plate. This is placed into a fluid-carrying pipe to measure the flow rate, which is a function of the pressure that is measured either side of the orifice plate. This measurement procedure causes a permanent pressure loss in the flowing fluid. The disturbance of the measured system can often be very significant.

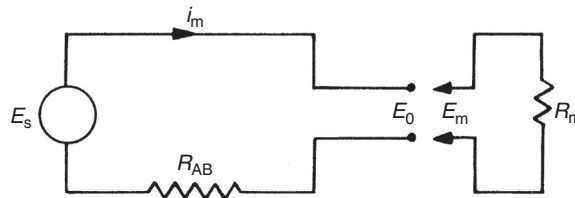
Thus, as a general rule, the process of measurement always disturbs the system being measured. The magnitude of the disturbance varies from one measurement system to the next and is affected particularly by the type of instrument used for measurement. Ways of minimizing disturbance of measured systems is an important consideration in instrument design. However, an accurate understanding of the mechanisms of system disturbance is a prerequisite for this.

Measurements in electric circuits

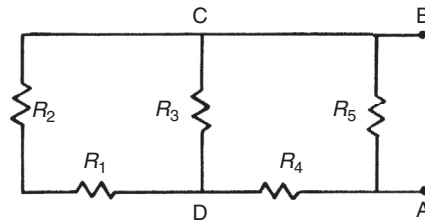
In analyzing system disturbance during measurements in electric circuits, Thévenin's theorem (see appendix 2) is often of great assistance. For instance, consider the circuit



(a)



(b)



(c)

Figure 3.1

Analysis of circuit loading: (a) A circuit in which the voltage across R_5 is to be measured; (b) Equivalent circuit by Thévenin's theorem; (c) The circuit used to find the equivalent single resistance R_{AB} .

shown in Fig. 3.1a in which the voltage across resistor R_5 is to be measured by a voltmeter with resistance R_m . Here, R_m acts as a shunt resistance across R_5 , decreasing the resistance between points AB and so disturbing the circuit. Therefore, the voltage E_m measured by the meter is not the value of the voltage E_o that existed prior to measurement. The extent of the disturbance can be assessed by calculating the open-circuit voltage E_o and comparing it with E_m .

Thévenin's theorem allows the circuit of Fig. 3.1a comprising two voltage sources and five resistors to be replaced by an equivalent circuit containing a single resistance and one voltage source, as shown in Fig. 3.1b. For the purpose of defining the equivalent single

resistance of a circuit by Thévenin's theorem, all voltage sources are represented just by their internal resistance, which can be approximated to zero, as shown in Fig. 3.1c. Analysis proceeds by calculating the equivalent resistances of sections of the circuit and building these up until the required equivalent resistance of the whole of the circuit is obtained. Starting at C and D , the circuit to the left of C and D consists of a series pair of resistances (R_1 and R_2) in parallel with R_3 , and the equivalent resistance can be written as:

$$\frac{1}{R_{CD}} = \frac{1}{R_1 + R_2} + \frac{1}{R_3} \quad \text{or} \quad R_{CD} = \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3}$$

Moving now to A and B , the circuit to the left consists of a pair of series resistances (R_{CD} and R_4) in parallel with R_5 . The equivalent circuit resistance R_{AB} can thus be written as:

$$\frac{1}{R_{AB}} = \frac{1}{R_{CD} + R_4} + \frac{1}{R_5} \quad \text{or} \quad R_{AB} = \frac{(R_4 + R_{CD})R_5}{R_4 + R_{CD} + R_5}$$

Substituting for R_{CD} using the expression derived previously, we obtain:

$$R_{AB} = \frac{\left[\frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3} + R_4 \right] R_5}{\frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3} + R_4 + R_5} \quad (3.1)$$

Defining I as the current flowing in the circuit when the measuring instrument is connected to it, we can write: $I = \frac{E_o}{R_{AB} + R_m}$, and the voltage measured by the meter is then given

$$\text{by: } E_m = \frac{R_m E_o}{R_{AB} + R_m}.$$

In the absence of the measuring instrument and its resistance R_m , the voltage across AB would be the equivalent circuit voltage source whose value is E_o . The effect of measurement is therefore to reduce the voltage across AB by the ratio given by:

$$\frac{E_m}{E_o} = \frac{R_m}{R_{AB} + R_m} \quad (3.2)$$

It is thus obvious that as R_m gets larger, the ratio E_m/E_o gets closer to unity, showing that the design strategy should be to make R_m as high as possible to minimize disturbance of the measured system. (Note that we did not calculate the value of E_o , since this is not required in quantifying the effect of R_m .)

■ Example 3.1

Suppose that the components of the circuit shown in Fig. 3.1a have the following values: $R_1 = 400\Omega$; $R_2 = 600\Omega$; $R_3 = 1000\Omega$, $R_4 = 500\Omega$; $R_5 = 1000\Omega$. The voltage across AB is measured by a voltmeter whose internal resistance is 9500Ω . What is the measurement error caused by the resistance of the measuring instrument?

■ Solution

Proceeding by applying Thévenin's theorem to find an equivalent circuit to that of Fig. 3.1a of the form shown in Fig. 3.1b, and substituting the given component values into the equation for R_{AB} (Eq. 3.1), we obtain:

$$R_{AB} = \frac{[(1000^2/2000) + 500]1000}{(1000^2/2000) + 500 + 1000} = \frac{1000^2}{2000} = 500\Omega$$

From Eq. (3.2), we have:

$$\frac{E_m}{E_o} = \frac{R_m}{R_{AB} + R_m}$$

The measurement error is given by $(E_o - E_m)$: $E_o - E_m = E_o$

$$\left(1 - \frac{R_m}{R_{AB} + R_m}\right)$$

Substituting in values: $E_o - E_m = E_o$

$$\left(1 - \frac{9500}{10000}\right) = 0.95E_o$$

Thus, the error in the measured value is 5%.

At this point, it is interesting to note the constraints that exist when practical attempts are made to achieve a high internal resistance in the design of a moving-coil voltmeter. Such an instrument consists of a coil carrying a pointer mounted in a fixed magnetic field. As current flows through the coil, the interaction between the field generated and the fixed field causes the pointer it carries to turn in proportion to the applied current (see Chapter 10 for further details). The simplest way of increasing the input impedance (the resistance) of the meter is either to increase the number of turns in the coil or to construct the same number of coil turns with a higher-resistance material. However, either of these solutions decreases the current flowing in the coil, giving less magnetic torque and thus decreasing the measurement sensitivity of the instrument (i.e., for a given applied voltage, we get less

deflection of the pointer). This problem can be overcome by changing the spring constant of the restraining springs of the instrument, such that less torque is required to turn the pointer by a given amount. However, this reduces the ruggedness of the instrument and also demands better pivot design to reduce friction. This highlights a very important but tiresome principle in instrument design: any attempt to improve the performance of an instrument in one respect generally decreases the performance in some other aspect. This is an inescapable fact of life with passive instruments such as the type of voltmeter mentioned, and is often the reason for the use of alternative active instruments such as digital voltmeters, where the inclusion of auxiliary power greatly improves performance.

Similar errors due to system loading are also caused when an ammeter is inserted to measure the current flowing in a branch of a circuit. For instance, consider the circuit shown in Fig. 3.2a, in which the current flowing in the branch of the circuit labeled A-B is measured by an ammeter with resistance R_m . Here, R_m acts as a resistor in series with the resistor R_5 in branch A-B, thereby increasing the resistance between points AB and so disturbing the circuit. Therefore, the current I_m measured by the meter is not the value of the current I_o that existed prior to measurement. The extent of the disturbance can be assessed by calculating the open-circuit current I_o and comparing it with I_m .

Thévenin's theorem is again a useful tool in analyzing the effect of inserting the ammeter. To apply Thevenin's theorem, the voltage sources are represented just by their internal resistance, which can be approximated to zero as in Fig. 3.2b. This allows the circuit of Fig. 3.2a, comprising two voltage sources and five resistors, to be replaced by an equivalent circuit containing just two resistances and a single voltage source, as shown in Fig. 3.2c. Analysis proceeds by calculating the equivalent resistances of sections of the circuit and building these up until the required equivalent resistance of the whole of the circuit is obtained. Starting at C and D, the circuit to the left of C and D consists of a series pair of resistances (R_1 and R_2) in parallel with R_3 , and the equivalent resistance can be written as:

$$\frac{1}{R_{CD}} = \frac{1}{R_1 + R_2} + \frac{1}{R_3} \quad \text{or} \quad R_{CD} = \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3}$$

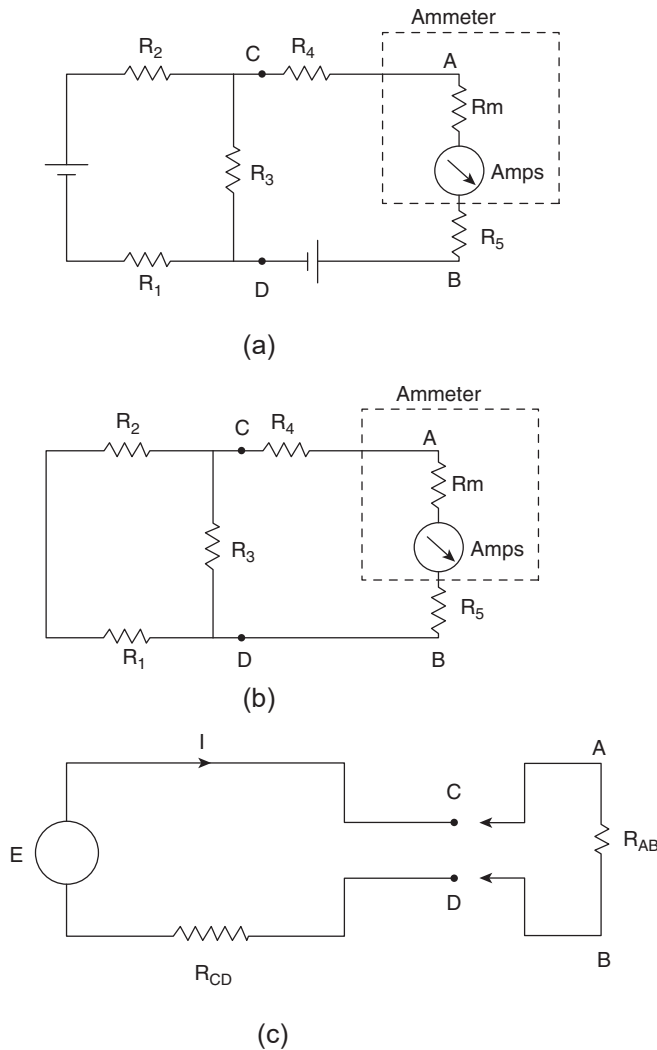
The current flowing between A and B can be calculated simply by Ohm's law as:

$$I = \frac{E}{R_{CB} + R_{CD}}$$

When the ammeter is not in the circuit, $R_{CB} = R_4 + R_5$ and $I = I_o$, where I_o is the normal (circuit-unloaded) current flowing.

Hence,

$$I_o = \frac{E}{R_4 + R_5 + \left[\frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3} \right]} = \frac{E[R_1 + R_2 + R_3]}{[R_4 + R_5][R_1 + R_2 + R_3] + [(R_1 + R_2)R_3]}$$


Figure 3.2

Analysis of circuit loading: (a) A circuit in which the current flowing in branch A-B of the circuit is to be measured; (b) The circuit with all voltage sources represented by their approximately zero resistance; (c) Equivalent circuit by Thévenin's theorem.

With the ammeter in the circuit, $R_{CB} = R_4 + R_5 + R_m$ and $I = I_m$, where I_m is the measured current.

Hence,

$$I_m = \frac{E}{R_4 + R_5 + R_m + \left[\frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3} \right]} = \frac{E[R_1 + R_2 + R_3]}{[R_4 + R_5 + R_m][R_1 + R_2 + R_3] + [(R_1 + R_2)R_3]}$$

The measurement error is given by the ratio I_m/I_0 .

$$\frac{I_m}{I_0} = \frac{[R_4 + R_5][R_1 + R_2 + R_3] + [(R_1 + R_2)R_3]}{[R_4 + R_5 + R_m][R_1 + R_2 + R_3] + [(R_1 + R_2)R_3]} \quad (3.3)$$

■ Example 3.2

Suppose that the components of the circuit shown in Fig. 3.2a have the following values: $R_1 = 250\Omega$; $R_2 = 750\Omega$; $R_3 = 1000\Omega$, $R_4 = 500\Omega$; $R_5 = 500\Omega$. The current between A and B is measured by an ammeter whose internal resistance is 50Ω . What is the measurement error caused by the resistance of the measuring instrument?

■ Solution

Substituting the parameter values into Eq. (3.3):

$$\begin{aligned} \frac{I_m}{I_0} &= \frac{[R_4 + R_5][R_1 + R_2 + R_3] + [(R_1 + R_2)R_3]}{[R_4 + R_5 + R_m][R_1 + R_2 + R_3] + [(R_1 + R_2)R_3]} \\ &= \frac{[1000][2000] + [1000 \times 1000]}{[1050][1000] + [1000 \times 1000]} = \frac{3000}{3100} = 0.968 \end{aligned}$$

The error is $1 - I_m/I_0 = 1 - 0.968 = 0.032$ or 3.2%.

Thus, the error in the measured current is 3.2%.

Bridge circuits for measuring resistance values are a further example of the need for careful design of the measurement system. The impedance of the instrument measuring the bridge output voltage must be very large in comparison with the component resistances in the bridge circuit. Otherwise, the measuring instrument will load the circuit and draw current from it. This is discussed more fully in Chapter 6.

3.2.2 Errors due to environmental inputs

An environmental input is defined as an apparently real input to a measurement system that is actually caused by a change in the environmental conditions surrounding the measurement system. The fact that the static and dynamic characteristics specified for measuring instruments are only valid for particular environmental conditions (e.g., of temperature and pressure) has already been discussed at considerable length in Chapter 2. These specified conditions must be reproduced as closely as possible during calibration exercises because, away from the specified calibration conditions, the characteristics of measuring instruments vary to some extent and cause measurement errors. The magnitude

of this environment-induced variation is quantified by the two constants known as sensitivity drift and zero drift, both of which are generally included in the published specifications for an instrument. Such variations of environmental conditions away from the calibration conditions are sometimes described as *modifying inputs* to the measurement system because they modify the output of the system. When such modifying inputs are present, it is often difficult to determine how much of the output change in a measurement system is due to a change in the measured variable and how much is due to a change in environmental conditions. This is illustrated by the following example. Suppose we are given a small closed box and told that it may contain either a mouse or a rat. We are also told that the box weighs 0.1 Kg when empty. If we put the box onto bathroom scales and observe a reading of 1.0 Kg, this does not immediately tell us what is in the box because the reading may be due to one of three things:

- (a) a 0.9 Kg rat in the box (real input)
- (b) an empty box with a 0.9 Kg bias on the scales due to a temperature change (environmental input)
- (c) A 0.4 Kg mouse in the box together with a 0.5 Kg bias (real + environmental inputs)

Thus, the magnitude of any environmental input must be measured before the value of the measured quantity (the real input) can be determined from the output reading of an instrument.

In any general measurement situation, it is very difficult to avoid environmental inputs, because it is either impractical or impossible to control the environmental conditions surrounding the measurement system. System designers are therefore charged with the task of either reducing the susceptibility of measuring instruments to environmental inputs or, alternatively, quantifying the effect of environmental inputs and correcting for them in the instrument output reading. The techniques used to deal with environmental inputs and minimize their effect on the final output measurement follow a number of routes as discussed next.

3.2.3 Wear in instrument components

Systematic errors can frequently develop over a period of time because of wear in instrument components. Recalibration often provides a full solution to this problem.

3.2.4 Connecting leads

In connecting together the components of a measurement system, a common source of error is the failure to take proper account of the resistance of connecting leads (or pipes in the case of pneumatically or hydraulically actuated measurement systems). For instance, in typical applications of a resistance thermometer, it is common to find that the thermometer is separated from other parts of the measurement system by perhaps 100 m. The resistance of such a length of 20-gauge copper wire is 7Ω , and there is a further complication that such wire has a temperature coefficient of $1\text{ m}\Omega/^\circ\text{C}$.

Therefore, careful consideration needs to be given to the choice of connecting leads. Not only should they be of adequate cross section so that their resistance is minimized, but they should be adequately screened if they are thought likely to be subject to electrical or magnetic fields that could otherwise cause induced noise. Where screening is thought essential, then the routing of cables also needs careful planning. In one application in the author's personal experience involving instrumentation of an electric-arc steelmaking furnace, screened signal-carrying cables between transducers on the arc furnace and a control room at the side of the furnace were initially corrupted by high-amplitude 50 Hz noise. However, by changing the route of the cables between the transducers and the control room, the magnitude of this induced noise was reduced by a factor of about 10.

3.3 Reduction of systematic errors

The prerequisite for the reduction of systematic errors is a complete analysis of the measurement system that identifies all sources of error. Simple faults within a system, such as bent meter needles and poor cabling practices, can usually be readily and cheaply rectified once they have been identified. However, other error sources require more detailed analysis and treatment. Various approaches to error reduction are considered below.

Careful instrument design: Careful instrument design is the most useful weapon in the battle against environmental inputs, by reducing the sensitivity of an instrument to environmental inputs to as low a level as possible. For instance, in the design of strain gauges, the element should be constructed from a material whose resistance has a very low temperature coefficient (i.e., the variation of the resistance with temperature is very small). However, errors due to the way in which an instrument is designed are not always easy to correct, and a choice often has to be made between the high cost of redesign and the alternative of accepting the reduced measurement accuracy if redesign is not undertaken.

Calibration: Instrument calibration is a very important consideration in measurement systems and therefore calibration procedures are considered in detail in Chapter 5. All instruments suffer drift in their characteristics, and the rate at which this happens depends on many factors, such as the environmental conditions in which instruments are used and the frequency of their use. The error due to an instrument being out of calibration is never zero, even immediately after the instrument has been calibrated, because there is always some inherent error in the reference instrument that a working instrument is calibrated against during the calibration exercise. Nevertheless, the error immediately after calibration is of low magnitude. The calibration error then grows steadily with the drift in instrument characteristics until the time of the next calibration. The maximum error that exists just before an instrument is recalibrated can therefore be made smaller by increasing the frequency of recalibration so that the amount of drift between calibrations is reduced.

Method of opposing inputs: The method of opposing inputs compensates for the effect of an environmental input in a measurement system by introducing an equal and opposite environmental input that cancels it out. One example of how this technique is applied is in the type of millivoltmeter shown in Fig. 3.3. This consists of a coil suspended in a fixed magnetic field produced by a permanent magnet. When an unknown voltage is applied to the coil, the magnetic field due to the current interacts with the fixed field and causes the coil (and a pointer attached to the coil) to turn. If the coil resistance R_{coil} is sensitive to temperature, then any environmental input to the system in the form of a temperature change will alter the value of the coil current for a given applied voltage and so alter the pointer output reading. Compensation for this is made by introducing a compensating resistance R_{comp} into the circuit, where R_{comp} has a temperature coefficient that is equal in magnitude but opposite in sign to that of the coil. Thus, in response to an increase in temperature, R_{coil} increases but R_{comp} decreases, and so the total resistance remains approximately the same.

High-gain feedback: The benefit of adding high-gain feedback to many measurement systems is illustrated by considering the case of the voltage-measuring instrument whose block diagram is shown in Fig. 3.4. In this system, the unknown voltage E_i is applied to a coil of torque constant K_c , and the induced torque turns a pointer against the restraining action of a spring with spring constant K_s . The effect of environmental inputs on the coil and spring constants is represented by variables D_c and D_s . In the absence of environmental inputs, the displacement of the pointer X_o is given by: $X_o = K_c K_s E_i$.

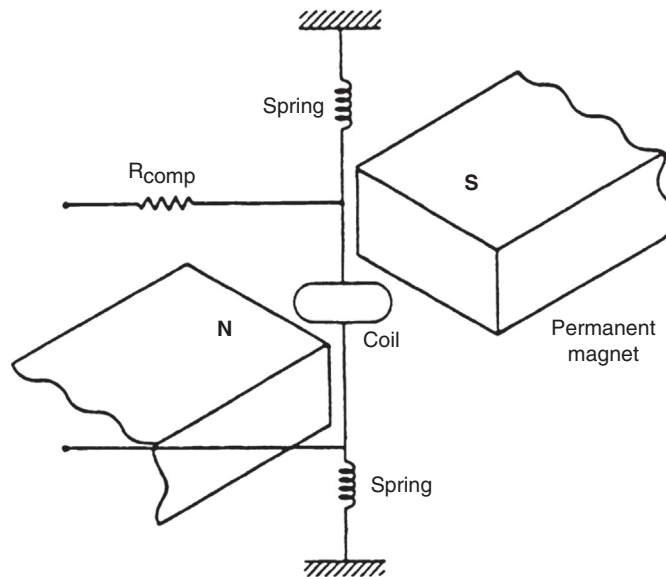


Figure 3.3

Analog millivoltmeter mechanism.

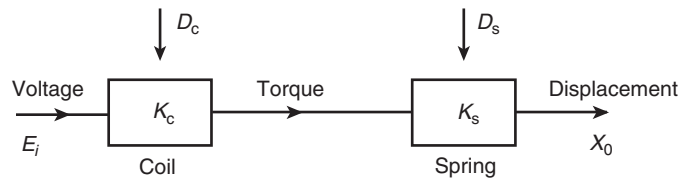


Figure 3.4
Block diagram for voltage measuring instrument.

However, in the presence of environmental inputs, both K_c and K_s change, and the relationship between X_o and E_i can be affected greatly. Therefore, it becomes difficult or impossible to calculate E_i from the measured value of X_o . Consider now what happens if the system is converted into a high-gain, closed-loop one, as shown in Fig. 3.5, by adding an amplifier of gain constant K_a and a feedback device with gain constant K_f . Assume also that the effects of environmental inputs on the values of K_a and K_f are represented by D_a and D_f . The feedback device feeds back a voltage E_o proportional to the pointer displacement X_o . This is compared with the unknown voltage E_i by a comparator and the error is amplified. Writing down the equations of the system, we have:

$$E_o = K_f X_o; \quad X_o = (E_i - E_o) K_a K_c K_s = (E_i - K_f X_o) K_a K_c K_s$$

Thus:

$$E_i K_a K_c K_s = (1 + K_f \cdot K_a \cdot K_c \cdot K_s) X_o \quad \text{i.e.} \quad X_o = \frac{K_a K_c K_s}{1 + K_f \cdot K_a \cdot K_c \cdot K_s} E_i \quad (3.4)$$

Because K_a is very large (it is a high-gain amplifier), $K_f \cdot K_a \cdot K_c \cdot K_s \gg 1$, and Eq. (3.4) reduces to:

$$X_o = E_i / K_f$$

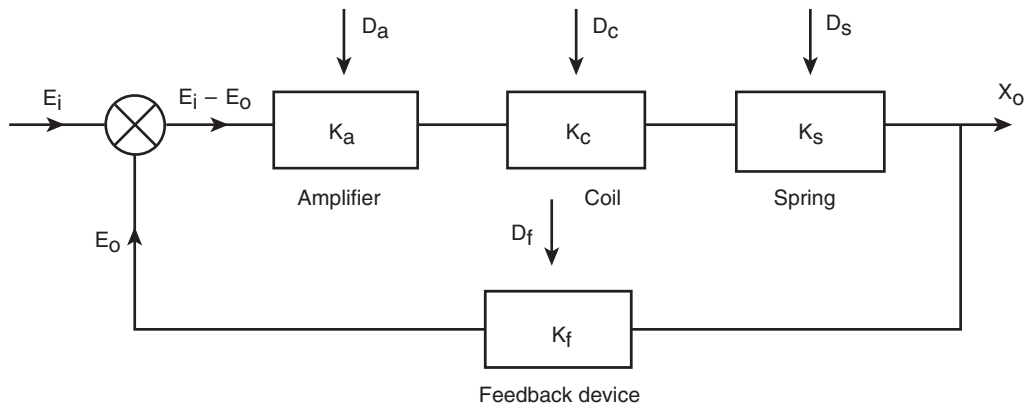


Figure 3.5
Block diagram of voltage measuring instrument with high-gain feedback.

This is a highly important result because we have reduced the relationship between X_o and E_i to one that involves only K_f . The sensitivity of the gain constants K_a , K_c , and K_s to the environmental inputs D_a , D_c , and D_s has thereby been rendered irrelevant, and we only have to be concerned with one environmental input, D_f . Conveniently, it is usually easy to design a feedback device that is insensitive to environmental inputs: this is much easier than trying to make a coil or spring insensitive. Thus, high-gain feedback techniques are often a very effective way of reducing a measurement system's sensitivity to environmental inputs. However, one potential problem that must be mentioned is that there is a possibility that high-gain feedback will cause instability in the system. Therefore, any application of this method must include careful stability analysis of the system.

Signal filtering: One frequent problem in measurement systems is corruption of the output reading by periodic noise, often at a frequency of 50 Hz¹ caused by pickup through the close proximity of the measurement system to apparatus or current-carrying cables operating on a mains supply. Periodic noise corruption at higher frequencies is also often introduced by mechanical oscillation or vibration within some component of a measurement system. The amplitude of all such noise components can be substantially attenuated by the inclusion of filtering of an appropriate form in the system, as discussed at greater length in Chapter 8. Band-stop filters can be especially useful where corruption is of one particular known frequency, or, more generally, low-pass filters are employed to attenuate all noise in the frequency range of 50 Hz¹ and above. Measurement systems with a low-level output, such as a bridge circuit measuring a strain-gauge resistance, are particularly prone to noise, and Fig. 3.6 shows the typical corruption of a bridge output by 50 Hz pickup. The beneficial effect of putting a simple passive RC low-pass filter across the output is shown in Fig. 3.6.

Manual correction of output reading: In the case of errors that are due either to system disturbance during the act of measurement or due to environmental changes, a good measurement technician can substantially reduce errors at the output of a measurement system by calculating the effect of such systematic errors and making appropriate correction to the instrument readings. This is not necessarily an easy task, and requires all disturbances in the measurement system to be quantified. This procedure is carried out automatically by intelligent instruments.

Intelligent instruments: Intelligent instruments contain extra sensors that measure the value of environmental inputs and automatically compensate the value of the output reading. They have the ability to deal very effectively with systematic errors in measurement systems, and errors can be attenuated to very low levels in many cases. A more detailed discussion of intelligent instruments can be found in Chapter 11.

¹50 Hz is the frequency of the mains electricity supply in most countries around the world, but it is 60 Hz in the USA rather than 50 Hz.

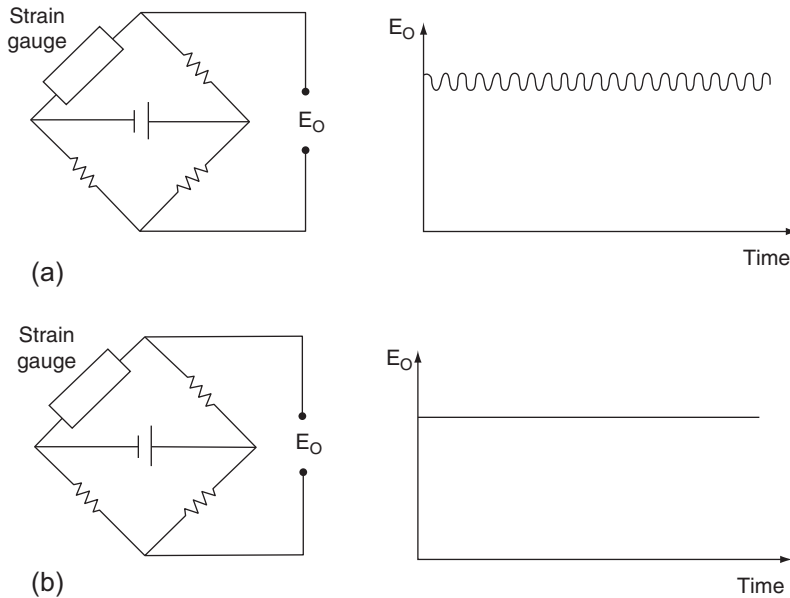


Figure 3.6
Signal filtering.

3.4 Quantification of systematic errors

Once all practical steps have been taken to eliminate or reduce the magnitude of systematic errors, the final action required is to estimate the maximum remaining error that may exist in a measurement due to systematic errors. This quantification of the maximum likely systematic error in a measurement requires careful analysis.

3.4.1 Quantification of individual systematic error components

The first complication in the quantification of systematic errors is that it is not usually possible to specify an exact value for a component of systematic error, and the quantification has to be in terms of a “best estimate.” Once systematic errors have been reduced as far as reasonably possible using the techniques explained in [Section 3.3](#), a sensible approach to estimate the various kinds of remaining systematic error would be as follows.

Environmental condition errors

If a measurement is subject to unpredictable environmental conditions, the usual course of action is to assume midpoint environmental conditions and specify the maximum measurement error as $\pm x\%$ of the output reading to allow for the maximum expected deviation in environmental conditions away from this midpoint. Of course, this only refers to the case where the environmental conditions remain essentially constant during a period

of measurement but vary unpredictably on perhaps a daily basis. If random fluctuations occur over a short period of time from causes like random draughts of hot or cold air, this is a random error rather than a systematic error that has to be quantified according to the techniques explained later in Chapter 4.

Calibration errors

All measuring instruments suffer from drift in their characteristics over a period of time. The schedule for recalibration is set so that the frequency at which an instrument is calibrated means that the drift in characteristics by the time just before the instrument is due for recalibration is kept within an acceptable limit. The maximum error just before the instrument is due for recalibration becomes the basis for estimating the maximum likely error. This error due to the instrument being out of calibration is usually in the form of a bias. The best way to express this is to assume some midpoint value of calibration error and compensate all measurements by this midpoint error. The maximum measurement error over the full period of time between when the instrument has just been calibrated and time just before the next calibration is due can then be expressed as $\pm x\%$ of the output reading.

■ Example 3.3

The recalibration frequency of a pressure transducer with a range of 0–10 bar is set so that it is recalibrated once the measurement error has grown to +1% of the full-scale reading. How can its inaccuracy be expressed in the form of a $\pm x\%$ error in the output reading?

■ Solution

Just before the instrument is due for recalibration, the measurement error will have grown to +0.1 bar (1% of 10 bar). An amount of half this maximum error, i.e., 0.05 bar, should be subtracted from all measurements. Having done this, the error just after the instrument has been calibrated will be –0.05 bar (–0.5% of full-scale reading), and the error just before the next recalibration will be +0.05 bar (+0.5% of full-scale reading). The inaccuracy due to calibration error can then be expressed as $\pm 0.05\%$ of full-scale reading.

System disturbance errors

Disturbance of the measured system by the act of measurement itself introduces a systematic error that can be quantified for any given set of measurement conditions.

However, if the quantity being measured and/or the conditions of measurement can vary, the best approach is to calculate the maximum likely error under worst-case system loading and then to express the likely error as a plus or minus value of half this calculated maximum error, as suggested for calibration errors.

Measurement system loading errors

These have a similar effect to system disturbance errors and are expressed in the form of $\pm x\%$ of the output reading, where x is half the magnitude of the maximum predicted error under the most adverse loading conditions expected.

3.4.2 Calculation of overall systematic error

The second complication in the analysis to quantify systematic errors in a measurement system is the fact that the total systemic error in a measurement is often composed of several separate components, for example, measurement system loading, environmental factors, and calibration errors. A worst-case prediction of maximum error would be to simply add up each separate systematic error. For example, if there are three components of systematic error with a magnitude of $\pm 1\%$ each, a worst-case prediction error would be the sum of the separate errors, i.e., $\pm 3\%$. However, it is very unlikely that all components of error would be at their maximum or minimum values simultaneously. The usual course of action is therefore to combine separate sources of systematic error using a **root-sum-squares method**. Applying this method for n systematic component errors of magnitude $\pm x_1\%$, $\pm x_2\%$, $\pm x_3\%$, \dots , $\pm x_n\%$, the best prediction of likely maximum systematic error by the root-sum-squares method is:

$$\text{error} = \pm \sqrt{x_1^2 + x_2^2 + x_3^2 + \dots + x_n^2}$$

■ **Example 3.4**

Three separate sources of systematic error are identified in a measurement system, and after reducing the magnitude of these errors as much as possible, the magnitudes of the three errors are estimated to be:

System loading: $\pm 1.2\%$

Environmental changes: $\pm 0.8\%$

Calibration error: $\pm 0.5\%$

Calculate the maximum possible total systematic error and the likely system error by the root-mean-square method.



■ Solution

The maximum possible system error is $\pm(1.2 + 0.8 + 0.5)\% = \pm 2.5\%$

Applying the root-mean-square method,

$$\text{likely error} = \pm \sqrt{1.2^2 + 0.8^2 + 0.5^2} = \pm 1.53\%$$

Before closing this discussion on quantifying systematic errors, a word of warning must be given about the use of manufacturers' data sheets. When instrument manufacturers provide data sheets with an instrument that they have made, the measurement uncertainty or inaccuracy value quoted in the data sheets is the best estimate that the manufacturer can give about the way that the instrument will perform when it is new, used under specified conditions, and recalibrated at the recommended frequency. Therefore, this can only be a starting point in estimating the measurement accuracy that will be achieved when the instrument is actually used. Many sources of systematic error may apply in a particular measurement situation that are not included in the accuracy calculation in the manufacturer's data sheet, and careful quantification and analysis of all systematic errors is necessary, as described above. ■

3.5 Sources and treatment of random errors

Random errors in measurements are caused by unpredictable variations in the measurement system. In some books, they are known by the alternative name *precision errors*. Typical sources of random error are:

- (i) measurements are taken by human observation of an analog meter, especially where this involves interpolation between scale points
- (ii) electrical noise
- (iii) random environmental changes, e.g., sudden draught of air.

Random errors are usually observed as small perturbations of the measurement either side of the correct value, i.e., positive errors and negative errors occur in approximately equal numbers for a series of measurements made of the same constant quantity. Therefore, random errors can largely be eliminated by calculating the average of a number of repeated measurements. Of course, this is only possible if the quantity being measured remains at a constant value during the repeated measurements. This averaging process of repeated measurements can be done automatically by intelligent instruments, as discussed in Chapter 11.

While the process of averaging over a large number of measurements substantially reduces the magnitude of random errors, it would be entirely incorrect to assume that this totally eliminates random errors. This is because the mean of a number of measurements would only be equal to the correct value of the measured quantity if the measurement set contained an infinite number of values. In practice, it is impossible to take an infinite number of measurements. Therefore, in any practical situation, the process of averaging over a finite number of measurements only reduces the magnitude of random error to a small (but nonzero) value. The degree of confidence that the calculated mean value is close to the correct value of the measured quantity can be indicated by calculating the standard deviation or variance of the data, these being parameters that describe how the measurements are distributed about the mean value (see Section 4.3). This leads on to a more formal quantification of this degree of confidence in terms of the standard error of the mean (see Section 4.7).

3.6 Induced measurement noise

The earlier sections in this chapter have already provided a detailed analysis of error sources that arise during the measurement process of sensing the value of a physical variable and generating an output signal. However, further errors are often created in measurement systems when electrical signals from measurement sensors and transducers are corrupted by induced noise. This induced noise arises both within the measurement circuit itself and also during the transmission of measurement signals to remote points. The aim when designing measurement systems is always to reduce such induced noise voltage levels as far as possible. However, it is usually not possible to eliminate all such noise, and signal processing has to be applied to deal with any noise that remains.

Noise voltages can exist either in serial mode or common mode forms. Serial mode noise voltages act in series with the output voltage from a measurement sensor or transducer, which can cause very significant errors in the output measurement signal. The extent to which series mode noise corrupts measurement signals is measured by a quantity known as the *signal-to-noise ratio*. This is defined as:

$$\text{Signal-to-noise ratio} = 20 \log_{10} \left(\frac{V_s}{V_n} \right)$$

where V_s is the mean voltage level of the signal and V_n is the mean voltage level of the noise. In the case of a.c. noise voltages, the root-mean-squared value is used as the mean.

Common mode noise voltages are less serious, because they cause the potential of both sides of a signal circuit to be raised by the same level, and thus the level of the output measurement signal is unchanged. However, common mode voltages do have to be

considered carefully, since they can be converted into series mode voltages in certain circumstances.

Noise can be generated from sources both external and internal to the measurement system. Induced noise from external sources arises in measurement systems for a number of reasons that include their proximity to mains-powered equipment and cables (causing noise at the mains frequency), proximity to florescent-lighting circuits (causing noise at twice the mains frequency), proximity to equipment operating at audio and radio frequencies (causing noise at corresponding frequency), switching of nearby d.c. and a.c. circuits, and corona discharge (both of the latter causing induced spikes and transients). Internal noise includes thermoelectric potentials, shot noise, and potentials due to electrochemical action.

3.6.1 Inductive coupling

The primary mechanism by which external devices such as mains cables and equipment, fluorescent lighting, and circuits operating at audio or radio frequencies generate noise is through inductive coupling. If signal-carrying cables are close to such external cables or equipment, a significant mutual inductance M can exist between them, as shown in Fig. 3.7a, and this can generate a series mode noise voltage of several millivolts given by $V_n = MI$, where I is the rate of change of current in the mains circuit.

3.6.2 Capacitive (electrostatic) coupling

Capacitive coupling, also known as electrostatic coupling, can also occur between the signal wires in a measurement circuit and a nearby mains-carrying conductor. The magnitude of the capacitance between each signal wire and the mains conductor is represented by the quantities C_1 and C_2 in Fig. 3.7b. In addition to these capacitances, a capacitance can also exist between the signal wires and earth, represented by C_3 and C_4

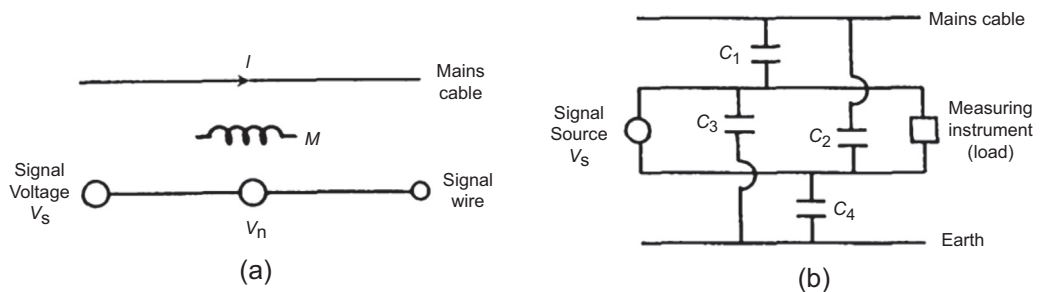


Figure 3.7

Noise induced by coupling: (a) inductive coupling; (b) capacitive (electrostatic) coupling.

in the figure. It can be shown that the series mode noise voltage V_n is zero if the coupling capacitances are perfectly balanced, i.e., if $C_1=C_2$ and $C_3=C_4$. However, exact balance is unlikely in practice, since the signal wires are not perfectly straight, causing the distances and thus the capacitances to the mains cable and to earth to vary. Thus, some series mode noise voltage induced by capacitive coupling usually exists.

3.6.3 Noise due to multiple earths

As far as possible, measurement signal circuits are isolated from earth. However, leakage paths often exist between measurement circuit signal wires and earth at both the source (sensor) end of the circuit and also the load (measuring instrument) end. This does not cause a problem as long as the earth potential at both ends is the same. However, it is common to find that other machinery and equipment carrying large currents is connected to the same earth plane. This can cause the potential to vary between different points on the earth plane. This situation, which is known as *multiple earths*, can cause a series mode noise voltage in the measurement circuit.

3.6.4 Noise in the form of voltage transients

When motors and other electrical equipment (both a.c. and d.c.) are switched on and off, large changes of power consumption suddenly occur in the electricity supply system. This can cause voltage transients (“spikes”) in measurement circuits connected to the same power supply. Such noise voltages are of large magnitude but short time duration. *Corona discharge* can also cause voltage transients on the mains power supply. This occurs when the air in the vicinity of high-voltage d.c. circuits becomes ionized and discharges to earth at random times.

3.6.5 Thermoelectric potentials

Whenever metals of two different types are connected together, a thermoelectric potential (sometimes called a *thermal e.m.f.*) is generated according to the temperature of the joint. This is known as the *thermoelectric effect* and is the physical principle on which temperature-measuring thermocouples operate (see Chapter 14). Such thermoelectric potentials are only a few millivolts in magnitude and so the effect is only significant when typical voltage output signals of a measurement system are of a similar low magnitude.

One such situation is where one e.m.f.-measuring instrument is used to monitor the output of several thermocouples measuring the temperatures at different points in a process control system. This requires a means of automatically switching the output of each thermocouple to the measuring instrument in turn. Nickel-iron reed relays with copper connecting leads are commonly used to provide this switching function. This introduces a

thermocouple effect of magnitude $40 \mu\text{V}/^\circ\text{C}$ between the reed relay and the copper connecting leads. There is no problem if both ends of the reed relay are at the same temperature because then the thermoelectric potentials will be equal and opposite and so cancel out. However, there are several recorded instances where, because of lack of awareness of the problem, poor design has resulted in the two ends of a reed relay being at different temperatures and causing a net thermoelectric potential. The serious error that this introduces is clear. For a temperature difference between the two ends of only 2°C , the thermoelectric potential is $80 \mu\text{V}$, which is very large compared with a typical thermocouple output level of $400 \mu\text{V}$.

Another example of the difficulties that thermoelectric potentials can create becomes apparent in considering the following problem that was reported in a current-measuring system. This system had been designed such that the current flowing in a particular part of a circuit was calculated by applying it to an accurately calibrated wire-wound resistance of value 100Ω and measuring the voltage drop across the resistance. In calibration of the system, a known current of $20 \mu\text{A}$ was applied to the resistance and a voltage of 2.20 mV was measured by an accurate high-impedance instrument. Simple application of Ohm's law reveals that such a voltage reading indicates a current value of $22 \mu\text{A}$. What then was the explanation for this discrepancy? The answer once again is a thermoelectric potential. Because the designer was not aware of thermoelectric potentials, the circuit had been constructed such that one side of the standard resistance was close to a power transistor, creating a difference in temperature between the two ends of the resistor of 2°C . The thermoelectric potential associated with this was sufficient to account for the 10% measurement error found.

3.6.6 Shot noise

Shot noise occurs in transistors, integrated circuits, and other semiconductor devices. It consists of random fluctuations in the rate of transfer of carriers across junctions within such devices.

3.6.7 Electrochemical potentials

These are potentials that arise within measurement systems due to electrochemical action. Poorly soldered joints are a common source.

3.7 Techniques for reducing induced measurement noise

Prevention is always better than cure, and much can be done to reduce the level of measurement noise by taking appropriate steps when designing the measurement system.

3.7.1 Location and design of signal wires

Both the mutual inductance and capacitance between signal wires and other cables are inversely proportional to the square of the distance between the wires and the cable. Thus, noise due to inductive and capacitive coupling can be minimized by ensuring that signal wires are positioned as far away as possible from such noise sources. A minimum separation of 0.3 m is essential, and a separation of at least 1 m is preferable. Noise due to inductive coupling is also substantially reduced if each pair of signal wires is twisted together along its length. This design is known as a *twisted pair*, and is illustrated in Fig. 3.8. In the first loop, wire A is closest to the noise source and has a voltage V_1 induced in it, while wire B has an induced noise voltage V_2 . For loop 2, wire B is closest to the noise source and has an induced voltage V_1 , while wire A has an induced voltage V_2 . Thus the total voltage induced in wire A is $V_1 + V_2$ and in wire B it is $V_2 + V_1$ over these two loops. This pattern continues for all the loops, and hence the two wires have an identical voltage induced in them.

3.7.2 Earthing

Noise due to multiple earths can be avoided by good earthing practices. In particular, this means keeping earths for signal wires and earths for high-current equipment entirely separate. Recommended practice is to install four completely isolated earth circuits as follows:

Power earth: provides a path for fault currents due to power faults.

Logic earth: provides a common line for all logic circuit potentials.

Analog earth (ground): provides a common reference for all analog signals.

Safety earth: connected to all metal parts of equipment to protect personnel should power lines come into contact with metal enclosures.

3.7.3 Shielding

Shielding consists of enclosing the signal wires in an earthed, metal shield that is itself isolated electrically from the signal wires. The shield should be earthed at only one point,

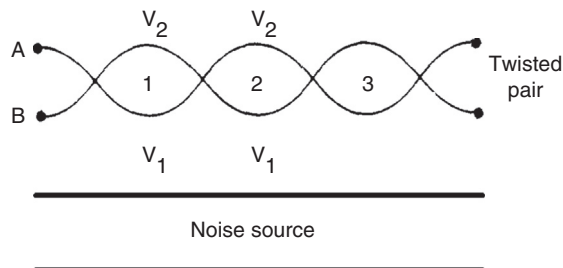


Figure 3.8
Cancellation of induced noise.

preferably the signal source end. A shield consisting of braided metal eliminates 85% of noise due to capacitive coupling, while a lapped metal foil shield eliminates noise almost entirely. The wires inside such a shield are normally formed as a twisted pair so that protection is also provided against induced noise due to nearby electromagnetic fields. Metal conduit is also sometimes used to provide shielding from capacitive-coupled noise, but the necessary supports for the conduit provide multiple earth points and lead to the problem of earth loops.

3.7.4 Other techniques

The *phase-locked loop* is often used as a signal-processing element to clean up poor-quality signals. Although this is primarily a circuit for measuring the frequency of a signal, as described in Chapter 6, it is also useful for noise removal because its output waveform is a pure (i.e., perfectly clean) square wave at the same frequency as the input signal, irrespective of the amount of noise, modulation, or distortion on the input signal.

Lock-in amplifiers are also commonly used to extract d.c. or slowly varying measurement signals from noise. The input measurement signal is modulated into a square-wave a.c. signal whose amplitude varies with the level of the input signal. This is normally achieved by either a relay or a field-effect transistor. As a relay is subject to wear, the transistor is better. An alternative method is to use an analog multiplier. Also, in the case of optical signals, the square wave can be produced by chopping the measurement signals using a set of windows in a rotating disk. This technique is frequently used with transducers like photodiodes that often generate large quantities of noise.

3.8 Summary

This chapter has introduced the subject of measurement uncertainty. We have learned that measurement errors are a fact of life, and, although we can do much to reduce the magnitude of errors, we can never reduce errors entirely to zero. We also learned that errors occur both during the measurement process and also during transmission of measurement signals from the point of measurement to some other point through induced noise. We started the chapter off by noting that uncertainty during the measurement process comes in two distinct forms, known, respectively, as *systematic errors* and *random errors*. We learned that the nature of systematic errors was such that the effect on a measurement reading was to make it either consistently greater than or consistently less than the true value of the measured quantity. Random errors on the other hand are entirely random in nature, such that successive measurements of a constant quantity are randomly both greater than and less than the true value of the measured quantity.

In our subsequent study of systematic measurement errors, we firstly examined all the sources of this kind of error. Following this, we looked at all the techniques that are

available for reducing the magnitude of systematic errors arising from the various error sources identified. Finally, we examined ways of quantifying the remaining systemic measurement error after all reasonable means of reducing error magnitude had been applied. We also briefly discussed the nature of random errors and the use of averaging over a number of measurements to reduce their effect. However, since the detailed analysis of random errors is a lengthy subject, this was deferred until Chapter 4.

This chapter then went on to consider the additional measurement errors that are generated when electrical signals from measurement sensors and transducers are corrupted by induced noise during transmission of the measurement signal from the point of measurement to some other point. We examined ways of reducing induced noise voltage levels as far as possible but noted that it is usually not possible to eliminate all such noise, and that signal processing has to be applied to deal with any noise that remains.

3.9 Problems

- 3.1 Explain the difference between systematic and random errors. What are the typical sources of these two types of error?
- 3.2 In what ways can the act of measurement cause a disturbance in the system being measured?
- 3.3 In the circuit shown in Fig. 3.9, the resistor values are given by $R_1 = 1000\Omega$; $R_2 = 1000\Omega$; $V = 20$ V. The voltage across AB (i.e., across R_2) is measured by a voltmeter whose internal resistance is given by: $R_m = 9500\Omega$.
 - (a) What will be the reading on the voltmeter?
 - (b) What would the voltage across AB be if the voltmeter was not loading the circuit (i.e., if $R_m = \text{infinity}$)?
 - (c) What is the measurement error due to the loading effect of the voltmeter?
- 3.4 Suppose that the components in the circuit shown in Fig. 3.1a have the following values:

$$R_1 = 330\Omega; \quad R_2 = 1000\Omega; \quad R_3 = 1200\Omega; \quad R_4 = 220\Omega; \quad R_5 = 270\Omega.$$

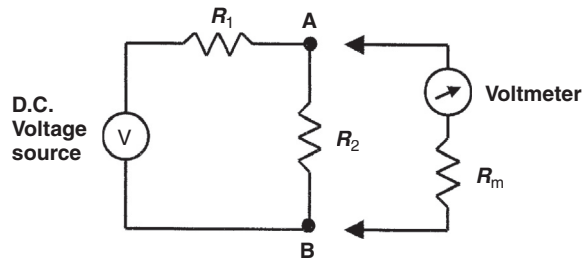


Figure 3.9

Circuit for Problems 3.3, 3.5 and 3.13.

If the instrument measuring the output voltage across AB has a resistance of 5000Ω , what is the measurement error caused by the loading effect of this instrument?

- 3.5 (a) Why does a fully calibrated voltmeter never give the correct value when measuring the voltage in an electrical circuit (assume that it is used at the same temperature that it was calibrated at)?
- (b) What steps can be taken to reduce the measurement error?
- (c) In the circuit shown in Fig. 3.9, the resistor values are given by $R_1 = 500\Omega$; $R_2 = 500\Omega$; $V = 12$ V. The voltage across AB (i.e., across R_2) is measured by a voltmeter whose internal resistance is given by: $R_m = 7500\Omega$.
- (i) What will be the reading on the voltmeter?
- (ii) What would the voltage across AB be if the voltmeter was not loading the circuit (i.e., if $R_m = \text{infinity}$)?
- (iii) What is the measurement error due to the loading effect of the voltmeter?
- 3.6 (a) Explain what is meant by the term “modifying inputs.”
- (b) Explain briefly what measures can be taken to reduce or eliminate the effect of modifying inputs.
- 3.7 (a) Explain what a “thermoelectric potential” is. Discuss the circumstances in which thermoelectric potentials do and do not cause problems in electrical circuits.
- (b) Why does a voltmeter load an electric circuit when it is placed into it to measure a voltage value and what is the effect of this loading?
- (c) Suppose that the components in the circuit shown in Fig. 3.1a have the following values:
- $$R_1 = 470\Omega; \quad R_2 = 1200\Omega; \quad R_3 = 1600\Omega; \quad R_4 = 330\Omega; \quad R_5 = 180\Omega.$$

If the instrument measuring the output voltage across AB has a resistance of $10,000\Omega$, what is the measurement error caused by the loading effect of this instrument?

- 3.8 Instruments are normally calibrated and their characteristics defined for particular standard ambient conditions. What procedures are normally taken to avoid measurement errors when using instruments that are subjected to changing ambient conditions?
- 3.9 The voltage across a resistance R_5 in the circuit of Fig. 3.10 is to be measured by a voltmeter connected across it.
- (a) If the voltmeter has an internal resistance (R_m) of 4750Ω , what is the measurement error?
- (b) What value would the voltmeter internal resistance need to be in order to reduce the measurement error to 1%?
- 3.10 In the circuit shown in Fig. 3.11, the current flowing between A and B is measured by an ammeter whose internal resistance is 100Ω . What is the measurement error caused by the resistance of the measuring instrument?

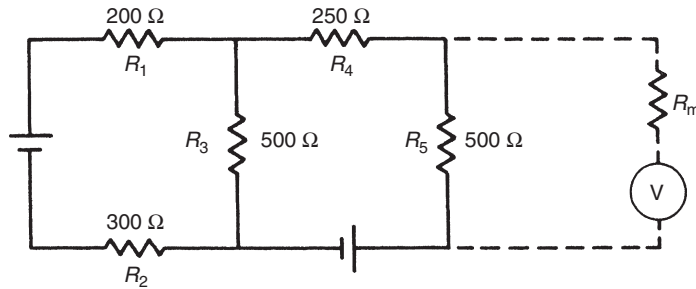


Figure 3.10

Circuit for Problems 3.9 and 3.12.

- 3.11 What steps can be taken to reduce the effect of environmental inputs in measurement systems?
- 3.12 (a) Why is there an error in the measured voltage when a voltmeter is inserted into an electrical circuit to measure the voltage across a component in the circuit?
- (b) The voltage across a resistance R_5 in the circuit of Fig. 3.10 is to be measured by a voltmeter connected across it.
- (i) If the voltmeter has an internal resistance (R_m) of 8950Ω , what is the measurement error?
- (ii) What value would the voltmeter internal resistance need to be in order to reduce the measurement error to 0.5%?
- 3.13 (a) Explain why a voltmeter never reads exactly the correct value when it is applied to an electrical circuit to measure the voltage between two points.
- (b) For the circuit shown in Fig. 3.9, show that the voltage E_m measured across points AB by the voltmeter is related to the true voltage E_o by the following expression:

$$\frac{E_m}{E_o} = \frac{R_m(R_1 + R_2)}{R_1(R_2 + R_m) + R_2R_m}$$

- (c) If the parameters in Fig. 3.9 have the following values, $R_1 = 500\Omega$; $R_2 = 500\Omega$; $R_m = 4750\Omega$, calculate the percentage error in the voltage value measured across points AB by the voltmeter.
- 3.14 (a) What is the root-mean-square method for estimating the overall error in a measurement system that has several components of error? Why does it give a better estimate of the likely overall measurement error than that given by simply adding up all the components of error?
- (b) A measurement system has three significant errors consisting of a calibration error, an error due to environmental condition changes, and an error due to system loading. The calculated magnitudes of these errors are: calibration error

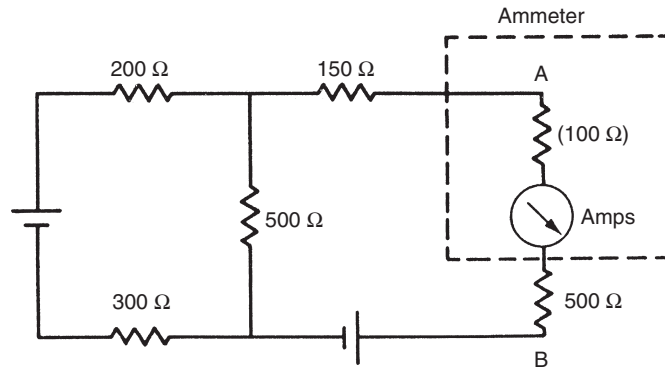


Figure 3.11
Circuit for Problems 3.10 and 3.14.

$\pm 0.3\%$, error due to environmental changes $\pm 0.7\%$, and error due to system loading $\pm 1.1\%$. Calculate the maximum possible total measurement error and then apply the root-mean-square method to calculate the most likely error.

- 3.15 (a) Why is there a measurement error when an ammeter is inserted to measure the current in a branch of a circuit?
- (b) In the circuit shown in Fig. 3.2, the current flowing between A and B is measured by an ammeter whose internal resistance is $85\ \Omega$. The resistance values are given by:

$$R_1 = 300\ \Omega; R_2 = 200\ \Omega; R_3 = 500\ \Omega; R_4 = 150\ \Omega; R_5 = 500\ \Omega.$$

What is the measurement error caused by the insertion of the measuring instrument into the circuit?

- 3.16 The output of a potentiometer is measured by a voltmeter having a resistance R_m , as shown in Fig. 3.12. R_t is the resistance of the total length X_t of the potentiometer and

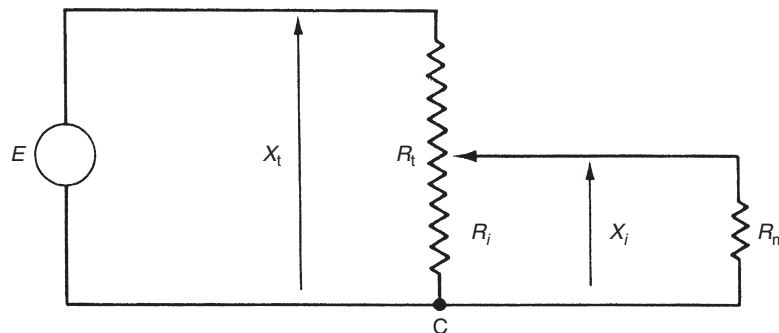


Figure 3.12
Circuit for Problem 3.15.

R_i is the resistance between the wiper and common point C for a general wiper position X_i . Show that the measurement error due to the resistance R_m of the measuring instrument is given by:

$$Error = E \frac{R_i^2(R_t - R_i)}{R_t(R_i R_t + R_m R_t - R_i^2)}$$

Hence show that the maximum error occurs when X_i is approximately equal to $2X_t/3$. (Hint: differentiate the error expression with respect to R_i and set to 0. Note that maximum error does not occur exactly at $X_i = 2X_t/3$, but this value is very close to the position where the maximum error occurs.)

- 3.17 What are the main sources of noise in measurement systems?
- 3.18 What techniques can be used to reduce noise in measurement systems?

Statistical analysis of measurements subject to random errors

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4.1 Introduction

Having introduced the subject of measurement uncertainty in Chapter 2 and its effect on the quality and accuracy of measurements, the last chapter went on to consider the subject

of measurement errors in more detail. We learned in the previous chapter that errors in measurement systems can be divided into two classes : systematic errors and random (precision) errors. Furthermore, we learned that the methods for quantifying these two types of error are fundamentally different. In the case of systematic errors, it is necessary to know first the magnitudes of the physical effects that affect a measurement system, but then the measurement error can be estimated as an absolute value. However, in the case of random errors, where the error is caused by random and unpredictable effects, it is possible to quantify the error only in probabilistic terms.

The appropriate means of quantifying and reducing systematic errors were covered in the previous chapter. This chapter examines the nature of random errors in more detail and explains the various means available for quantifying random errors. The discussion starts with an explanation of how the approximate value of a measurement subject to random errors can be calculated as a mean or median value. The discussion then introduces the concepts of quantifying the spread of measurements about the mean value as either the standard deviation or the variance. After this, graphical techniques for displaying the spread of random errors are introduced, particularly the Gaussian distribution. This includes an explanation of Gaussian (z-function) tables and how these can be used to quantify this spread in statistical terms. The chapter then explains various other statistical means of analyzing measurements subject to random errors, including the standard error of the mean, chi-squared distribution, goodness of fit tests, identification of rogue data points (data outliers), and the Student t distribution.

So far, we have covered the means of quantifying both systematic and random errors, the former in the last chapter and the latter in the first part of this chapter. However, an important question remains: If the final output from a measurement system is calculated by combining two or more measurements of separate physical variables, how should an overall system measurement error be optimally calculated from the error level in the individual system components? This question is answered in the final part of this chapter.

4.2 Mean and median values

The average value of a set of measurements of a constant quantity can be expressed as either the mean value or the median value. Historically, the median value was easier for a computer to compute than the mean value because median computation involves a series of logic operations, whereas the mean computation requires addition and division. Many years ago, a computer performed logic operations much faster than arithmetic operations, and there were computational speed advantages in calculating average values by computing the median rather than the mean. However, computer power increased many years ago to a point where this advantage disappeared.

As the number of measurements increases, the difference between the mean and median values becomes very small. However, the average calculated in terms of the mean value is always slightly closer to the correct value of the measured quantity than the average calculated as the median value for any finite set of measurements. Given the loss of any computational speed advantage because of the massive power of modern computers, this means that there is now little argument for calculating average values in terms of the median.

For any set of n measurements $x_1, x_2 \cdots x_n$ of a constant quantity, the most likely true value is the **mean**, given by:

$$x_{mean} = \frac{x_1 + x_2 + \cdots x_n}{n} \quad (4.1)$$

This is valid for all data sets where the measurement errors are distributed equally about the zero error value (i.e., where the positive errors are balanced in quantity and magnitude by the negative errors).

The **median** is an approximation to the mean, which can be written down without having to sum the measurements. The median is the middle value when the measurements in the data set are written down in ascending order of magnitude. For a set of n measurements $x_1, x_2 \cdots x_n$ of a constant quantity, written down in ascending order of magnitude, the median value is given by:

$$x_{median} = x_{(n+1)/2} \quad (4.2)$$

Thus, for a set of 9 measurements $x_1, x_2 \cdots x_9$ arranged in order of magnitude, the median value is x_5 . For an even number of measurements, the median value is midway between the two center values, i.e., for 10 measurements $x_1 \cdots x_{10}$, the median value is given by $(x_5 + x_6)/2$.

Suppose the length of a steel bar is measured by a number of different observers and the following set of 11 measurements are recorded (units are in millimeters). We will call this measurement set A:

398 420 394 416 404 408 400 420 396 413 430 (Measurement set A)

Using Eqs. (4.1) and (4.2), mean = 409.0 and median = 408. Suppose now that the measurements are taken again using a better measuring rule and with the observers taking more care, producing the following measurement set B:

409 406 402 407 405 404 407 404 407 407 408 (Measurement set B)

For these measurements, mean = 406.0 and median = 407. In which of the two measurement sets, A and B, and the corresponding mean and median values should we

have most confidence? Intuitively, we can regard measurement set B as being more reliable because the measurements are much closer together. In set A, the spread between the smallest (396) and largest (430) value is 34, whereas in set B, the spread is only 6.

- ***Thus, the smaller the spread of the measurements, the more confidence we have in the mean or median value calculated.***

Let us now see what happens if we increase the number of measurements by extending measurement set B to 23 measurements. We will call this measurement set C:

409 406 402 407 405 404 407 404 407 407 408 406 410
 406 405 408 406 409 406 405 409 406 407 (Measurement set C)

Now, mean = 406.5 and median = 406.

- ***This confirms our earlier statement that the median value tends toward the mean value as the number of measurements increases.***

4.3 Standard deviation and variance

Expressing the spread of measurements simply as the range between the largest and smallest value is not in fact a good way to examine how the measurement values are distributed about the mean value. A much better way to express the distribution is to calculate the variance or standard deviation of the measurements. The starting point for calculating these parameters is to calculate the deviation (error) d_i of each measurement x_i from the mean value x_{mean} in a set of measurements x_1, x_2, \dots, x_n :

$$d_i = x_i - x_{mean} \quad (4.3)$$

The **variance** (V_s) of the set of measurements is formally defined as the mean of the squares of the deviations:

$$V_s = \frac{d_1^2 + d_2^2 \cdots d_n^2}{n} \quad (4.4)$$

The **standard deviation** (σ_s) of the set of measurements is defined as the square root of the variance:

$$\sigma_s = \sqrt{V_s} = \sqrt{\frac{d_1^2 + d_2^2 \cdots d_n^2}{n}} \quad (4.5)$$

Unfortunately, these formal definitions for the variance and standard deviation of data are made with respect to an infinite population of data values, whereas in all practical

situations, we can only have a finite set of measurements. We previously observed that the mean value x_m of a finite set of measurements will differ from the true mean μ of the theoretical infinite population of measurements of which the finite set is a part. This means that there is an error in the mean value x_{mean} used to calculate d_i in Eq. (4.3). Because of this, Eqs. (4.4) and (4.5) give a biased estimate that tends to underestimate the variance and standard deviation of the infinite set of measurements. A better prediction of the variance of the infinite population can be obtained by applying the Bessel correction factor $(n/n - 1)$ to the formula for V_s in Eq. (4.4):

$$V = \left(\frac{n}{n-1}\right) V_s = \frac{d_1^2 + d_2^2 \cdots d_n^2}{n-1} \quad (4.6)$$

where V_s is the variance of the finite set of measurements and V is the variance of the infinite population of measurements.

This leads to a similar better prediction of the standard deviation by taking the square root of the variance in Eq. (4.6):

$$\sigma = \sqrt{V} = \sqrt{\frac{d_1^2 + d_2^2 \cdots d_n^2}{n-1}} \quad (4.7)$$

■ Example 4.1

Calculate σ and V for measurement sets A, B, and C. ■

■ Solution

First, draw a table of measurements and deviations for set A (mean = 409 as calculated earlier):

Measurement	398	420	394	416	404	408	400	420	396	413	430
Deviation from mean	-11	+11	-15	+7	-5	-1	-9	+11	-13	+4	+21
(Deviations) ²	121	121	225	49	25	1	81	121	169	16	441

$\sum (\text{deviations}^2) = 1370$; $n = \text{number of measurements} = 11$.

Then, from Eqs. (4.6) and (4.7), $V = \sum (\text{deviations}^2) / n - 1$; $= 1370 / 10 = 137$;
 $\sigma = \sqrt{V} = 11.7$.

The measurements and deviations for set B are (mean = 406 as calculated earlier):

Measurement	409	406	402	407	405	404	407	404	407	407	408
Deviation from mean	+3	0	-4	+1	-1	-2	+1	-2	+1	+1	+2
(Deviations) ²	9	0	16	1	1	4	1	4	1	1	4

From this data, using Eqs. (4.6) and (4.7), $V = 4.2$ and $\sigma = 2.05$.

The measurements and deviations for set C are (mean = 406.5 as calculated earlier):

Measurement	409	406	402	407	405	404	407	404	407	407	408
Deviation from mean	+2.5	-0.5	-4.5	+0.5	-1.5	-2.5	+0.5	-2.5	+0.5	+0.5	+1.5
(Deviations) ²	6.25	0.25	20.25	0.25	2.25	6.25	0.25	6.25	0.25	0.25	2.25

Measurement	406	410	406	405	408	406	409	406	405	409	406	407
Deviation from mean	-0.5	+3.5	-0.5	-1.5	+1.5	-0.5	+2.5	-0.5	-1.5	+2.5	-0.5	+0.5
(Deviations) ²	0.25	12.25	0.25	2.25	2.25	0.25	6.25	0.25	2.25	6.25	0.25	0.25

From this data, using Eqs. (4.6) and (4.7), $V = 3.53$ and $\sigma = 1.88$.



Note that the smaller values of V and σ for measurement set B compared with A correspond with the respective size of the spread in the range between maximum and minimum values for the two sets.

- *Thus, as V and σ decrease for a measurement set, we are able to express greater confidence that the calculated mean or median value is close to the true value (i.e., that the averaging process has reduced the random error value close to zero).*
- *Comparing V and σ for measurement sets B and C, V and σ get smaller as the number of measurements increases, confirming that confidence in the mean value increases as the number of measurements increases).*

We have observed so far that random errors can be reduced by taking the average (mean or median) of a number of measurements. However, although the mean or median value is close to the true value, it would become exactly equal to the true value only if we could average an infinite number of measurements. Because we can only make a finite number of measurements in a practical situation, the average value will still have some error. This error can be quantified as the *standard error of the mean*, which will be discussed in detail a little later. However, before that, the subject of graphical analysis of random measurement errors needs to be covered.

4.4 Graphical data analysis techniques: frequency distributions

Graphical techniques are useful for analyzing the way in which random measurement errors are distributed. The simplest method for doing this is to draw a *histogram* in which bands of equal width across the range of measurement values are defined and the number

of measurements within each band is counted. The bands are often given the name *data bins*. There are two alternative rules for calculating the best number of data bins to use: The *Sturgis rule* calculates the number of bands as:

Number of bands = $1 + 3.3 \log_{10}(n)$,
 where n is the number of measurement values.

The *Rice rule* calculates the number of bands as: $2n^{1/3}$

Obviously the result produced has to be rounded to the nearest integer in both cases.

When n is relatively small, the two rules suggest the same number of bins. However, for larger values of n , the Rice rule calculates a larger number of bins than the Sturgis rule. This is summarised in the table below:

Number of measurements	Number of bins calculated by Sturgis rule	Number of bins by Sturgis (rounded)	Number of bins calculated by Rice rule	Number of bins by Rice (rounded)
10	4.3	4	4.3	4
15	4.9	5	4.9	5
20	5.3	5	5.4	5
25	5.6	6	5.8	6
30	5.9	6	6.2	7
50	6.6	7	7.4	7
100	7.6	8	9.3	9
200	8.6	9	11.7	12

These rules should be regarded as a good guide, but in any given situation, there may be a good reason for varying the number of bins away from the number suggested. For larger numbers of measurements, it is common practice to use a bin number between the numbers recommended by the two rules. Special consideration is also needed when dealing with measurement data with random errors. For these, a symmetrical histogram is expected, and this is shown better if the number of bins is odd rather than even.

■ Example 4.2

Draw a histogram for the 23 measurements in set C of the length measurement data given in [Section 4.2](#)

■ Solution

For 23 measurements, the recommended number of bands calculated according to the Sturgis rule is $1 + 3.3 \log_{10}(23) = 5.49$. This rounds to 5, because the number of bands must be an integer number.

To cover the span of measurements in data set C with 5 bands, the data bands need to be 2 mm wide. The boundaries of these bands must be carefully chosen so that no measurements fall on the boundary between different bands and cause ambiguity about which band to put them in. Because the measurements are integer numbers, this can be easily accomplished by defining the range of the first band as 401.5–403.5, and so on. A histogram can now be drawn as in Fig. 4.1 by counting the number of measurements in each band.

In the first band from 401.5 to 403.5, there is just 1 measurement, and so the height of the histogram in this band is 1 unit.

In the next band from 403.5 to 405.5, there are 5 measurements, and so the height of the histogram in this band is $1 = 5$ units.

The rest of the histogram is completed in a similar fashion.



When a histogram is drawn using a sufficiently large number of measurements, it will have the characteristic shape shown by truly random data, with symmetry about the mean value of the measurements. However, for a relatively small number of measurements, only approximate symmetry in the histogram can be expected about the mean value. It is a matter of judgment as to whether the shape of a histogram is close enough to symmetry to justify a conclusion that the data on which it is based are truly random. It should be noted that the 23 measurements used to draw the histogram in Fig. 4.1 were carefully chosen to produce a

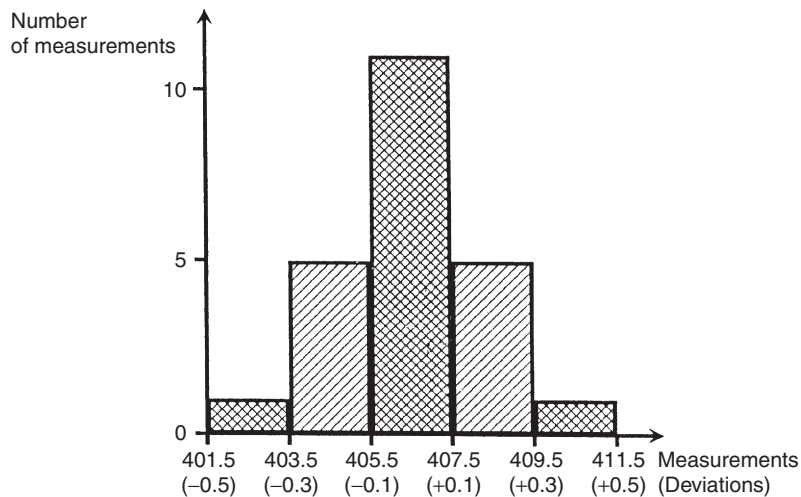


Figure 4.1
Histogram of measurements and deviations.

symmetrical histogram, but exact symmetry would not normally be expected for a measurement data set as small as 23.

Because the actual value of measurement error is usually of most concern, it is often more useful to draw a histogram of the deviations of the measurements from the mean value rather than to draw a histogram of the measurements themselves. The starting point for this is to calculate the deviation of each measurement away from the calculated mean value. Then, a *histogram of deviations* can be drawn by defining deviation bands of equal width and counting the number of deviation values in each band. This histogram has exactly the same shape as the histogram of the raw measurements, except that the scaling of the horizontal axis has to be redefined in terms of the deviation values (these units are shown in brackets in Fig. 4.1).

Let us now explore what happens to the histogram of deviations as the number of measurements increases. As the number of measurements increases, smaller bands can be defined for the histogram, which retains its basic shape but then consists of a larger number of smaller steps on each side of the peak. In the limit, as the number of measurements approaches infinity, the histogram becomes a smooth curve known as a *frequency distribution curve*, as shown in Fig. 4.2. The ordinate of this curve is the frequency of occurrence of each deviation value, $F(D)$, and the abscissa is the magnitude of deviation, D .

The symmetry of Figs. 4.1 and 4.2 about the zero deviation value is useful for showing graphically that the measurement data have only random errors. Although these figures

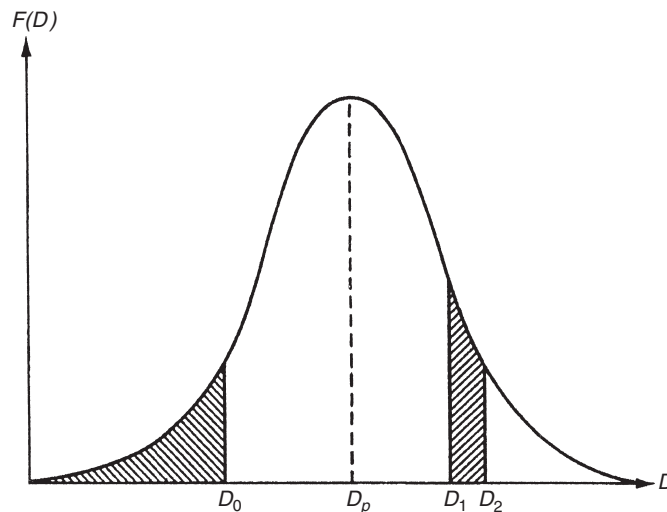


Figure 4.2
Frequency distribution curve of deviations.

cannot easily be used to quantify the magnitude and distribution of the errors, similar graphical techniques achieve this. If the height of the frequency distribution curve is normalized such that the area under it is unity, the curve in this form is known as a **probability curve**, and the height $F(D)$ at any particular deviation magnitude D is known as the **probability density function**. The condition in which the area under the curve is unity can be expressed mathematically as:

$$\int_{-\infty}^{\infty} F(D)dD = 1$$

The probability that the error in any one particular measurement lies between two levels D_1 and D_2 can be calculated by measuring the area under the curve contained between two vertical lines drawn through D_1 and D_2 , as shown by the right-hand hatched area in Fig. 4.2. This can be expressed mathematically as:

$$P(D_1 \leq D \leq D_2) = \int_{D_1}^{D_2} F(D)dD \quad (4.8)$$

Of particular importance for assessing the maximum error likely in any one measurement is the **cumulative distribution function** (cdf). This is defined as the probability of observing a value less than or equal to D_o and is expressed mathematically as:

$$P(D \leq D_0) = \int_{-\infty}^{D_0} F(D)dD \quad (4.9)$$

Thus, the cdf is the area under the curve to the left of a vertical line drawn through D_o , as shown by the left-hand hatched area in Fig. 4.2.

The deviation magnitude D_p corresponding to the peak of the frequency distribution curve (Fig. 4.2) is the value of deviation that has the greatest probability. If the errors are entirely random in nature, then the value of D_p will equal zero. Any nonzero value of D_p indicates systematic errors in the data in the form of a bias that is often removable by recalibration.

4.5 Gaussian (Normal) distribution

Measurement sets that contain only random errors usually conform to a distribution with a particular shape that is called **Gaussian**, although this conformance must always be tested (see the section Goodness of Fit). The shape of a Gaussian curve is such that the frequency of small deviations from the mean value is much greater than the frequency of large deviations. This coincides with the usual expectation in measurements subject to random errors that the number of measurements with a small error is much larger than the

number of measurements with a large error. Alternative names for the Gaussian distribution are the **normal distribution** or **bell-shaped distribution**. A Gaussian curve is formally defined as a normalized frequency distribution that is symmetrical about the line of zero error and in which the frequency and magnitude of quantities are related by the expression:

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{[-(x-m)^2/2\sigma^2]} \quad (4.10)$$

where m is the mean value of the data set x and the other quantities are as defined before. Eq. (4.10) is particularly useful for analyzing a Gaussian set of measurements and predicting how many measurements lie within some particular defined range. If the measurement deviations D are calculated for all measurements such that $D = x - m$, then the curve of deviation frequency $F(D)$ plotted against deviation magnitude D is a Gaussian curve known as the **error frequency distribution curve**. The mathematical relationship between $F(D)$ and D can then be derived by modifying Eq. (4.10) to give:

$$F(D) = \frac{1}{\sigma\sqrt{2\pi}} e^{[-D^2/2\sigma^2]} \quad (4.11)$$

The shape of a Gaussian curve is strongly influenced by the value of σ , with the width of the curve decreasing as σ becomes smaller. Because a smaller σ corresponds to typical deviations of measurements from the mean value becoming smaller, this confirms the earlier observation that the mean value of a set of measurements gets closer to the true value as σ decreases.

If the standard deviation is used as a unit of error, the Gaussian curve can be used to determine the probability that the deviation in any particular measurement in a Gaussian data set is greater than a certain value. By substituting the expression for $F(D)$ in (Eq. 4.11) into the probability expression Eq. (4.8), the probability that the error lies in a band between error levels D_1 and D_2 can be expressed as:

$$P(D_1 \leq D \leq D_2) = \int_{D_1}^{D_2} \frac{1}{\sigma\sqrt{2\pi}} e^{(-D^2/2\sigma^2)} dD \quad (4.12)$$

Solution of this expression is simplified by the substitution:

$$z = D/\sigma \quad (4.13)$$

The effect of this is to change the error distribution curve into a new Gaussian distribution that has a standard deviation of 1 ($\sigma = 1$) and a mean of 0. This new form, shown in

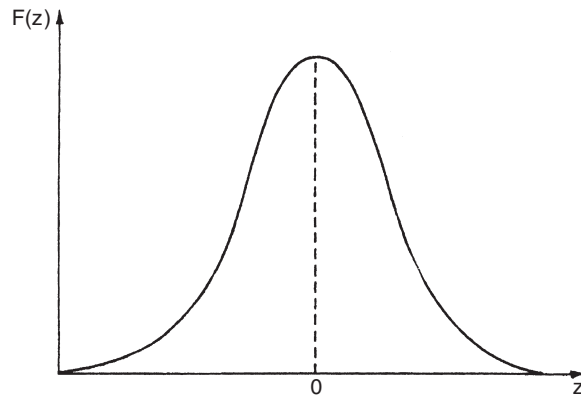


Figure 4.3

Standard Gaussian curve ($F[z]$ versus z).

Fig. 4.3, is known as a *standard Gaussian curve* (or sometimes as a *z distribution*), and the dependent variable is now z instead of D . Eq. (4.12) can now be re-expressed as:

$$P(D_1 \leq D \leq D_2) = P(z_1 \leq z \leq z_2) = \int_{z_1}^{z_2} \frac{1}{\sigma\sqrt{2\pi}} e^{(-z^2/2)} dz \quad (4.14)$$

Unfortunately, neither Eq. (4.12) nor (Eq. 4.14) can be solved analytically using tables of standard integrals; numerical integration provides the only method of solution. However, in practice, the tedium of numerical integration can be avoided when analyzing data because the standard form of Eq. (4.14) and its independence from the particular values of the mean and standard deviation of the data mean that standard Gaussian tables that tabulate $F(z)$ for various value of z can be used.

4.6 Standard Gaussian tables (*z distribution*)

A standard Gaussian table (sometimes called the *z distribution*), such as that shown in Table 4.1, tabulates the area under the Gaussian curve $F(z)$ for various values of z , where $F(z)$ is given by:

$$F(z) = \int_{-\infty}^z \frac{1}{\sigma\sqrt{2\pi}} e^{(-z^2/2)} dz \quad (4.15)$$

Thus, $F(z)$ gives the proportion of data values that are less than or equal to z . This proportion is the area under the curve of $F(z)$ against z that is to the left of z . Therefore, the expression given in (Eq. 4.14) has to be evaluated as $[F(z_2) - F(z_1)]$. Study of Table 4.1 shows that $F(z) = 0.5$ for $z = 0$. This confirms that, as expected, the number of data values ≤ 0 is 50% of the total. This must be so if the data have only random errors. Moreover, Table 4.1, in common with most published standard Gaussian tables, gives only

Table 4.1: Error function table (area under a Gaussian curve or z distribution).

F(z)										
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5000	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7793	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8906	0.8925	0.8943	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9648	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9926	0.9928	0.9930	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

$F(z)$ for positive values of z . For negative values of z , we can use the following relationship because the frequency distribution curve is normalized:

$$F(-z) = 1 - F(z) \tag{4.16}$$

where $F(-z)$ is the area under the curve to the left of $(-z)$ (i.e., it represents the proportion of data values $\leq -z$).

■ Example 4.3

How many measurements in a data set subject to random errors lie outside deviation boundaries of $+\sigma$ and $-\sigma$ (i.e., how many measurements have a deviation greater than $|\sigma|$)?

■ Solution

The required number is represented by the sum of the two shaded areas in Fig. 4.4. This can be expressed mathematically as: $P(E < -\sigma \text{ or } E > +\sigma) = P(E < -\sigma) + P(E > +\sigma)$

For $E = -\sigma$, $z = -1.0$ (from Eq. (4.13)).

Using Table 4.1: $P(E < -\sigma) = F(-1) = 1 - F(1) = 1 - 0.8413 = 0.1587$

Similarly, for $E = +\sigma$, $z = +1.0$, Table 4.1 gives:

$P(E > +\sigma) = 1 - P(E < +\sigma) = 1 - F(1) = 1 - 0.8413 = 0.1587$. (This last step is valid because the frequency distribution curve is normalized such that the total area under it is unity.)

Thus, $P(E < -\sigma) + P(E > +\sigma) = 0.1587 + 0.1587 = 0.3174 \sim 32\%$ (i.e., 32% of measurements lie outside the $\pm\sigma$ boundaries, and thus 68% of measurements lie inside).

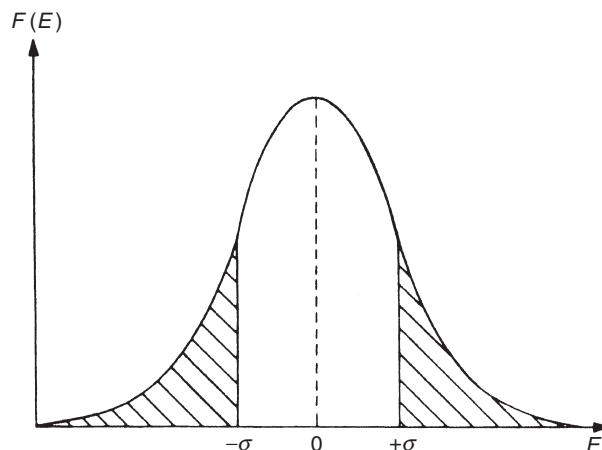


Figure 4.4
 $\pm\sigma$ boundaries.

This analysis shows that for Gaussian-distributed data values, 68% of measurements have deviations that lie within the bounds of $\pm\sigma$. Similar analysis shows that boundaries of $\pm 2\sigma$ contain 95.4% of data points and that extending the boundaries to $\pm 3\sigma$ encompasses 99.7% of data points. The probability of any data point lying outside particular deviation boundaries can therefore be expressed by the following table:

Deviation boundaries	% data points within boundary	Probability of any particular data point being outside boundary
$\pm\sigma$	68.0	32.0%
$\pm 2\sigma$	95.4	4.6%
$\pm 3\sigma$	99.7	0.3%

4.7 *Standard error of the mean*

The previous analysis examined the way in which measurements with random errors are distributed about the mean value. However, we already observed that some error exists between the mean value of a finite set of measurements and the true value (i.e., averaging a number of measurements will yield the true value only if the number of measurements is infinite). If several subsets are taken from an infinite data population with a Gaussian distribution, the central limit theorem states that the means of the subsets will form a Gaussian distribution about the mean of the infinite data set. The standard deviation of the mean values of a series of finite sets of measurements relative to the true mean (the mean of the infinite population from which the finite set of measurements is drawn) is defined as the *standard error of the mean*, α . This is calculated as:

$$\alpha = \sigma / \sqrt{n} \tag{4.17}$$

Clearly, α tends toward zero as the number of measurements (n) in the data set expands toward infinity.

The next question is, how do we use the standard error of the mean to predict the error between the calculated mean of a finite set of measurements and the mean of the infinite population? In other words, if we use the mean value of a finite set of measurements to predict the true value of the measured quantity, what is the likely error in this prediction? This likely error can be expressed only in probabilistic terms. All we know for certain is the standard deviation of the error, which is expressed as α in Eq. (4.17). We also know that a range of ± 1 standard deviation (i.e., $\pm\alpha$) encompasses 68% of the deviations of sample means on either side of the true value. Thus, we can say that the measurement value obtained by calculating the mean of a set of n measurements, x_1, x_2, \dots, x_n , can be expressed as:

$$x = x_{mean} \pm \alpha$$

with 68% certainty that the magnitude of the error does not exceed $|\alpha|$. For the data set C of length measurements used earlier, $n = 23$, $\sigma = 1.88$, and $\alpha = 0.39$. The length can therefore be expressed as 406.5 ± 0.4 (68% confidence limit).

The problem of expressing the error with 68% certainty is that there is a 32% chance that the error is greater than α . Such a high probability of the error being greater than α may not be acceptable in many situations. If this is the case, we can use the fact that a range of ± 2 standard deviations (i.e., $\pm 2\alpha$) encompasses 95.4% of the deviations of sample means on either side of the true value. Thus, we can express the measurement value as:

$$x = x_{mean} \pm 2\alpha$$

with 95.4% certainty that the magnitude of the error does not exceed $|2\alpha|$. This means that there is only a 4.6% chance that the error exceeds 2α . Referring again to set C of length measurements, $2\sigma = 3.76$, $2\alpha = 0.78$ and the length can be expressed as 406.5 ± 0.8 (95.4% confidence limits).

If we wish to express the maximum error with even greater probability that the value is correct, we could use $\pm 3\alpha$ limits (99.7% confidence). In this case, for the length measurements again, $3\sigma = 5.64$, $3\alpha = 1.17$, and the length should be expressed as 406.5 ± 1.2 (99.7% confidence limits). There is now only a 0.3% chance (3 in 1000) that the error exceeds this value of 1.2.

■ Example 4.4

In a practical exercise to determine the freezing point of a metal alloy, the following 10 measurements of the freezing point temperature were obtained:

519.5 521.7 518.9 520.3 521.4 520.1 519.8 520.2 518.6 521.5

Express the mean value and the error boundaries expressed to (a) 68% confidence limits, (b) 95.4% confidence limits, and (c) 99.7% confidence limits

■ Solution

First calculate the mean value of the measurements:

$$\text{Mean} = \left[\sum (519.5 \ 521.7 \ 518.9 \ 520.3 \ 521.4 \ 520.1 \ 519.8 \ 520.2 \ 518.6 \ 521.5) \right] / 10 = 520.2$$

Next, calculate the deviations of the measurements from the mean, and hence the standard deviation:

Measurement	519.5	521.7	518.9	520.3	521.4	520.0	519.8	520.3	518.6	521.5
Deviation from mean	-0.7	+1.5	-1.3	+0.1	+1.2	-0.2	-0.4	+0.1	-1.6	+1.3
(Deviations) ²	0.49	2.25	1.69	0.01	1.44	0.04	0.16	0.01	2.56	1.69

$\sum(\text{deviations})^2 = 10.34$; $n = \text{number of measurements} = 10$.

Then, from Eqs. (4.6) and (4.7), $\sigma = \sqrt{\frac{\sum(\text{deviations}^2)}{n-1}} = \sqrt{\frac{10.34}{9}} = \sqrt{1.149} = 1.072$.

The standard error of the mean is given by:

$$\alpha = \sigma / \sqrt{n} = 1.072 / \sqrt{10} = 0.339.$$

The mean of the measurements expressed to 68% confidence limits is given by:

$$x = x_{\text{mean}} \pm \alpha = 520.2 \pm 0.3$$

The mean of the measurements expressed to 95.4% confidence limits is given by:

$$x = x_{\text{mean}} \pm 2\alpha = 520.2 \pm 0.7$$

The mean of the measurements expressed to 99.7% confidence limits is given by:

$$x = x_{\text{mean}} \pm 3\alpha = 520.2 \pm 1.0$$



4.8 Estimation of random error in a single measurement

In many situations where measurements are subject to random errors, it is not practical to take repeated measurements and find the average value. Also, the averaging process becomes invalid if the measured quantity does not remain at a constant value, as is usually the case when process variables are being measured. Thus, if only one measurement can be made, some means of estimating the likely magnitude of error in it is required. The normal approach to this is to calculate the error within 95% confidence limits (i.e., to calculate the value of the deviation D such that 95% of the area under the probability curve lies within limits of $\pm D$). These limits correspond to a deviation of $\pm 1.96\sigma$. Thus, it is necessary to maintain the measured quantity at a constant value while a number of measurements are taken to create a reference measurement set from which σ can be calculated. Subsequently, the maximum likely deviation in a single measurement can be expressed as: Deviation = $\pm 1.96\sigma$. However, this expresses only the maximum likely deviation of the measurement from the calculated mean of the reference measurement set, which is not the true value, as observed earlier. Thus, the calculated value for the standard

error of the mean has to be added to the likely maximum deviation value. To be consistent, this should be expressed to the same 95% confidence limits. Thus, the maximum likely error in a single measurement can be expressed as:

$$\text{Error} = \pm 1.96(\sigma + \alpha) \quad (4.18)$$

■ Example 4.5

Suppose a standard mass is measured 30 times with the same instrument to create a reference data set, and the calculated values of σ and α are $\sigma = 0.46$ and $\alpha = 0.08$. If the instrument is then used to measure an unknown mass and the reading is 105.6 kg, how should the mass value be expressed?

■ Solution

Using (Eq. 4.18), $1.96(\sigma + \alpha) = 1.06$. The mass value should therefore be expressed as: 105.6 ± 1.1 kg.

Before leaving this matter, it must be emphasized that the maximum error specified for a measurement is specified only for the confidence limits defined. Thus, if the maximum error is specified as $\pm 1\%$ with 95% confidence limits, this means that there is still 1 chance in 20 that the error will exceed $\pm 1\%$.

4.9 Distribution of manufacturing tolerances

Many aspects of manufacturing processes are subject to random variations caused by factors similar to those that cause random errors in measurements. In most cases, these random variations in manufacturing, which are known as *tolerances*, fit a Gaussian distribution, and the previous analysis of random measurement errors can be applied to analyze the distribution of these variations in manufacturing parameters.

■ Example 4.6

An integrated circuit chip contains 10^5 transistors. The transistors have a mean current gain of 20 and a standard deviation of 2. Calculate the following:

- the number of transistors with a current gain between 19.8 and 20.2
- the number of transistors with a current gain greater than 17

■ Solution

(a) The proportion of transistors in which $19.8 < \text{gain} < 20.2$ is:

$$P[X < 20.2] - P[X < 19.8] = P[z < 0.2] - P[z < -0.2] \quad (\text{for } z = (X - \mu) / \sigma)$$

For $X = 20.2$; $z = 0.1$ and for $X = 19.8$; $z = -0.1$

From the tables, $P[z < 0.1] = 0.5398$ and thus $P[z < -0.1] = 1 - P[z < 0.1] = 1 - 0.5398 = 0.4602$

Hence, $P[z < 0.1] - P[z < -0.1] = 0.5398 - 0.4602 = 0.0796$

Thus $0.0796 \times 10^5 = 7960$ transistors have a current gain in the range 19.8–20.2.

(b) The number of transistors with gain > 17 is given by:

$$P[x > 17] = 1 - P[x < 17] = 1 - P[z < -1.5] = P[z < +1.5] = 0.9332$$

Thus, 93.32% (i.e., 93,320 transistors) has a gain > 17 .

4.10 Chi-squared (χ^2) distribution

We have already observed the fact that, if we calculate the mean value of successive sets of samples of N measurements, the means of those samples form a Gaussian distribution about the true value of the measured quantity (the true value being the mean of the infinite data set of which the set of samples are part). The standard deviation of the distribution of the mean values was quantified as the standard error of the mean.

It is also useful for many purposes to look at the distribution of the variance of successive sets of samples of N measurements that form part of a Gaussian distribution. This is expressed as the chi-squared distribution, $F(\chi^2)$, in which χ^2 is given by:

$$\chi^2 = k\sigma_x^2 / \sigma^2 \quad (4.19)$$

where σ_x^2 is the variance of a sample of N measurements and σ^2 is the variance of the infinite data set of which the sets of N samples are part. k is a constant known as the number of degrees of freedom and is equal to $(N - 1)$.

The shape of the chi-squared distribution depends on the value of k ; typical shapes are shown in Fig. 4.5. The area under the chi-squared distribution curve is unity, but unlike the Gaussian distribution, the chi-squared distribution is not symmetrical. However, it tends toward the symmetrical shape of a Gaussian distribution as k becomes very large.

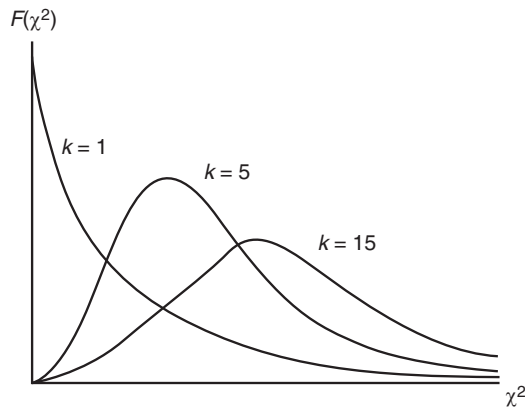


Figure 4.5
Typical chi-squared distributions.

The chi-squared distribution expresses the expected variation due to random chance of the variance of a sample away from the variance of the infinite population of which the sample is part. The magnitude of this expected variation depends on what level of random chance we set. The level of random chance is normally expressed as a **level of significance**, usually denoted by the symbol α . Referring to the chi-squared distribution shown in Fig. 4.6, the value χ^2_α denotes the χ^2 value to the left of which lies $100(1 - \alpha)\%$ of the area under the χ^2 distribution curve. Thus, the area of the curve to the right of χ^2_α is α and that to the left is $(1 - \alpha)$.

Numerical values for χ^2 are obtained from tables that express the value of χ^2 for various degrees of freedom k and for various levels of significance α . Published tables differ in the number of degrees of freedom and the number of levels of significance covered. A typical table is shown as Table 4.2. The first column in Table 4.2 gives various values of the

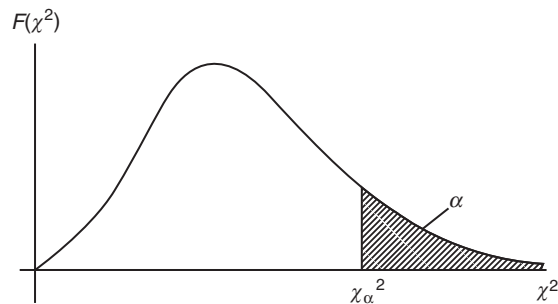


Figure 4.6
Meaning of symbol α for chi-squared distribution.

Table 4.2: Chi-squared (χ^2) distribution.

k	$\chi_{0.995}^2$	$\chi_{0.990}^2$	$\chi_{0.975}^2$	$\chi_{0.950}^2$	$\chi_{0.900}^2$	$\chi_{0.100}^2$	$\chi_{0.050}^2$	$\chi_{0.025}^2$	$\chi_{0.010}^2$	$\chi_{0.005}^2$
1	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21	10.6
3	0.07	0.12	0.22	0.35	0.58	6.25	7.81	9.35	11.3	12.8
4	0.21	0.30	0.48	0.71	1.06	7.78	9.49	11.1	13.3	14.9
5	0.41	0.55	0.83	1.15	1.61	9.24	11.1	12.8	15.1	16.7
6	0.68	0.87	1.24	1.64	2.20	10.6	12.6	14.4	16.8	18.5
7	0.99	1.24	1.69	2.17	2.83	12.0	14.1	16.0	18.5	20.3
8	1.34	1.65	2.18	2.73	3.49	13.4	15.5	17.5	20.1	22.0
9	1.73	2.09	2.70	3.33	4.17	14.7	16.9	19.0	21.7	23.6
10	2.16	2.56	3.25	3.94	4.87	16.0	18.3	20.5	23.2	25.2
11	2.60	3.05	3.82	4.57	5.58	17.3	19.7	21.9	24.7	26.8
12	3.07	3.57	4.40	5.23	6.30	18.5	21.0	23.3	26.2	28.3
13	3.57	4.11	5.01	5.89	7.04	19.8	22.4	24.7	27.7	29.8
14	4.07	4.66	5.63	6.57	7.79	21.1	23.7	26.1	29.1	31.3
15	4.60	5.23	6.26	7.26	8.55	22.3	25.0	27.5	30.6	32.8
16	5.14	5.81	6.91	7.96	9.31	23.5	26.3	28.8	32.0	34.3
17	5.70	6.41	7.56	8.67	10.1	24.8	27.6	30.2	33.4	35.7
18	6.26	7.02	8.23	9.39	10.9	26.0	28.9	31.5	34.8	37.2
19	6.84	7.63	8.91	10.1	11.7	27.2	30.1	32.9	36.2	38.6
20	7.43	8.26	9.59	10.9	12.4	28.4	31.4	34.2	37.6	40.0
21	8.03	8.90	10.3	11.6	13.2	29.6	32.7	35.5	38.9	41.4
22	8.64	9.54	11.0	12.3	14.0	30.8	33.9	36.8	40.3	42.8
23	9.26	10.2	11.7	13.1	14.9	32.0	35.2	38.1	41.6	44.2
24	9.89	10.9	12.4	13.8	15.7	33.2	36.4	39.4	43.0	45.6
25	10.5	11.5	13.1	14.6	16.5	34.4	37.7	40.6	44.3	46.9
26	11.2	12.2	13.8	15.4	17.3	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	36.7	40.1	43.2	47.0	49.6
28	12.5	13.6	15.3	16.9	18.9	37.9	41.3	44.5	48.3	51.0
29	13.1	14.3	16.0	17.7	19.8	39.1	42.6	45.7	49.6	52.3
30	13.8	15.0	16.8	18.5	20.6	40.3	43.8	47.0	50.9	53.7
35	17.2	18.5	20.6	22.5	24.8	46.1	49.8	53.2	57.3	60.3
40	20.7	22.2	24.4	26.5	29.1	51.8	55.8	59.3	63.7	66.8
45	24.3	25.9	28.4	30.6	33.4	57.5	61.7	65.4	70.0	73.2
50	28.0	29.7	32.4	34.8	37.7	63.2	67.5	71.4	76.2	79.5
75	47.2	49.5	52.9	56.1	59.8	91.1	96.2	100.8	106.4	110.3
100	67.3	70.1	74.2	77.9	82.4	118.5	124.3	129.6	135.8	140.2

degrees of freedom (k). The rest of the columns give the values of χ_{α}^2 for each value of k corresponding to the values of α at the top of each column.

■ **Example 4.7**

Calculate χ^2 for a sample of 10 measurements and a level of significance of 0.05



■ Solution

Referring to [Table 4.2](#), the values of χ^2 corresponding to a value $\alpha = 0.05$ are to be found in column 8 of the table counting from the left. For a sample size $N = 10$, the number of degrees of freedom k is given by $(N - 1)$ (i.e., $k = 9$). Now, reading along the row of the table corresponding to $k = 9$ as far as the eighth column gives a value for χ^2 of 16.9

■ Example 4.8

Calculate χ^2 for a sample of 21 measurements and a level of significance of 0.005

■ Solution

Referring to [Table 4.2](#), the values of χ^2 corresponding to a value $\alpha = 0.005$ are to be found in the final (right-most) column of the table. For a sample size $N = 21$, the number of degrees of freedom k is given by $(N - 1)$ (i.e., $k = 20$). Now, reading along the row of the table corresponding to $k = 20$ as far as the last column gives a value for χ^2 of 40.0

One major use of the χ^2 distribution is to predict the variance σ^2 of an infinite data set, given the measured variance σ_x^2 of a sample of N measurements drawn from the infinite population. The boundaries of the range of χ^2 values expected for a particular level of significance α can be expressed by the probability expression:

$$P\left[\chi_{1-\alpha/2}^2 \leq \chi^2 \leq \chi_{\alpha/2}^2\right] = 1 - \alpha \quad (4.20)$$

To put this in simpler terms, we are saying that there is a probability of $(1 - \alpha)\%$ that χ^2 lies within the range bounded by $\chi_{1-\alpha/2}^2$ and $\chi_{\alpha/2}^2$ for a level of significance of α . For example, for a level of significance $\alpha = 0.05$, there is a 95% probability (95% confidence level) that χ^2 lies between $\chi_{0.975}^2$ and $\chi_{0.025}^2$.

Substituting into Eq. (4.20) using the expression for χ^2 given in [Eq. \(4.19\)](#):

$$P\left[\chi_{1-\alpha/2}^2 \leq \frac{k\sigma_x^2}{\sigma^2} \leq \chi_{\alpha/2}^2\right] = 1 - \alpha$$

This can be expressed in an alternative but equivalent form by inverting the terms and changing the \leq relationships to \geq ones:

$$P \left[\frac{1}{\chi_{1-\alpha/2}^2} \geq \frac{\sigma^2}{k\sigma_x^2} \geq \frac{1}{\chi_{\alpha/2}^2} \right] = 1 - \alpha$$

Now, multiplying the expression through by $k\sigma_x^2$ gives the following expression for the boundaries of the variance σ^2 :

$$P \left[\frac{k\sigma_x^2}{\chi_{1-\alpha/2}^2} \geq \sigma^2 \geq \frac{k\sigma_x^2}{\chi_{\alpha/2}^2} \right] = 1 - \alpha \quad (4.21)$$

■ Example 4.9

The length of each rod in a sample of 10 brass rods is measured and the variance of the length measurement in the sample is found to be 16.3 mm. Estimate the true variance and standard deviation for the whole batch of rods from which the sample of 10 was drawn, expressed to a confidence level of 95%.

■ Solution

Degrees of freedom (k) = $N - 1 = 9$.

For $\sigma_x^2 = 16.3$, $k\sigma_x^2 = 146.7$

For a confidence level of 95%, the level of significance, α , = 0.05

Applying Eq. (4.21), the true variance is bounded by the values of $146.7/\chi_{0.975}^2$ and $146.7/\chi_{0.025}^2$.

Looking up the appropriate values in the χ^2 distribution table for $k = 9$ gives:

$$\chi_{0.975}^2 = 2.70; \chi_{0.025}^2 = 19.0; 146.7/\chi_{0.975}^2 = 54.3; 146.7/\chi_{0.025}^2 = 7.7$$

The true variance can therefore be expressed as $7.7 \leq \sigma^2 \leq 54.3$

The true standard deviation can be expressed as $\sqrt{7.7} \leq \sigma \leq \sqrt{54.3}$, i.e., $2.8 \leq \sigma \leq 7.4$.

What is immediately evident in this solution is that the range within which the true variance and standard deviation lies is wide. This is a consequence of the relatively small number of measurements (10) in the sample. It is therefore highly desirable whenever possible to use a considerably larger sample when making predictions of the true variance and standard deviation of some measured quantity.

■ Example 4.10

The length of a sample of 25 bricks is measured and the variance of the sample is calculated as 6.8 mm. Estimate the true variance for the whole batch of bricks from which the sample of 25 was drawn, expressed to confidence levels of (a) 90%, (b) 95%, and (c) 99%.

■ Solution

Degrees of freedom (k) = $N - 1 = 24$

For $\sigma_x^2 = 6.8$, $k\sigma_x^2 = 163.2$.

(a) For a confidence level of 90%, the level of significance, $\alpha = 0.10$ and $\alpha/2 = 0.05$.

Applying Eq. (4.21), the true variance is bounded by the values of $163.2/\chi_{0.95}^2$ and $163.2/\chi_{0.05}^2$.

Looking up the appropriate values in the χ^2 distribution table for $k = 24$ gives:

$$\chi_{0.95}^2 = 13.8; \chi_{0.05}^2 = 36.4; 163.2/\chi_{0.95}^2 = 11.8; 163.2/\chi_{0.05}^2 = 4.5$$

The true variance can therefore be expressed as: $4.5 \leq \sigma^2 \leq 11.8$.

(b) For a confidence level of 95%, the level of significance, $\alpha = 0.05$ and $\alpha/2 = 0.025$.

Applying Eq. (4.21), the true variance is bounded by the values of $163.2/\chi_{0.975}^2$ and $163.2/\chi_{0.025}^2$.

Looking up the appropriate values in the χ^2 distribution table for $k = 24$ gives:

$$\chi_{0.975}^2 = 12.4; \chi_{0.025}^2 = 39.4; 163.2/\chi_{0.975}^2 = 13.2; 163.2/\chi_{0.025}^2 = 4.1$$

The true variance can therefore be expressed as: $4.1 \leq \sigma^2 \leq 13.2$.

(c) For a confidence level of 99%, the level of significance, $\alpha = 0.01$ and $\alpha/2 = 0.005$.

Applying Eq. (4.21), the true variance is bounded by the values of $146.7/\chi_{0.995}^2$ and $146.7/\chi_{0.005}^2$.

Looking up the appropriate values in the χ^2 distribution table for $k = 24$ gives:

$$\chi_{0.995}^2 = 9.89; \chi_{0.005}^2 = 45.6; 146.7/\chi_{0.995}^2 = 16.5; 146.7/\chi_{0.005}^2 = 3.6$$

The true variance can therefore be expressed as: $3.6 \leq \sigma^2 \leq 16.5$.

The solution to Example (4.10) shows that, as expected, the width of the estimated range in which the true value of standard deviation lies widens as we increase the confidence

level from 90% to 99%. It is also interesting to compare the results in Examples (4.9) and (4.10) for the same confidence level of 95%. The ratio between the maximum and minimum values of estimated variance is much greater for the 10 samples in Example (4.9) compared with the 25 samples in Example (4.10). This shows the benefit of having a larger sample size when predicting the variance of the whole population from which the sample is drawn.

4.11 Goodness of fit to a Gaussian distribution

All of the analyses of random deviations presented so far apply only when the data being analyzed belong to a Gaussian distribution. Hence, the degree to which a set of data fits a Gaussian distribution should always be tested before any analysis is carried out, which can be done in one of three ways.

4.11.1 Inspecting shape of histogram

The simplest way to test for the Gaussian distribution of data is to plot a histogram and look for a bell shape of the form shown earlier in Fig. 4.1. Deciding whether the histogram confirms a Gaussian distribution is a matter of judgment. For a Gaussian distribution, there must always be approximate symmetry about the line through the center of the histogram, the highest point of the histogram must always coincide with this line of symmetry, and the histogram must get progressively smaller on either side of this point. However, because the histogram can be drawn only with a finite set of measurements, some deviation from the perfect shape of histogram as described earlier is to be expected even if the data really are Gaussian.

Fig. 4.7 shows sample histograms of two sets of measurements. Histogram (a) is clearly not even close to being symmetrical about the vertical line through the center of the histogram and shows that this data set is not Gaussian. In the case of histogram (b), there is approximate symmetry about the line through the center of the histogram. This indicates that the data probably follow a Gaussian distribution, but further tests, by drawing a normal probability test or applying the chi-squared test, are necessary to confirm whether the data set really is Gaussian.

4.11.2 Using a normal probability plot

A normal probability plot involves dividing data values into a number of ranges and plotting the cumulative probability of summed data frequencies against the data values on special graph paper.¹ This line should be a straight line if the data distribution is Gaussian.

¹This is available from specialized stationery suppliers.

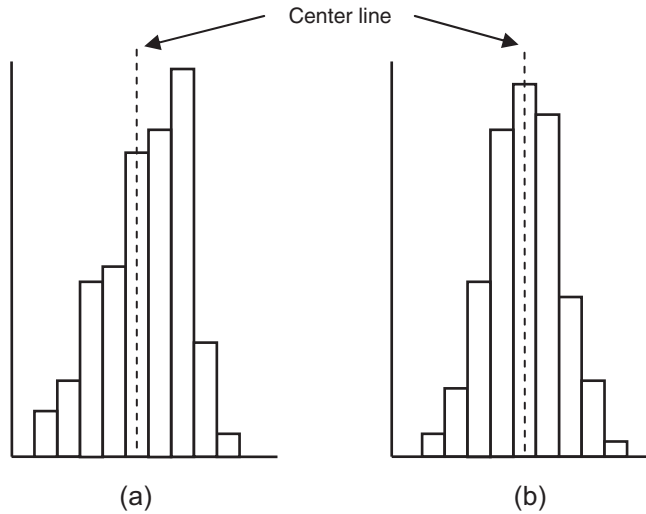


Figure 4.7

Histograms of data drawn to test goodness of fit to a Gaussian distribution. (a) Typical shape for non-Gaussian measurements; (b) typical shape for Gaussian measurements.

However, careful judgment is required, because only a finite number of data values can be used, and therefore the line drawn will not be entirely straight even if the distribution is Gaussian. Considerable experience is needed to judge whether the line is straight enough to indicate a Gaussian distribution. This will be easier to understand if the data in measurement set C are used as an example. Using the same five ranges as used to draw the histogram, the following table is first drawn:

Range:	401.5–403.5	403.5–405.5	405.5–407.5	407.5–409.5	409.5–411.5
Number of data items in range	1	5	11	5	1
Cumulative number of data items	1	6	17	22	23
Cumulative number of data items as percentage	4.3	26.1	73.9	95.7	100.0

The normal probability plot drawn from the above table is shown in Fig. 4.8. This is sufficiently straight to indicate that the data in measurement set C are Gaussian.

4.11.3 Chi-squared test

The chi-squared distribution provides a more formal method for testing whether data follow a Gaussian distribution. The principle of the chi-squared test is to divide the data into p equal-width bins. The appropriate number for p is usually determined by applying the Sturgis rule (see section 4.4). The next step is to count the number of measurements n_i in each bin, using exactly the same procedure as done when drawing a histogram.

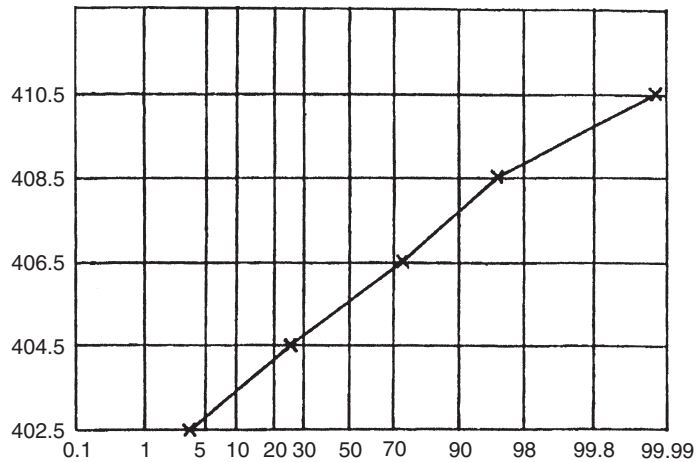


Figure 4.8
Normal probability plot.

The expected number of measurements n_i' in each bin for a Gaussian distribution is also calculated. Before proceeding further, a check must be made at this stage to confirm that at least 80% of bins have a data count greater than a minimum number for both n_i and n_i' . We will apply a minimum number of 4, although some statisticians use the smaller minimum of 3 and some use a larger minimum of 5. If this check reveals that too many bins have data counts less than the minimum number, it is necessary to reduce the number of bins by redefining their widths. Common practice when this is necessary is to effect this reduction by halving the number of bins, by combining them two at a time. In anticipation that this may be necessary, it is better to start initially with an even rather than odd number of bins. This means choosing the next highest even number if the Sturgis rule suggests an odd number of bins. If a reduction in the number of bins has been necessary, the test for at least 80% of the bins exceeding the minimum number has to be reapplied. Once the data count in the bins is satisfactory, a chi-squared value is calculated for the data in each bin. Then, all chi-squared values for the bins are added according to the following formula:

$$\chi^2 = \sum_{i=1}^p \frac{(n_i - n_i')^2}{n_i'} \quad (4.22)$$

The chi-squared test then examines whether the calculated value of χ^2 according to Eq. (4.22) is greater than would be expected for a Gaussian distribution according to some specified level of chance. This involves reading the expected value from the chi-squared distribution table (Table 4.2) for the specified confidence level and comparing this expected value with that calculated in Eq. (4.22). This procedure will become clearer if we work through an example.

■ Example 4.11

A sample of 100 pork pies produced in a bakery is taken and the mass of each pie (in grams) is measured. Apply the chi-squared test to examine whether the data set formed by the set of 100 mass measurements shown below conforms to a Gaussian distribution:

487 504 501 515 491 496 482 502 508 494 505 501 485 503 507 494 489 501 510 491
 503 492 483 501 500 493 505 501 517 500 494 503 500 488 496 500 519 499 495 490
 503 500 497 492 510 506 497 499 489 506 502 484 495 498 502 496 512 504 490 497
 488 503 512 497 480 509 496 513 499 502 487 499 505 493 498 508 492 498 486 511
 499 504 495 500 484 513 509 497 505 510 516 499 495 507 498 514 506 500 508 494

■ Solution

Applying the Sturgis rule, the recommended number of data bins p for N data points is given by:

$$p = 1 + 3.3 \log_{10} N = 1 + (3.3)(2.0000) = 7.6. \text{ This rounds to } 8.$$

The mass measurements span the range 480–519. Hence, we will choose data bin widths of 5 g, with bin boundaries set at 479.5, 484.5, 489.5, 494.5, 499.5, 504.5, 509.5, 514.5, and 519.5 (boundaries are set so that there is no ambiguity about which bin any particular data value fits in). The next step involves counting the number of measurements in each bin. These are the n_i values, $i = 1, \dots, 8$, for Eq. (4.22). The results of this counting are set out in the following table:

Bin number (i)	1	2	3	4	5	6	7	8
Data range	479.5 –484.5	484.5 –489.5	489.5 –494.5	494.5 –499.5	499.5 –504.5	504.5 –509.5	509.5 –514.5	514.5 –519.5
Measurements in range (n_i)	5	8	13	23	24	14	9	4

None of the bins have a count less than the stated minimum threshold of 4 and so we can now proceed to calculate the n_i' values. These are the expected numbers of measurements in each data bin for a Gaussian distribution. The starting point for this calculation is knowing the mean value (μ) and standard deviation of the 100 mass measurements. These are calculated using Eqs. (4.1) and (4.7) as $\mu = 499.53$ and $\sigma = 8.389$. We now calculate the z values corresponding to the measurement values (x) at the upper end of each data bin using Eq. (4.13), and then use the error

function table (Table 4.1) to calculate $F(z)$. $F(z)$ gives the proportion of z values that are $\leq z$, which gives the proportion of measurements less than the corresponding x values. This then allows calculation of the expected number of measurements (n'_i) in each data bin. These calculations are shown in the following table:

x	$z \left(\frac{x - \mu}{\sigma} \right)$	$F(z)$	Expected number of data in bin (n'_i)
484.5	-1.792	0.037	3.7
489.5	-1.195	0.116	7.9
494.5	-0.600	0.274	15.8
499.5	-0.004	0.498	22.4
504.5	0.592	0.723	22.5
509.5	1.188	0.883	16.0
514.5	1.784	0.963	8.0
519.5	2.381	0.991	2.8

In case there is confusion about the calculation of the numbers in the final column, let us consider rows 1 and 2. Row 1 shows that the proportion of data points less than 484.5 is 0.037. Because there are 100 data points in total, the actual estimated number of data points less than 484.5 is 3.7. Row 2 shows that the proportion of data points less than 489.5 is 0.116, and hence the total estimated number of data points less than 489.5 is 11.6. This total includes the 3.7 data points less than 484.5 calculated in the previous row. Hence, the number of data points in this bin between 484.5 and 489.5 is 11.6 minus 3.7 (i.e., 7.9).

We can now calculate the χ^2 value for the data using Eq. (4.22). The steps of the calculation are shown in the following table:

Bin number (p)	n_i	n'_i	$(n_i - n'_i)$	$(n_i - n'_i)^2$	$\frac{(n_i - n'_i)^2}{n'_i}$
1	5	3.7	1.3	1.69	0.46
2	8	7.9	0.1	0.01	0.00
3	13	15.8	-2.8	7.84	0.50
4	23	22.4	0.6	0.36	0.02
5	24	22.5	1.5	2.25	0.10
6	14	16.0	-2.0	4.00	0.25
7	9	8.0	1.0	1.00	0.12
8	4	2.8	1.2	1.44	0.51

The value of χ^2 is now found by summing the values in the final column to give $\chi^2 = 1.96$. The final step is to check whether this value of χ^2 is greater than would be expected for a Gaussian distribution. This involves looking up χ^2 in Table 4.2. Before doing this, we have to specify the number of degrees of freedom, k . In this case, k is the number of bins minus 2, because the data are manipulated twice to obtain the μ and σ statistical values used to calculate n_i' . Hence, $k = 8 - 2 = 6$.

Table 4.2 shows that for $k = 6$, $\chi^2 = 1.64$ for a 95% confidence level and $\chi^2 = 2.20$ for a 90% confidence level. Hence, our calculated value for χ^2 of 1.96 shows that the confidence level that the data follows a Gaussian distribution is between 90 and 95%.

We will now look at a slightly different example in which we meet the problem that the initial division of the data into bins produces too many bins that do not contain the minimum number of data points necessary for the chi-squared test to work reliably.

■ Example 4.12

Suppose the production machinery used to produce the pork pies featured in Example (4.11) is modified to try and reduce the amount of variation in mass. The mass of a new sample of 100 pork pies is then measured. Apply the chi-squared test to examine whether the data set formed by the set of 100 new mass measurements shown below conforms to a Gaussian distribution.

503	509	495	500	504	491	496	499	501	489	507	501	486	497	500	493	499	505	501	495
499	515	505	492	499	502	507	500	498	507	494	499	506	501	493	498	505	499	496	512
498	502	508	500	497	485	504	499	502	496	483	501	510	494	498	505	491	499	503	495
502	481	498	503	508	497	511	490	506	500	508	504	517	494	487	505	499	509	492	484
500	507	501	496	510	503	498	490	501	492	497	489	502	495	491	500	513	499	494	498

■ Solution

The recommended number of data bins for 100 measurements according to the Sturges rule is 8, as calculated in Example (4.11). The mass measurements in this new data set span the range 481–517. Hence, data bin widths of 5 g are still suggested, with bin boundaries set at 479.5, 484.5, 489.5, 494.5, 499.5, 504.5, 509.5, 514.5,

and 519.5. The number of measurements in each bin is then counted, with the counts given in the following table:

Bin number (i)	1	2	3	4	5	6	7	8
Data range	479.5 –484.5	484.5 –489.5	489.5 –494.5	494.5 –499.5	499.5 –504.5	504.5 –509.5	509.5 –514.5	514.5 –519.5
Measurements in range (n_i)	3	5	14	29	26	16	5	2

Looking at these counts, we see that there two bins have a count less than four. This amounts to 25% of the data bins. We previously said that not more than 20% of the data bins can have a data count less than the threshold of 4 if the chi-squared test is to operate reliably. Hence, we must combine the bins and count the measurements again. The usual approach is to combine pairs of bins, which in this case reduces the number of bins from eight to four. The boundaries of the new set of four bins are now 479.5, 489.5, 499.5, 509.5, and 519.5. The new data ranges and counts are shown in the following table:

Bin number (i)	1	2	3	4
Data range	479.5–489.5	489.5–499.5	499.5–509.5	509.5–519.5
Measurements in range (n_i)	8	43	42	7

Now, none of the bins have a count less than the stated minimum threshold of 4 and so we can proceed to calculate the n_i' values as before. The mean value (μ) and standard deviation of the new mass measurements are $\mu = 499.39$ and $\sigma = 6.979$. We now calculate the z values corresponding to the measurement values (x) at the upper end of each data bin, read the corresponding $F(z)$ values from [Table 4.1](#), and so calculate the expected number of measurements (n_i') in each data bin:

x	$z\left(\frac{x-\mu}{\sigma}\right)$	$F(z)$	Expected number of data in bin (n_i')
489.5	–1.417	0.078	7.8
499.5	–0.016	0.494	41.6
509.5	1.449	0.926	43.2
519.5	2.882	0.998	7.2

(Note: Because the values for z are calculated to an accuracy of 3 figures after the decimal point, but the error function table used to calculate $F[z]$ gives values only for z values with

a maximum of two figures after the decimal point, interpolation between the nearest $F[z]$ values has to be carried out, the details of which are explained in Appendix 4.)

In case there is confusion about the calculation of numbers in the final column, let us consider rows 1 and 2. Row 1 shows that the proportion of data points less than 489.5 is 0.078. Because there are 100 data points in total, the actual estimated number of data points less than 484.5 is 7.8. Row 2 shows that the proportion of data points less than 499.5 is 0.494, and hence the total estimated number of data points less than 499.5 is 49.4. This total includes the 7.8 data points less than 489.5 calculated in the previous row. Hence, the number of data points in this bin between 489.5 and 499.5 is $49.4 - 7.8$ (i.e., 41.6).

We now calculate the χ^2 value for the data using Eq. (4.22). The steps of the calculation are shown in the table below:

Bin number (p)	n_i	n'_i	$(n_i - n'_i)$	$(n_i - n'_i)^2$	$\frac{(n_i - n'_i)^2}{n'_i}$
1	8	7.8	0.2	0.04	0.005
2	43	41.6	1.4	1.96	0.047
3	42	43.2	-1.2	1.44	0.033
4	7	7.2	-0.2	0.04	0.006

The value of χ^2 is found by summing the values in the final column to give $\chi^2 = 0.091$. The final step is to check whether this value of χ^2 is greater than would be expected for a Gaussian distribution. This involves looking up χ^2 in Table 4.2. This time, $k = 2$, because there are 4 bins and k is the number of bins $- 2$ (as explained in Example 4.11, the data was manipulated twice to obtain the μ and σ statistical values used to calculate n'_i).

Table 4.2 shows that for $k = 2$, $\chi^2 = 0.10$ for a 95% confidence level. Hence, our calculated value for χ^2 of 0.91 shows that the confidence level that the data follow a Gaussian distribution is slightly better than 95%. ■

Out of interest, if the two bin counts less than 4 had been ignored and χ^2 had been calculated for the 8 original data bins, a value of $\chi^2 = 2.97$ would have been obtained. (It would be a useful exercise for the reader to check this for himself or herself). For six degrees of freedom ($k = 8 - 2$), the predicted value of χ^2 for a Gaussian population from Table 4.2 is 2.20 at a 90% confidence level. Thus, the confidence that the data fit a Gaussian distribution is substantially less than 90% given the χ^2 value of 2.97 calculated for the data. This result arises because of the unreliability associated with calculating χ^2 from data bin counts of less than four.

4.12 Rogue data points (data outliers)

In a set of measurements subject to random error, measurements with a very large error sometimes occur at random and unpredictable times, in which the magnitude of the error is much larger than could reasonably be attributed to the expected random variations in measurement value. These are often called *rogue data points* or *data outliers*. Sources of such abnormal error include sudden transient voltage surges on the mains power supply and incorrect recording of data (e.g., writing down 146.1 when the actual measured value was 164.1). It is accepted practice in such cases to discard these rogue measurements, and a threshold level of a $\pm 3\sigma$ deviation is often used to determine what should be discarded. It is rare for measurement errors to exceed $\pm 3\sigma$ limits when only normal random effects affect the measured value.

Although this represents a reasonable theoretical approach to identifying and eliminating rogue data points, practical implementation of such a procedure needs to be done carefully. The main practical difficulty that exists in dealing with rogue data points is in establishing what the expected standard deviation of the measurements is. When a new set of measurements is being taken in which the expected standard deviation is unknown, the possibility exists that a rogue data point exists within the measurements. Simply applying a computer program to the measurements to calculate the standard deviation will produce an erroneous result because the calculated value will be biased by the rogue data point. The simplest way to overcome this difficulty is to plot a histogram of any new set of measurements and examine this manually to spot any data outliers. If no outliers are apparent, the standard deviation can be calculated and then used in a $\pm 3\sigma$ threshold against which to test all future measurements. However, if this initial data histogram shows up any outliers, these should be excluded from calculation of the standard deviation.

■ Example 4.13

A set of measurements is made with a new pressure transducer. Inspection of a histogram of the first 20 measurements shows no data outliers. The standard deviation of the measurements is calculated as 0.05 bar after this check for data outliers, and the mean value is calculated as 4.41. After this, the following further set of measurements is obtained:

4.35 4.46 4.39 4.34 4.41 4.52 4.44 4.37 4.41 4.33 4.39 4.47 4.42 4.59 4.45 4.38 4.43 4.36 4.48 4.45

Use the $\pm 3\sigma$ threshold to determine whether there are rogue data points in the measurement set.

■ Solution

Because the calculated σ value for a set of good measurements is given as 0.05, the $\pm 3\sigma$ threshold is ± 0.15 . With a mean data value of 4.41, the threshold for rogue data points is values below 4.26 (mean value $- 3\sigma$) or above 4.56 (mean value $+ 3\sigma$). Looking at the set of measurements, we observe that the measurement of 4.59 is outside the $\pm 3\sigma$ threshold, indicating that this is a rogue data point. ■

It is interesting to return to the problem of ensuring there are no outliers in the set of data used to calculate the standard deviation of the data and hence the threshold for rejecting outliers. We have suggested a histogram of some initial measurements be drawn and examined for outliers. What would happen if the set of data given in Example 4.13 was the initial data set that was examined for outliers by drawing a histogram? What would happen if we did not spot the outlier of 4.59? This question can be answered by looking at the effect on the calculated value of standard deviation if this rogue data point of 4.59 is included in the calculation. The standard deviation calculated over the 19 values excluding the 4.59 measurement is 0.052. The standard deviation calculated over the 20 values including the 4.59 measurement is 0.063 and the mean data value is changed to 4.42. This gives a 3σ threshold of 0.19, and the boundaries for the $\pm 3\sigma$ threshold operation are now 4.23 and 4.61. This does not exclude the data value of 4.59, which we previously identified as a being a rogue data point! This confirms the necessity of carefully looking at the initial set of data used to calculate the thresholds for rejection of rogue data point to ensure that the initial data do not contain rogue data points. If drawing and examining a histogram does not clearly show that there are no rogue data points in the reference set of data, it is worth taking another set of measurements to see whether a reference set of data can be obtained that is more clearly free of rogue data points.

The discussion so far has suggested that it is always necessary to have a reference data set for which the absence of rogue data points has been confirmed by drawing a histogram. However, there are some occasions when a rogue data point can be spotted even without the tedium of drawing a histogram, as demonstrated by Example 4.14.

■ Example 4.14

Consider the following data set of 20 measurements and examine it for rogue data points:

10.4 9.9 9.7 9.6 10.1 10.3 9.8 10.0 10.2 9.5 9.8 10.1 10.3 8.1 9.7 10.2 10.3 9.7 9.9 10.2

 ■

■ Solution

The measurement of 8.1 clearly looks like a rogue data point. However, this must be confirmed. The mean and standard deviation of the other 19 data points (excluding the value of 8.1) are calculated as: mean = 9.98 and standard Deviation (σ) = 0.273. This mean that $3\sigma = 0.819$. Hence, the $\pm 3\sigma$ limits on either side of the mean value are at 9.16 and 10.80. This confirms that the data point at 8.1 is well outside the $\pm 3\sigma$ limits and is therefore confirmed as a rogue data point. ■

4.13 Student *t* distribution

When the number of measurements of a quantity is particularly small (less than about 30 samples) and statistical analysis of the distribution of error values is required, possible deviation of the mean of the measurements from the true measurement value (the mean of the infinite population of which the sample is part) may be significantly greater than is suggested by analysis based on a *z* distribution. In response to this, a statistician called William Gosset developed an alternative distribution function that gives a more accurate prediction of the error distribution when the number of samples is small. He published this under the pseudonym Student, and the distribution is commonly called the ***Student t distribution***. The *t* distribution has the same requirement as the *z* distribution in terms of the necessity for the data to belong to a Gaussian distribution.

The Student *t* variable expresses the difference between the mean of a small sample (x_{mean}) and the population mean (μ) in terms of the following ratio:

$$t = \frac{|\text{error in mean}|}{\text{standard error of the mean}} = \frac{|\mu - x_{mean}|}{\sigma/\sqrt{N}} \quad (4.23)$$

where N is the number of samples and σ is the standard error of the mean of the infinite population of which the N samples are part. Because we do not know the exact value of σ , we have to use the best approximation to σ that we have, which is the standard deviation of the sample σ_x . Substituting this value for σ in Eq. (4.23) gives:

$$t = \frac{|\mu - x_{mean}|}{\sigma_x/\sqrt{N}} \quad (4.24)$$

Note that the modulus operation ($|\dots|$) on the error in the mean in Eqs. (4.23) and (4.24) means that t is always positive.

The shape of the probability distribution curve $F(t)$ of the *t* variable varies according to the value of the number of degrees of freedom, k ($= N - 1$), with typical curves being shown

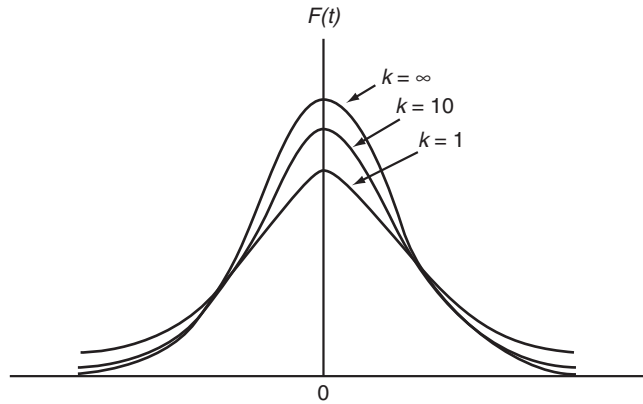


Figure 4.9
Typical t distribution curves.

in Fig. 4.9. As $k \rightarrow \infty$, $F(t) \rightarrow F(z)$ (i.e., the distribution becomes a standard Gaussian one). For values of $k < \infty$, the curve of $F(t)$ against t is both narrower and less high in the center than a standard Gaussian curve, but it has the same properties of symmetry about $t = 0$ and a total area under the curve of unity.

In a similar way to the z distribution, the probability that t will lie between two values t_1 and t_2 is given by the area under the $F(t)$ curve between t_1 and t_2 . The t distribution is published in the form of a standard table (Table 4.3) that gives values of the area under the curve α for various values of the number of degrees of freedom (k), where:

$$\alpha = \int_{t_\alpha}^{\infty} F(t) dt \quad (4.25)$$

The area α is shown in Fig. 4.10. α corresponds to the probability that t will have a value greater than t_α to some specified confidence level. Because the total area under the $F(t)$ curve is unity, there is also a probability of $(1 - \alpha)$ that t will have a value less than t_α . Thus, for a value $\alpha = 0.05$, there is a 95% probability (i.e., a 95% confidence level) that $t < t_\alpha$.

Because of the symmetry of the t distribution, α is also given by:

$$\alpha = \int_{-\infty}^{-t_\alpha} F(t) dt \quad (4.26)$$

as shown in Fig. 4.11. Here, α corresponds to the probability that t will have a value less than $-t_\alpha$, with a probability of $(1 - \alpha)$ that t will have a value greater than $-t_\alpha$.

Table 4.3: *t* distribution.

<i>k</i>	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	$t_{0.001}$
1	3.078	6.314	12.71	31.82	63.66	318.3
2	1.886	2.920	4.303	6.965	9.925	23.33
3	1.638	2.353	3.182	4.541	5.841	10.21
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385

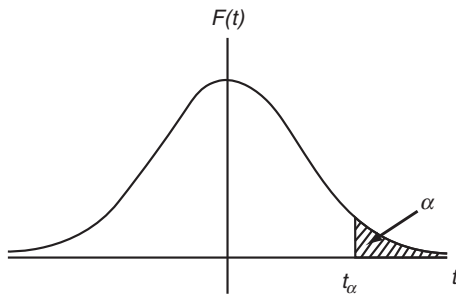


Figure 4.10

Meaning of area α for *t* distribution curve.

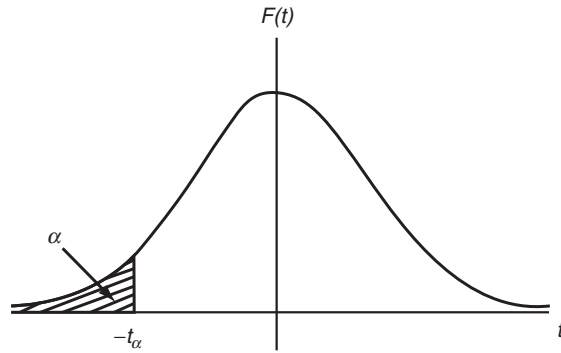


Figure 4.11

Alternative interpretation of area α for t distribution curve.

■ Example 4.15

Calculate the value of the variable t for a set of 15 measurements and a 97.5% confidence level.

■ Solution

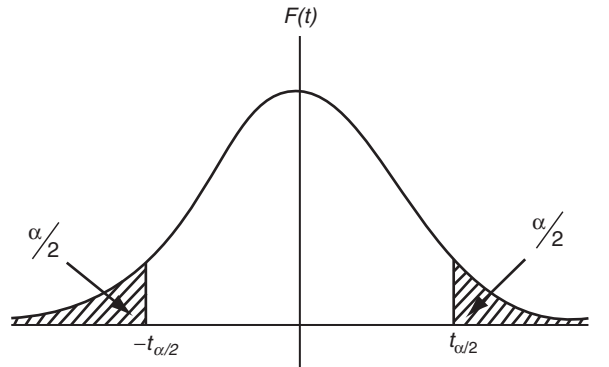
The number of degrees of freedom (k) is one less than the number of measurements. Thus, $k = 14$.

For a 97.5% confidence level, $\alpha = 1 - 0.975 = 0.025$

The values of t corresponding to a value $\alpha = 0.025$ are to be found in the fourth column of [Table 4.3](#). Reading along the row in the table corresponding to $k = 14$ as far as the fourth column gives a value of t of 2.145

[Eqs. \(4.25\) and \(4.26\)](#) can be combined to express the probability $(1 - \alpha)$ that t lies between two values, $-t_4$ and $+t_4$. In this case, α is the sum of two areas of $\alpha/2$, as shown in [Fig. 4.12](#). These two areas can be represented mathematically as:

$$\frac{\alpha}{2} = \int_{-\infty}^{-t_4} F(t) dt \quad (\text{left-hand area}) \quad \text{and} \quad \frac{\alpha}{2} = \int_{t_4}^{\infty} F(t) dt \quad (\text{right-hand area})$$


Figure 4.12

Area between $-\alpha/2$ and $+\alpha/2$ on t distribution curve.

The values of t_4 can be found in any t distribution table, such as [Table 4.3](#).

Referring back to [Eq. \(4.24\)](#), this can be expressed in the form:

$$\left| \mu - x_{\text{mean}} = \frac{t\sigma_x}{\sqrt{N}} \right|$$

Hence, the upper and lower bounds on the expected value of the population mean μ (the true value of x) can be expressed as:

$$-\frac{t_4\sigma_x}{\sqrt{N}} \leq \mu - x_{\text{mean}} \leq +\frac{t_4\sigma_x}{\sqrt{N}}$$

or

$$x_{\text{mean}} - \frac{t_4\sigma_x}{\sqrt{N}} \leq \mu \leq x_{\text{mean}} + \frac{t_4\sigma_x}{\sqrt{N}} \quad (4.27)$$

■ Example 4.16

The internal diameter of a sample of hollow castings is measured by destructive testing of 15 samples taken randomly from a large batch of castings. If the sample mean is 105.4 mm with a standard deviation of 1.9 mm, express the upper and lower bounds to a confidence level of 95% on the range in which the mean value lies for the internal diameter of the whole batch. ■

■ Solution

For 15 samples ($N = 15$), the number of degrees of freedom (k) = 14.

For a confidence level of 95%, $\alpha = 1 - 0.95 = 0.05$. Looking up the value of t in [Table 4.3](#) for $k = 14$ and $\alpha/2 = 0.025$ gives $t = 2.145$. Thus, applying [Eq. \(4.27\)](#):

$$105.4 - \frac{(2.145)(1.9)}{\sqrt{15}} \leq \mu \leq 105.4 + \frac{(2.145)(1.9)}{\sqrt{15}}, \text{ i.e. } 104.3 \leq \mu \leq 106.5$$

Thus, we would express the mean internal diameter of the whole batch of castings as 105.4 ± 1.1 mm. ■

■ Example 4.17

The width of the air gap in some double glazing panels is measured by destructive testing of 6 panels. If the mean gap for the sample is 19.7 mm with a standard deviation of 0.6 mm, express the upper and lower bounds to a confidence level of 99% on the range in which the mean value lies for the air gap in the whole batch. ■

■ Solution

For six samples ($N = 6$), the number of degrees of freedom (k) = 5.

For a confidence level of 99%, $\alpha = 1 - 0.99 = 0.01$. Looking up the value of t in [Table 4.3](#) for $k = 5$ and $\alpha/2 = 0.005$ gives $t = 4.032$. Thus, applying [Eq. \(4.27\)](#):

$$19.7 - \frac{(4.032)(0.6)}{\sqrt{6}} \leq \mu \leq 19.7 + \frac{(4.032)(0.6)}{\sqrt{6}}, \text{ i.e. } 18.7 \leq \mu \leq 20.7$$

Thus, we would express the mean internal diameter of the whole batch of panels as 19.7 ± 1.0 mm. ■

Out of interest, let us examine what would have happened if we had calculated the error bounds on μ using standard Gaussian (z distribution) tables. For 95% confidence, the maximum error is given as $\pm 1.96\sigma/\sqrt{N}$ (i.e., ± 0.48 , which rounds to ± 0.5 mm, meaning the mean internal diameter is given as 19.7 ± 0.5 mm). The effect of using the t distribution instead of the z distribution clearly expands the magnitude of the likely error in the mean value to compensate for the fact that our calculations are based on a relatively small number of measurements.

4.14 Aggregation of measurement system errors

Errors in measurement systems often arise from two or more different sources, and these must be aggregated in the correct way to obtain a prediction of the total likely error in output readings from the measurement system. Two different forms of aggregation are required. First, a single measurement component may have both systematic and random errors; second, a measurement system may consist of several measurement components that each have separate errors.

4.14.1 Combined effect of systematic and random errors

If a measurement is affected by both systematic and random errors that are quantified as $\pm x$ (systematic errors) and $\pm y$ (random errors), some means of expressing the combined effect of both types of error is needed. One way to express the combined error would be to sum the two separate components of error (i.e., to say that the total possible error is $e = \pm [x + y]$). However, a more usual course of action is to express the likely maximum error as:

$$e = \sqrt{(x^2 + y^2)} \quad (4.28)$$

It can be shown that this is the best expression for the error statistically, because it considers the reasonable assumption that the systematic and random errors are independent, and so it is unlikely that both will be at their maximum or minimum value simultaneously.

4.14.2 Aggregation of errors from separate measurement system components

A measurement system often consists of several separate components, each of which is subject to errors. Therefore, what remains to be investigated is how the errors associated with each measurement system component combine, so that a total error calculation can be made for the complete measurement system. All four mathematical operations of addition, subtraction, multiplication, and division may be performed on measurements derived from different instruments/transducers in a measurement system. Appropriate techniques for the various situations that arise are covered next.

Error in a sum

If the two outputs y and z of separate measurement system components are to be added together, we can write the sum as: $S = y + z$. If the maximum errors in y and z are $\pm ay$ and $\pm bz$, respectively, we can express the maximum and minimum possible values of S as:

$$S_{\max} = (y + ay) + (z + bz); S_{\min} = (y - ay) + (z - bz); \text{ or } S = y + z \pm (ay + bz)$$

This relationship for S is inconvenient because in this form the error term cannot be expressed as a fraction or percentage of the calculated value for S . Fortunately, statistical analysis can be applied that expresses S in an alternative form such that the most probable maximum error in S is represented by a quantity e , in which e is calculated in terms of the *absolute* errors as:

$$e = \sqrt{(ay)^2 + (bz)^2} \quad (4.29)$$

Thus: $S = (y + z) \pm e$. This can be expressed in the alternative form:

$$S = (y + z)(1 \pm f) \quad \text{where } f = e/(y + z) \quad (4.30)$$

Eqs. (4.29) and (4.30) are valid only provided that the measurements are uncorrelated (i.e., each measurement is entirely independent of the others).

■ Example 4.18

A circuit requirement for a resistance of 550Ω is satisfied by connecting together two resistors of nominal values 220 and 330Ω in series. If each resistor has a tolerance of $\pm 2\%$, the error in the sum calculated according to Eqs. (4.29) and (4.30) is given by:

$$e = \sqrt{(0.02 \times 220)^2 + (0.02 \times 330)^2} = 7.93; \quad f = 7.93/550 = 0.0144$$

Thus, the total resistance S can be expressed as: $S = 550 \Omega \pm 7.93 \Omega$

or $S = 550 (1 \pm 0.0144) \Omega$ (i.e., $S = 550 \Omega \pm 1.4\%$).

Error in a difference

If the two outputs y and z of separate measurement systems are to be subtracted from one another, and the possible errors are $\pm ay$ and $\pm bz$, the difference S can be expressed (using statistical analysis as for calculating the error in a sum and assuming that the measurements are uncorrelated) as:

$$S = (y - z) \pm e \quad \text{or} \quad S = (y - z)(1 \pm f)$$

where e is calculated as in (Eq. 4.29) and $f = e/(y - z)$.

■ Example 4.19

A fluid flow rate is calculated from the difference in pressure measured on both sides of an orifice plate. If the pressure measurements are 10.0 and 9.5 bar and the error

in the pressure measuring instruments is specified as $\pm 0.1\%$, values for e and f can be calculated as:

$$e = \sqrt{(0.001 \times 10)^2 + (0.001 \times 9.5)^2} = 0.0138 ; f = 0.0138/0.5 = 0.0276$$

- *This example illustrates very poignantly the relatively large error that can arise when calculations are made based on the difference between two measurements.*

Error in a product

If outputs y and z of two measurement system components are multiplied, the product can be written as: $P = yz$. If the possible error in y is $\pm ay$ and in z is $\pm bz$, then the maximum and minimum values possible in P can be written as:

$$P_{\max} = (y + ay)(z + bz) = yz + ayz + byz + aybz;$$

$$P_{\min} = (y - ay)(z - bz) = yz - ayz - byz + aybz$$

For typical measurement system components with output errors of up to 1% or 2% in magnitude, both a and b are much less than 1 in magnitude and thus terms in $aybz$ are negligible compared with other terms. Therefore, we have: $P_{\max} = yz(1 + a + b)$; $P_{\min} = yz(1 - a - b)$. Thus, the maximum error in product P is $\pm(a + b)$. Although this expresses the maximum possible error in P , it tends to overestimate the likely maximum error, because it is unlikely that the errors in y and z will both be at the maximum or minimum value at the same time. A statistically better estimate of the likely maximum error e in the product P , provided that the measurements are uncorrelated, is given by:

$$e = \sqrt{a^2 + b^2} \quad (4.31)$$

In the case of multiplicative errors, e is calculated in terms of the *fractional* errors in y and z (as opposed to the *absolute* error values used to calculate additive errors).

■ Example 4.20

If the power in a circuit is calculated from measurements of voltage and current in which the calculated maximum errors are, respectively, $\pm 1\%$ and $\pm 2\%$, then the maximum likely error in the calculated power value, calculated using (Eq. 4.31) is $\pm \sqrt{0.01^2 + 0.02^2} = \pm 0.022$ or $\pm 2.2\%$.

Error in a quotient

If the output measurement y of one system component with possible error $\pm ay$ is divided by the output measurement z of another system component with possible error $\pm bz$, then the maximum and minimum possible values for the quotient can be written as:

$$Q_{\max} = \frac{y + ay}{z - bz} = \frac{(y + ay)(z + bz)}{(z - bz)(z + bz)} = \frac{yz + ayz + byz + aybz}{z^2 - b^2z^2};$$

$$Q_{\min} = \frac{y - ay}{z + bz} = \frac{(y - ay)(z - bz)}{(z + bz)(z - bz)} = \frac{yz - ayz - byz + aybz}{z^2 - b^2z^2}$$

For $a \ll 1$ and $b \ll 1$, terms in ab and b^2 are negligible compared with the other terms.

Hence: $Q_{\max} = \frac{yz(1 + a + b)}{z^2}$; $Q_{\min} = \frac{yz(1 - a - b)}{z^2}$; i.e., $Q = \frac{y}{z} \pm \frac{y}{z}(a + b)$.

Thus, the maximum error in the quotient is $\pm(a + b)$. However, using the same argument as made earlier for the product of measurements, a statistically better estimate of the likely maximum error in quotient Q , provided the measurements are uncorrelated, is that given in Eq. (4.31).

■ Example 4.21

If the density of a substance is calculated from measurements of its mass and volume, in which the respective errors are $\pm 2\%$ and $\pm 3\%$, then the maximum likely error in the density value using (Eq. 4.31) is $\pm\sqrt{0.02^2 + 0.003^2} = \pm 0.036$ or $\pm 3.6\%$. ■

4.14.3 Total error when combining multiple measurements

The final case to be covered is that in which the final measurement is calculated from several measurements that are combined together in a way that involves more than one type of arithmetic operation. For example, the density of a rectangular-sided solid block of material can be calculated from measurements of its mass divided by the product of measurements of its length, height, and width. The errors involved in each stage of arithmetic are cumulative, and so the total measurement error can be calculated by adding together the two error values associated with the two multiplication stages involved in calculating the volume and then calculating the error in the final arithmetic operation when the mass is divided by the volume.

■ Example 4.22

A rectangular-sided block has edges of lengths a , b , and c , and its mass is m . If the values and possible errors in quantities a , b , c , and m are as shown next, calculate the value of density and the possible error in this value:

$a = 100 \text{ mm} \pm 1\%$, $b = 200 \text{ mm} \pm 1\%$, $c = 300 \text{ mm} \pm 1\%$, and $m = 20 \text{ kg} \pm 0.5\%$.

■ Solution

Value of $ab = 0.02 \text{ m}^2 \pm 2\%$ (possible error = $1\% + 1\% = 2\%$)

Value of $(ab)c = 0.006 \text{ m}^3 \pm 3\%$ (possible error = $2\% + 1\% = 3\%$)

Value of $\frac{m}{abc} = \frac{20}{0.006} = 3330 \text{ kg/m}^3 \pm 3.5\%$ (possible error = $3\% + 0.5\% = 3.5\%$)

4.15 Summary

This chapter has been the second of two chapters dealing with measurement uncertainty. Chapter 3 introduced the subject and established that there were two distinct forms of measurement uncertainty, known respectively as *systematic error* and *random error*. This last chapter then went on to consider systematic errors in detail, especially the sources of such errors, ways to reduce their magnitude, and the techniques for quantifying the remaining error after all reasonable means of reducing error magnitude had been applied. This chapter then went on to introduce random errors and explain their nature and typical sources. There was also a brief introduction to the means available for quantifying random errors. The current chapter continued where Chapter 3 left off in examining random errors in much greater detail and describing the various means available for analyzing random errors.

The starting point in this chapter was to examine the two alternative ways to calculate the average value of a set of measurements of a constant quantity: the *mean* and *median* values. We then explained how the way in which measurements are spread about the average value affects the degree of confidence that the calculated mean value is close to the correct value of the measured quantity. We explained the calculation of the two quantities of *standard deviation* and *variance* of measurement data, these being parameters that express how the measurements are distributed about the mean value.

Following from this, we started to look at graphical ways to express the spread. Initially, we considered representations of spread as a *histogram*, then, we showed how histograms expand into *frequency distributions* in the form of a smooth curve. We found that truly random data are described by a particular form of frequency distribution known as *Gaussian* (or *normal*). We introduced the *z variable* and saw how this can be used to estimate the number of measurements in a set of measurements that have an error magnitude between two specified values. After this, we started to look at implications of the fact that we can only ever have a finite number of measurements. We saw that a variable called the *standard error of the mean* could be calculated, which estimates the difference between the mean value of a finite set of measurements and the true value of the measured quantity (the mean of an infinite data set). We went on to look at how this was useful for estimating the likely error in a single measurement subject to random errors, in the situation in which it is impossible to average over a number of measurements. As an aside, we then looked at how the *z* variable was useful for analyzing the tolerances of manufactured components subject to random variations, in a parallel way to the analysis of measurements subject to random variations. After this, we examined the *chi-squared distribution*. This can be used to quantify variation in the variance of a finite set of measurements with respect to the variance of the infinite set of which the finite set is part. Until this point in the chapter, all analysis of random errors assumed that the measurement set fitted a Gaussian distribution. However, this assumption must always be justified by applying *goodness of fit tests*, and so these were explained in the next section, in which we saw that a chi-squared test is the most rigorous test available for goodness of fit. A particular problem that can adversely affect the analysis of random errors is the presence of rogue data points (data outliers) in the measurement data. These were considered, and the conditions under which they can be justifiably excluded from the analyzed data set was explored. Finally, we saw that yet another problem that can affect the analysis of random errors is when the measurement set has only a small number of values. In this case, calculations based on the *z* distribution are inaccurate, and we explored the use of a better distribution called the *t distribution*.

The chapter ended with looking at how the effect of different measurement errors are aggregated to predict the total error in a measurement system. This process was considered in two parts. First, we looked at how systematic and random error magnitudes can be combined in an optimal way that best predicts the likely total error in a particular measurement. Second, we looked at situations in which two or more measurements of different quantities are combined to give a composite measurement value, and the best way to deal with each of the four arithmetic operations that can be carried out on different measurement components.

4.16 Problems

- 4.1 In a survey of 15 owners of a certain model of car, the following values for average fuel consumption were reported.
 25.5 30.3 31.1 29.6 32.4 39.4 28.9 30.0 33.3 31.4 29.5 30.5 31.7 33.0 29.2
 Calculate the mean value, median value, and standard deviation of the data set.
- 4.2 The following 10 measurements of the freezing point of aluminum were made using a platinum–rhodium thermocouple.
 658.2 659.8 661.7 662.1 659.3 660.5 657.9 662.4 659.6 662.2
 Find (a) the median, (b) the mean, (c) the standard deviation, and (d) the variance of the measurements.
- 4.3 The following 25 measurements were taken of the thickness of steel emerging from a rolling mill.
 3.97 3.99 4.04 4.00 3.98 4.03 4.00 3.98 3.99 3.96 4.02 3.99 4.01 3.97 4.02 3.99 3.95
 4.03 4.01 4.05 3.98 4.00 4.04 3.98 4.02
 Find (a) the median, (b) the mean, (c) the standard deviation, and (d) the variance of the measurements.
- 4.4 The following 10 measurements are made of the output voltage from a high-gain amplifier that is contaminated owing to noise fluctuations:
 1.53, 1.57, 1.54, 1.54, 1.50, 1.51, 1.55, 1.54, 1.56, 1.53
 Determine the mean value and standard deviation. Then, estimate the accuracy to which the mean value is determined from these 10 measurements. If 1000 measurements were taken instead of 10, but σ remained the same, by how much would the accuracy of the calculated mean value be improved?
- 4.5 The following measurements were taken with an analog meter of the current (mA) flowing in a circuit (the circuit was in steady-state and therefore, although the measurements varied owing to random errors, the current flowing was actually constant):
 21.5, 22.1, 21.3, 21.7, 22.0, 22.2, 21.8, 21.4, 21.9, 22.1mA.
 Calculate the mean value, the deviations from the mean, and the standard deviation.
- 4.6 A batch of digital garden thermometers is tested to determine their accuracy. They are placed in an environment maintained at 25°C, which is measured using an accurate calibrated resistance thermometer. The following readings in degrees Celsius are obtained from a batch of 10 of the garden thermometers:
 24.6 27.2 23.9 26.5 25.4 23.8 26.4 25.9 24.1 25.2
 Determine the mean value and standard deviation of these measurements.
- 4.7 A pressure control system in a distillation column is designed to maintain the pressure at a constant value of 10.9 bar. The following 10 pressure readings (in bars) were obtained at intervals of 10 minutes:
 10.7, 11.0, 11.2, 10.8, 10.6, 11.1, 10.9, 10.8, 10.7, 11.0
 Determine the mean and standard deviation of the pressure measurements.

- 4.8 Using the measurement data given in problem 4.3, draw a histogram of errors (use five error bands that are each 0.03 units wide [i.e., the center band will be from -0.015 to $+0.015$]).
- 4.9 (a) Calculate the recommended bin size using the Sturgis rule for a histogram of the following set of 25 measurements: 9.4, 10.1, 9.1, 12.3, 10.3, 10.0, 10.5, 9.0, 10.8, 9.9, 11.1, 9.8, 7.6, 9.2, 10.7, 8.4, 11.0, 9.7, 11.3, 8.7, 9.9, 11.5, 10.0, 9.5, 11.9
- (b) Draw a histogram of the set of measurements with a bin size of 6.
- (c) Draw a histogram of the set of measurements with a bin size of 5.
- (d) Which histogram looks most symmetrical?
- 4.10 (a) When drawing a histogram of measurements, how do you decide on the width of the data bins to use? How do you ensure that there is no ambiguity about which data bin into which to put any particular measurement?
- (b) Draw a histogram of the following 30 data values, which are believed to belong to a Gaussian distribution:
- 50.0, 51.4, 49.4, 50.6, 49.8, 50.9, 49.2, 47.9, 49.9, 50.7, 50.1, 49.0, 50.6, 51.8, 49.1, 50.6, 49.7, 49.2, 49.8, 52.3, 50.2, 48.3, 50.1, 50.9, 48.9, 49.8, 49.6, 50.3, 49.2, 50.8.
- 4.11 (a) What do you understand by the term *probability density function*? Write down an expression for a Gaussian probability density function of given mean value μ and standard deviation σ , and show how you would obtain the best estimate of these two quantities from a sample of population n .
- 4.12 The measurements in a data set are subject to random errors, but it is known that the data set fits a Gaussian distribution. Use standard Gaussian tables to determine the percentage of measurements that lie within the boundaries of $\pm 1.5\sigma$, where σ is the standard deviation of the measurements.
- 4.13 The measurements in a data set are subject to random errors, but it is known that the data set fits a Gaussian distribution. Use error function tables to determine the value of x required such that 95% of the measurements lie within the boundaries of $\pm x\sigma$, where σ is the standard deviation of the measurements.
- 4.14 By applying error function tables for the mean and standard deviation values calculated in problem 4.3, estimate:
- (a) How many measurements are less than 4.00
- (b) How many measurements are less than 3.95
- (c) How many measurements are between 3.98 and 4.02
- Check your answers against the real data.
- 4.15 A set of 25 measurement have the following values:
- 9.4, 10.1, 9.1, 12.3, 10.3, 10.0, 10.5, 9.0, 10.8, 10.0, 11.1, 9.8, 7.6, 9.2, 10.7, 8.4, 11.0, 9.7, 11.3, 8.7, 9.9, 11.5, 10.0, 9.5, 11.9

The mean value and standard deviation of these measurements are $x_{\text{mean}} = 10.072$ and standard deviation (σ) = 1.1108.

By applying error function tables to the mean and standard deviation values given, estimate:

- (a) How many measurements are less than 11.05
- (b) How many measurements are greater than 9.55
- (c) How many measurements are between 9.95 and 10.95

Check your answers against the real data.

- 4.16 The resolution of the instrument referred to in problem 4.3 is clearly 0.01. Because of the way in which error tables are presented, estimation of the number of measurements in a particular error band is likely to be closer to the real number if the boundaries of the error band are chosen to be between measurement values. In part (c) of problem 4.14, values greater than 3.98 are subtracted from values greater than 4.02, thus excluding measurements equal to 3.98. Test this hypothesis by estimating:

- (a) How many measurements are less than 3.995
- (b) How many measurements are less than 3.955
- (c) How many measurements are between 3.975 and 4.025

Check your answers against the real data.

- 4.17 The measurements in a data set are subject to random errors, but it is known that the data set fits a Gaussian distribution. Use error function tables to determine the percentage of measurements that lie within the boundaries of $\pm 2\sigma$, where σ is the standard deviation of the measurements.

- 4.18 A silicon integrated circuit chip contains 5000 ostensibly identical transistors. Measurements are made of the current gain of each transistor. The measurements have a mean of 20.0 and a standard deviation of 1.5. The probability distribution of the measurements is Gaussian.

- (a) Write down an expression for the number of transistors on the chip that have a current gain between 19.5 and 20.5.
- (b) Show that this number of transistors with a current gain between 19.5 and 20.5 is approximately 1300.
- (c) Calculate the number of transistors that have a current gain of 17 or more (this is the minimum current gain necessary for a transistor to be able to drive the succeeding stage of the circuit in which the chip is used).

- 4.19 In a survey of 12 owners of a certain model of car, the following values for average fuel consumption were reported:

25.5, 31.1, 29.6, 32.4, 39.4, 28.9, 33.3, 31.4, 29.5, 30.5, 31.7, 29.2

Calculate the mean value, the standard deviation, and the standard error of the mean for the reported consumption values. Express the mean consumption value and possible error in the mean expressed to a 95.4% confidence level.

- 4.20 In a particular manufacturing process, bricks are produced in batches of 10,000. Because of random variations in the manufacturing process, random errors occur in the target length of the bricks produced. If the bricks have a mean length of 200 mm with a standard deviation of 20 mm, show how the error function tables supplied can be used to calculate the following:
- the number of bricks with a length between 198 and 202 mm.
 - the number of bricks with a length greater than 170 mm.
- 4.21 A pressure microsensor is tested by applying a pressure to it of 200 bar, measured by an accurate, calibrated reference pressure-measuring instrument. A set of 12 measurements is made as a reference to assess the standard deviation and standard error of the mean for measurements made by the device. The measurements obtained for this reference set are given as:
- 199.7 202.0 200.9 195.7 200.2 199.9 204.4 198.0 203.1 199.1 200.5 196.9
- When the microsensor is subsequently used in a workplace to measure the pressure in an enclosed vessel, a reading of 183 bar is obtained. What is the likely error in this measurement, expressed to a 95.0% confidence limit?
- 4.22 The temperature-controlled environment in a hospital intensive care unit is monitored by an intelligent instrument that measures the temperature every minute and calculates the mean and standard deviation of the measurements. If the mean is 75°F and the standard deviation is 2.15°F,
- what percentage of the time is the temperature less than 70°F?
 - what percentage of the time is the temperature between 73°F and 77°F?
- 4.23 A semiconductor temperature measuring device is tested by placing it in an environment that is maintained at a constant temperature of 150°C, which is measured by an accurate, calibrated resistance thermometer. A set of 12 measurements is made as a reference to assess the standard deviation and standard error of the mean for measurements made by the device. The measurements obtained for this reference set are given as:
- 150.7 149.8 150.4 153.5 147.6 148.5 152.4 151.5 149.3 149.9 146.6 150.2
- When the sensor is subsequently used in a workplace to measure the temperature in an enclosed vessel, a reading of 137°C is obtained. What is the likely error in this measurement, expressed to a 95.0% confidence limit?
- 4.24 Calculate the standard error of the mean for the measurements given in problem 4.2. Hence, express the melting point of aluminum together with the possible error in the value expressed to a 68% confidence level.
- 4.25 The thickness of a set of gaskets varies because of random manufacturing disturbances, but the thickness values measured belong to a Gaussian distribution. If the mean thickness is 3 mm and the standard deviation is 0.25, calculate the percentage of gaskets that have a thickness greater than 2.5 mm.

- 4.26 If the measured variance of 25 samples of bread cakes taken from a large batch is 4.85 g, calculate the true variance of the mass for a whole batch of bread cakes to a 95% significance level.
- 4.27 Calculate the true standard deviation of the diameter of a large batch of tires to a confidence level of 99% if the measured standard deviation in the diameter for a sample of 30 tires is 0.795 cm.
- 4.28 A total of 150 measurements are taken of the thickness of a coil of rolled steel sheet measured at approximately equidistant points along the center line of its length. The measurements have a mean value of 11.291 mm and a standard deviation of 0.263 mm. The smallest and largest measurements in the sample are 10.73 and 11.89 mm. The measurements are divided into 8 data bins with boundaries at 10.695, 10.845, 10.995, 11.145, 11.295, 11.445, 11.595, 11.745, and 11.895. The first bin, containing measurements between 10.695 and 10.845, has 8 measurements in it, and the count of measurements in the following successive bins is 12, 21, 34, 31, 25, 14, and 5. Apply the chi-squared test to see whether the measurements fit a Gaussian distribution to a 95% confidence level.
- 4.29 In a foundry producing castings, the variance in the mass of a sample of 20 castings taken from a large batch is calculated to be 1.36 kg. Calculate the true variance of the mass for a whole batch of castings from which the sample was taken to (a) a 90% significance level and (b) a 95% significance level.
- 4.30 The temperature in a furnace is regulated by a control system that aims to keep the temperature close to 800°C. The temperature is measured every minute over a two-hour period, during which time the minimum and maximum temperatures measured are 782°C and 819°C. Analysis of the 120 measurements shows a mean value of 800.3°C and a standard deviation of 7.58°C. The measurements are divided into 8 data bins of 5°C width with boundaries at 780.5, 785.5, 790.5, 795.5, 800.5, 805.5, 810.5, 815.5, and 820.5. The measurement count in bin 1 from 780.5°C to 785.5°C was 3 and the count in the other successive bins was 8, 21, 30, 28, 19, 9, and 2. Apply the chi-squared test to see whether the measurements fit a Gaussian distribution to (a) a 90% confidence level and (b) a 95% confidence level (think carefully about whether the chi-squared test will be reliable for the measurement counts observed and whether there needs to be a change in the number of data bins used for the chi-squared test).
- 4.31 Lifetime tests for a sample of 51 tungsten light bulbs taken from a large batch are carried out, and the mean lifetime for the sample is found to be 984 hours, with a standard deviation of 9.4 hours. Calculate the true standard deviation for the whole batch of light bulbs from which the sample of 51 was taken to (a) a 95% significance level and (b) a 99% significance level.

4.32 A pressure control system is designed to maintain the pressure at a constant value of 1000 mbar inside a distillation column.

- (a) The following test set of initial pressure measurements is obtained: 1015, 1003, 1044, 1000 985, 971, 999, 990, 994, 1000, 1032, 1001 973, 1006 979, 961, 1010, 1021 997, 1026.

Show that this is free of rogue data points and therefore can be used as a reference set against which to test future pressure measurements.

- (b) The pressure is then measured at 5-minute intervals and the following 20 measurements are obtained: 961, 1010 990, 1039 997, 1015, 1000 949, 1000, 1051, 1001 976, 1003 985, 932, 1006 999, 1021, 1029 994.

Test whether there are any rogue measurements in this data set.

4.33 (a) What does the *Student t variable* express? Write down an expression for the Student t variable for a small sample (N) of measurements in terms of the mean of the sample (x_{mean}), the standard deviation (σ_x) of the sample, and the mean (μ) of the whole population of which the sample is part:

- (b) Sketch typical t distribution curves for sample sizes of 2, 11, and infinity.
(c) Sketch a t distribution curve that shows the meaning of the term α , which is commonly used to express the level of confidence that t is less than some value t_α .
(d) Use a t distribution table to calculate the value of t for a sample of 21 measurements and a confidence level of 99%.

The measurements are divided into 9 data bins with boundaries at 859.5, 876.5, 893.5, 910.5, 927.5, 944.5, 961.5, 978.5, 995.5, 1012.5.

The measurement count in bin 1 from 869.5 to 886.5 days is 3 and the count in the other successive bins is 11, 27, 39, 46, 40, 24, 8 and 2.

4.34 Accelerated lifetime tests are carried out for a batch of 200 thermocouples. The mean lifetime is found to be 936 hours with a standard deviation of 28 hours. The smallest and largest measurements in the sample are 860 and 1012 hours. The measurements are divided into 9 data bins with boundaries at 859.5, 876.5, 893.5, 910.5, 927.5, 944.5, 961.5, 978.5, 995.5, and 1012.5. The measurement count in bin 1 from 869.5 to 886.5 days is 3 and the count in the other successive bins is 8, 27, 39, 46, 40, 24, 11, and 2. Apply the chi-squared test to see whether the measurements fit a Gaussian distribution to (a) a 99% confidence level, (b) a 97.5% confidence level, and (c) a 95% confidence level.

4.35 A temperature control system is designed to maintain the temperature at a constant value of 500°C inside an induction furnace.

- (a) The following test set of 20 initial temperature measurements (once the furnace temperature has reached steady-state) is obtained: 507 502, 520, 500, 492, 485, 499, 495, 497, 500, 516, 501, 489, 503, 490, 481, 504, 511, 499, 512.

Show that this is free of rogue data points and can therefore be used as a reference set against which to test future temperature measurements.

- (b) The temperature is then measured at 10-minute intervals and the following 20 measurements are obtained:

481 505 495, 519, 498, 515, 508, 477, 500, 526, 501, 488, 502, 492, 464, 503, 499, 510, 515, 497

Test whether there are any rogue measurements in this data set.

- 4.36 The internal diameter of a large batch of pressure vessels is measured by destructive testing of 8 samples taken randomly from the batch. If the sample mean is 346 mm with a standard deviation of 7 mm, express the upper and lower bounds to a confidence level of 95% on the range in which the mean value lies for the internal diameter of the whole batch.
- 4.37 The volume contained in each of a sample of 10 bottles of expensive perfume is measured. If the mean volume of the sample measurements is 100.5 ml with a standard deviation of 0.64 ml, calculate the upper and lower bounds to a confidence level of 95% of the mean value of the whole batch of perfume from which the 10 samples were taken.
- 4.38 A 3-V direct current (dc) power source required for a circuit is obtained by connecting together two 1.5-V batteries in series. If the error in the voltage output of each battery is specified as $\pm 1\%$, calculate the likely maximum error in the 3-V power source that they make up.
- 4.39 A temperature measurement system consists of a thermocouple whose amplified output is measured by a voltmeter. The output relationship for the thermocouple is approximately linear over the temperature range of interest. The electromagnetic force–temperature relationship of the thermocouple has a possible error of $\pm 1\%$, the amplifier gain value has a possible error of $\pm 0.5\%$, and the voltmeter has a possible error of $\pm 2\%$. What is the possible error in the measured temperature?
- 4.40 A pressure measurement system consists of a monolithic piezoresistive pressure transducer and a bridge circuit to measure the resistance change of the transducer. The resistance (R) of the transducer is related to pressure (P) according to: $R = K_1 P$ and the output of the bridge circuit (V) is related to resistance (R) by: $V = K_2 R$. Thus, the output voltage is related to pressure according to: $V = K_1 K_2 P$. If the maximum error in K_1 is $\pm 2\%$, the maximum error in K_2 is $\pm 1.5\%$, and the voltmeter itself has a maximum measurement error of $\pm 1\%$, what is the likely maximum error in the pressure measurement?
- 4.41 A requirement for a resistance of 1220 Ω in a circuit is satisfied by connecting resistances of 1000 and 220 Ω in series. If each resistance has a tolerance of $\pm 5\%$, what is the likely tolerance in the total resistance?

- 4.42 To calculate the heat loss through the wall of a building, it is necessary to know the temperature difference between the inside and outside walls. Temperatures of 5°C and 20°C are measured on each side of the wall by mercury-in-glass thermometers with a range of $0\text{--}50^{\circ}\text{C}$ and a quoted inaccuracy of $\pm 1\%$ of full-scale reading. Calculate the likely maximum possible error in the calculated value for the temperature difference.
Briefly discuss how using measuring instruments with a different measurement range might improve measurement accuracy.
- 4.43 A fluid flow rate is calculated from the difference in pressure measured across a venturi. The flow rate is given by: $F = K(p_2 - p_1)$ where p_1, p_2 are the pressures either side of the venturi and K is a constant. The two pressure measurements are 15.2 and 14.7 bar.
- Calculate the possible error in flow measurement if the pressure measuring instruments have a quoted error of $\pm 0.2\%$ of their reading.
 - Briefly discuss why using a differential pressure sensor rather than two separate pressure sensors would improve measurement accuracy.
- 4.44 The power dissipated in a car headlight is calculated by measuring the dc voltage drop across it and the current flowing through it ($P = V \times I$). If the possible errors in the measured voltage and current values are $\pm 1\%$ and $\pm 2\%$ respectively, calculate the likely maximum possible error in the power value deduced.
- 4.45 The resistance of a carbon resistor is measured by applying a dc voltage across it and measuring the current flowing ($R = V/I$). If the voltage and current values are measured as 10 ± 0.1 V and 214 ± 5 mA, respectively, express the value of the carbon resistor.
- 4.46 The specific energy of a substance is calculated by measuring the energy content of a cubic meter volume of the substance. If the errors in energy measurement and volume measurement are $\pm 1\%$ and $\pm 2\%$, respectively, what is the possible error in the calculated value of specific energy? (Specific energy = energy per unit volume of material.)
- 4.47 (a) In a particular measurement system, a quantity x is calculated by subtracting a measurement of a quantity z from a measurement of a quantity y (i.e., $x = y - z$). If the possible measurement errors in y and z are $\pm ay$ and $\pm bz$, respectively, show that the value of x can be expressed as:

$$x = y - z \pm (ay - bz)$$

What is inconvenient about this expression for x and what is the basis for the following expression for x that is used more commonly?

$$x = (y - z) \pm e, \text{ where } e = \sqrt{(ay)^2 + (bz)^2}$$

- (b) In a different measurement system, a quantity p is calculated by multiplying together measurements of two quantities q and r such that $p = qr$. If the possible measurement errors in q and r are $\pm aq$ and $\pm br$, respectively, show that the value of p can be expressed as:

$$p = (qr)(1 \pm [a + b])$$

- (c) The volume flow rate of a liquid in a pipe (the volume flowing in unit time) is measured by allowing the end of the pipe to discharge into a vertical-sided tank with a rectangular base (Fig. 4.13). The depth of the liquid in the tank is measured at the start as h_1 meters, and 1 minute later it is measured as h_2 meters. If the length and width of the tank are l meters and w meters, respectively, write down an expression for the volume flow rate of the liquid in cubic meters per minute.

Calculate the volume flow rate of the liquid if the measured parameters have the following values:

$$h_1 = 0.8 \text{ m}; h_2 = 1.3 \text{ m}; l = 4.2 \text{ m}; w = 2.9 \text{ m}$$

If the possible errors in the measurements of h_1 , h_2 , l , and w are 1%, 1%, 0.5%, and 0.5%, respectively, calculate the possible error in the calculated value of the flow rate.

- 4.48 The density of a material is calculated by measuring the mass of a rectangular-sided block of the material whose edges have lengths of a , b , and c . What is the possible error in the calculated density if the possible error in mass measurement is $\pm 1.0\%$ and the possible errors in length measurement are $\pm 0.5\%$?

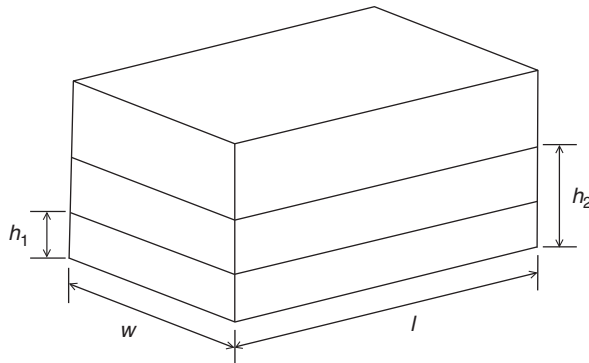


Figure 4.13
Diagram for Problem 4.32.

- 4.49 The density (d) of a liquid is calculated by measuring its depth (c) in a calibrated rectangular tank and then emptying it into a mass measuring system. The length and width of the tank are (a) and (b), respectively, and thus the density is given by:

$$d = m / (a \times b \times c)$$

where m is the measured mass of the liquid emptied out.

If the possible errors in the measurements of a , b , c , and m are 1%, 1%, 2%, and 0.5%, respectively, determine the likely maximum possible error in the calculated value of the density (d).

- 4.50 The volume flow rate of a liquid is calculated by allowing the liquid to flow into a cylindrical tank (stood on its flat end) and measuring the height of the liquid surface before and after the liquid has flowed for 10 minutes. The volume collected after 10 minutes is given by:

$$Volume = (h_2 - h_1)\pi(d/2)^2$$

where h_1 and h_2 are the starting and finishing surface heights and d is the measured diameter of the tank.

- (a) If $h_1 = 2$ m, $h_2 = 3$ m, and $d = 2$ m, calculate the volume flow rate in m^3/min .
- (b) If the possible error in each measurement h_1 , h_2 , and d is $\pm 1\%$, determine the likely maximum possible error in the calculated value of volume flow rate. (It is assumed that there is negligible error in the time measurement.)
- 4.51 The measurements in a data set are subject to random errors, but it is known that the data set fits a Gaussian distribution. Use error function tables to determine the percentage of measurements that lie within the boundaries of $\pm 2.5\sigma$, where σ is the standard deviation of the measurements.
- 4.52 The resistance of each resistor in a packet of 15 resistors with a nominal value of 560Ω was measured, and the following values in ohms were recorded:
546 558 585, 539, 542, 570, 566, 541, 563, 574, 547, 568, 550, 534, 572
Calculate the mean value, median value, standard deviation, and standard error of the mean for the set of resistance values.
- 4.53 A vortex flowmeter is tested by applying a constant flow rate of 100 litres/min to it, measured by an accurate, calibrated, reference positive displacement meter. A set of 12 measurements is made as a reference set of measurements to assess the standard deviation and standard error of the mean for measurements made by the device. The measurements obtained for this reference set are given as:
99.8, 99.3 100.2, 99.7 100.9, 99.9, 99.4 101.1, 99.5 100.8 100.4, 99.0
When the vortex flowmeter is subsequently used in a workplace to measure the flow rate of liquid through a pipe, a reading of 94.6 litres/min is obtained. What is the likely error in this measurement, expressed as a 95.4% confidence limit?

- 4.54 The thickness of a batch of steel plates varies because of random manufacturing disturbances, but the thickness values measured belong to a Gaussian distribution. If the mean thickness is 50 mm and the standard deviation is 3 mm, calculate the percentage of gaskets that have a thickness greater than 49 mm.
- 4.55 Calculate the true standard deviation of the diameter of a large batch of copper bars to a confidence level of 98% if the measured standard deviation in the diameter for a sample of 20 bars is 0.8 mm.
- 4.56 Measurements are taken of the thickness of 120 samples of ceramic tiles taken from a large batch of tiles. The measurements have a mean value of 16.937 mm and a standard deviation of 0.395 mm. The smallest and largest thickness measurements in the sample are 16.09 and 17.84 mm. The measurements are divided into 8 data bins with boundaries at 16.085, 16.305, 16.525, 16.745, 16.965, 17.185, 17.405, 17.625, and 17.845. The first bin, containing measurements between 16.085 and 16.305, has 6 measurements in it, and the count of measurements in the following successive bins is 10, 17, 27, 25, 20, 11 and 4. Apply the chi-squared test to see whether the measurements fit a Gaussian distribution to (a) a 95% confidence level and (b) a 97.5% confidence level.
- 4.57 Lifetime tests for a sample of 30 light bulbs taken from a large batch are carried out and the mean lifetime for the sample is found to be 427 hours, with a standard deviation of 26.5 hours. Calculate the true standard deviation for the whole batch of light bulbs from which the sample of 30 was taken to (a) a 90% significance level and (b) a 95% significance level.
- 4.58 A temperature control system is designed to maintain the temperature at a constant value of 253°C inside a kiln.
- (a) The following test set of 20 initial temperature measurements (once the temperature in the kiln has reached steady-state) is obtained:
256 251 249, 262, 253, 245, 252, 250, 261, 251, 253, 254, 247, 255, 248, 243, 255, 259, 252, 258
Show that this is free of rogue data points and can therefore be used as a reference set against which to test future temperature measurements.
- (b) The temperature is then measured at 15 minute intervals and the following 20 measurements are obtained:
240 253 247, 260, 249, 253, 254, 239, 250, 263, 251, 244, 251, 246, 232, 251, 249, 255, 258, 248
Test whether there are any rogue measurements in this data set.
- 4.59 The heat loss across the outer wall of a room in a glass-sided building is calculated according to the formula:
Heat loss = $U \times \text{area of wall} \times \text{temperature difference across the wall}$, where U is a constant known as the rate of heat transmittance.

The temperature inside the room is 20°C and the temperature outside the room is 5°C .

If the temperature measurements have an inaccuracy of $\pm 1\%$ and the dimension measurements (width and height of the wall) have an inaccuracy of $\pm 0.5\%$, calculate the possible error in the calculated heat loss.

- 4.60 The volume contained within a batch of bottles of malt whisky is estimated by destructive testing of 20 samples taken randomly from the batch. If the sample mean is 752 ml with a standard deviation of 8 ml, express the upper and lower bounds to a confidence level of 95% over the range in which the mean value lies for internal diameter of the whole batch.

Calibration of measuring sensors and instruments

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5.1 Introduction

We have just examined the various systematic and random measurement error sources in the last chapter. As far as systematic errors are concerned, we observed that recalibration at a suitable frequency was an important weapon in the quest to minimize errors due to drift in instrument characteristics. The use of proper and rigorous calibration procedures is essential in order to ensure that recalibration achieves its intended purpose, and, to reflect the importance of getting these procedures right, this whole chapter is dedicated to explaining the various facets of calibration.

We start in [Section 5.2](#) by formally defining what calibration means, explaining how it is performed and considering how to calculate the frequency with which the calibration exercise should be repeated. We then go on to look at the calibration environment in [Section 5.3](#), where we learn that proper control of the environment in which instruments are calibrated is an essential component in good calibration procedures. [Section 5.4](#) then continues with a review of how the calibration of working instruments against reference instruments is linked by the calibration chain to national and international reference standards relating to the quantity that the instrument being calibrated is designed to measure. Finally, [Section 5.5](#) emphasizes the importance of maintaining records of instrument calibrations and suggests appropriate formats for such records.

5.2 Principles of calibration

Calibration consists of comparing the output of the instrument or sensor under test against the output of an instrument of known accuracy when the same input (the measured quantity) is applied to both instruments. This procedure is carried out for a range of inputs covering the whole measurement range of the instrument or sensor. Calibration ensures that the measuring accuracy of all instruments and sensors used in a measurement system is known over the whole measurement range, provided that the calibrated instruments and sensors are used in environmental conditions that are the same as those under which they were calibrated. For use of instruments and sensors under different environmental conditions, appropriate correction has to be made for the ensuing modifying inputs, as described in Chapter 3. Whether applied to instruments or sensors, calibration procedures are identical, and hence only the term *instrument* will be used for the rest of this chapter, with the understanding that whatever is said for instruments applies equally well to single measurement sensors.

Instruments used as a standard in calibration procedures are usually chosen to be of greater inherent accuracy than the process instruments that they are used to calibrate. Because such instruments are only used for calibration purposes, greater accuracy can often be achieved by specifying a type of instrument that would be unsuitable for normal process measurements. For instance, ruggedness is not a requirement, and freedom from this constraint opens up a much wider range of possible instruments. In practice, high-accuracy, null-type instruments are very commonly used for calibration duties because the need for a human operator is not a problem in these circumstances.

Instrument calibration has to be repeated at prescribed intervals because the characteristics of any instrument change over a period. Changes in instrument characteristics are brought about by such factors as mechanical wear, and the effects of dirt, dust, fumes, chemicals, and temperature changes in the operating environment. To a great extent, the magnitude of the drift in characteristics depends on the amount of use an instrument receives and hence on the amount of wear and the length of time that it is subjected to the operating environment. However, some drift also occurs even in storage, as a result of aging effects in components within the instrument.

Determination of the frequency at which instruments should be calibrated is dependent upon several factors that require specialist knowledge. If an instrument is required to measure some quantity and an inaccuracy of $\pm 2\%$ is acceptable, then a certain amount of performance degradation can be allowed if its inaccuracy immediately after recalibration is $\pm 1\%$. What is important is that the pattern of performance degradation be quantified, such that the instrument can be recalibrated before its accuracy has reduced to the limit defined by the application.

Susceptibility to the various factors that can cause changes in instrument characteristics varies according to the type of instrument involved. Possession of an in-depth knowledge of the mechanical construction and other features involved in the instrument is necessary in order to be able to quantify the effect of these quantities on the accuracy and other characteristics of an instrument. The type of instrument, its frequency of use, and the prevailing environmental conditions all strongly influence the calibration frequency necessary, and because so many factors are involved, it is difficult or even impossible to determine the required frequency of instrument recalibration from theoretical considerations. Instead, practical experimentation has to be applied to determine the rate of such changes. Once the maximum permissible measurement error has been defined, knowledge of the rate at which the characteristics of an instrument change allows a time interval to be calculated that represents the moment in time when an instrument will have reached the bounds of its acceptable performance level. The instrument must be recalibrated either at this time or earlier. This measurement error level that an instrument reaches just before recalibration is the error bound that must be quoted in the documented specifications for the instrument.

A proper course of action must be defined that describes the procedures to be followed when an instrument is found to be out of calibration, i.e., when its output is different from that of the calibration instrument when the same input is applied. The required action depends very much upon the nature of the discrepancy and the type of instrument involved. In many cases, deviations in the form of a simple output bias can be corrected by a small adjustment to the instrument (following which the adjustment screws must be sealed to prevent tampering). In other cases, the output scale of the instrument may have to be redrawn, or scaling factors altered where the instrument output is part of some automatic control or inspection system. In extreme cases, where the calibration procedure shows up signs of instrument damage, it may be necessary to send the instrument for repair or even scrap it.

Whatever system and frequency of calibration is established, it is important to review this from time to time to ensure that the system remains effective and efficient. It may happen that a cheaper (but equally effective) method of calibration becomes available with the passage of time, and such an alternative system must clearly be adopted in the interests of cost-efficiency. However, the main item under scrutiny in this review is normally whether the calibration interval is still appropriate. Records of the calibration history of the instrument will be the primary basis on which this review is made. It may happen that an instrument starts to go out of calibration more quickly after a period of time, either because of aging factors within the instrument or because of changes in the operating environment. The conditions or mode of usage of the instrument may also be subject to change. As the environmental and usage conditions of an instrument may change beneficially as well as adversely, there is the possibility that the recommended calibration interval may decrease as well as increase.

5.3 Control of calibration environment

Any instrument that is used as a standard in calibration procedures must be kept solely for calibration duties and must never be used for other purposes. Most particularly, it must not be regarded as a spare instrument that can be used for process measurements if the instrument normally used for that purpose breaks down. Proper provision for process instrument failures must be made by keeping a spare set of process instruments. Standard calibration instruments must be totally separate.

To ensure that these conditions are met, the calibration function must be managed and executed in a professional manner. This will normally mean setting aside a particular place within the instrumentation department of a company where all calibration operations take place and where all instruments used for calibration are kept. As far as possible this should take the form of a separate room, rather than a sectioned-off area in a room used for other purposes as well. This will enable better environmental control to be applied in the calibration area and will also offer better protection against unauthorized handling or use of the calibration instruments. The level of environmental control required during calibration should be considered carefully with due regard to what level of accuracy is required in the calibration procedure, but should not be overspecified as this will lead to unnecessary expense. Full air-conditioning is not normally required for calibration at this level, as it is very expensive, but sensible precautions should be taken to guard the area from extremes of heat or cold, and also good standards of cleanliness should be maintained.

Whereas it is desirable that all calibration functions are performed in this carefully controlled environment, it is not always practical to achieve this. Sometimes, it is not convenient or possible to remove instruments from process plant, and in these cases, it is standard practice to calibrate them in situ. In these circumstances, appropriate corrections must be made for the deviation in the calibration environmental conditions away from those specified. This practice does not obviate the need to protect calibration instruments and maintain them in constant conditions in a calibration laboratory at all times other than when they are involved in such calibration duties on plant.

As far as management of calibration procedures is concerned, it is important that the performance of all calibration operations is assigned as the clear responsibility of just one person. That person should have total control over the calibration function, and be able to limit access to the calibration laboratory to designated, approved personnel only. Only by giving this appointed person total control over the calibration function can the function be expected to operate efficiently and effectively. Lack of such definite management can only lead to unintentional neglect of the calibration system, resulting in the use of equipment in an out-of-date state of calibration and subsequent loss of traceability to reference standards.

Professional management is essential so that the customer can be assured that an efficient calibration system is in operation and that the accuracy of measurements is guaranteed.

Calibration procedures that relate in any way to measurements that are used for quality control functions are controlled by the international standard [ISO 9000](#) (this subsumes the old British quality standard BS 5750), One of the clauses in ISO 9000 requires that all persons using calibration equipment be adequately trained. The manager in charge of the calibration function is clearly responsible for ensuring that this condition is met. Training must be adequate and targeted at the particular needs of the calibration systems involved. People must understand what they need to know and especially why they must have this information. Successful completion of training courses should be marked by the award of qualification certificates. These attest to the proficiency of personnel involved in calibration duties and are a convenient way of demonstrating that the ISO 9000 training requirement has been satisfied.

5.4 Calibration chain and traceability

The calibration facilities provided within the instrumentation department of a company provide the first link in the calibration chain. Instruments used for calibration at this level are known as ***working standards***. As such working standard instruments are kept by the instrumentation department of a company solely for calibration duties, and for no other purpose, then it can be assumed that they will maintain their accuracy over a reasonable period of time because use-related deterioration in accuracy is largely eliminated. However, over the longer term, the characteristics of even such standard instruments will drift, mainly due to aging effects in components within them. Therefore, over this longer term, a program must be instituted for calibrating working standard instruments at appropriate intervals of time against instruments of yet higher accuracy. The instrument used for calibrating working standard instruments is known as a ***secondary reference standard***. This must obviously be a very well-engineered instrument that gives high accuracy and is stabilized against drift in its performance with time. This implies that it will be an expensive instrument to buy. It also requires that the environmental conditions in which it is used be carefully controlled with respect to ambient temperature, humidity, etc.

When the working standard instrument has been calibrated by an authorized standards laboratory, a calibration certificate will be issued. This will contain at least the following information:

- the identification of the equipment calibrated
- the calibration results obtained
- the measurement uncertainty

- any use limitations on the equipment calibrated
- the date of calibration
- the authority under which the certificate is issued

The establishment of a company standards laboratory to provide a calibration facility of the required quality is economically viable only in the case of very large companies where large numbers of instruments need to be calibrated across several factories. In the case of small- to medium-sized companies, the cost of buying and maintaining such equipment is not justified. Instead, they would normally use the calibration service provided by various companies that specialize in offering a standards laboratory. What these specialist calibration companies effectively do is to share out the high cost of providing this highly accurate but infrequently used calibration service over a large number of companies. Such standards laboratories are closely monitored by national standards organizations.

In the United States, the appropriate national standards organization for validating standards laboratories is the National Bureau of Standards, whereas in the United Kingdom it is the National Physical Laboratory. An international standard now exists ([ISO/IEC 17025, 2017](#)) that sets down criteria that must be satisfied in order for a standards laboratory to be validated. These criteria cover the management requirements necessary to ensure proper operation and effectiveness of a quality management system within the calibration or testing laboratory and also some technical requirements that relate to the competence of staff, specification, and maintenance of calibration/test equipment and practical calibration procedures used.

National standards organizations usually monitor both instrument calibration and mechanical testing laboratories. Although each different country has its own structure for the maintenance of standards, each of these different frameworks tends to be equivalent in its effect in ensuring that the requirements of ISO/IEC 17025 are met. This provides confidence that the goods and services that cross national boundaries from one country to another have been measured by properly calibrated instruments.

The national standards organizations lay down strict conditions that a standards laboratory has to meet before it is approved. These conditions control laboratory management, environment, equipment, and documentation. The person appointed as head of the laboratory must be suitably qualified, and independence of operation of the laboratory must be guaranteed. The management structure must be such that any pressure to rush or skip calibration procedures for production reasons can be resisted. As far as the laboratory environment is concerned, proper temperature and humidity control must be provided, and high standards of cleanliness and housekeeping must be maintained. All equipment used for calibration purposes must be maintained to reference standards, and supported by

calibration certificates that establish this traceability. Finally, full documentation must be maintained. This should describe all calibration procedures, maintain an index system for recalibration of equipment, and include a full inventory of apparatus and traceability schedules. Having met these conditions, a standards laboratory becomes an accredited laboratory for providing calibration services and issuing calibration certificates. This accreditation is reviewed at approximately 12 monthly intervals to ensure that the laboratory is continuing to satisfy the conditions for approval laid down.

Primary reference standards, as listed in Table 2.1, describe the highest level of accuracy that is achievable in the measurement of any particular physical quantity. All items of equipment used in standards laboratories as secondary reference standards have to be calibrated themselves against primary reference standards at appropriate intervals of time. This procedure is acknowledged by the issue of a calibration certificate in the standard way. National standards organizations maintain suitable facilities for this calibration. In the United States this is the National Bureau of Standards and in the United Kingdom it is the National Physical Laboratory. Similar national standards organizations exist in many other countries. In certain cases, such primary reference standards can be located outside national standards organizations. For instance, the primary reference standard for dimension measurement is defined by the wavelength of the orange-red line of krypton light, and it can therefore be realized in any laboratory equipped with an interferometer. In certain cases (e.g., the measurement of viscosity), such primary reference standards are not available, and reference standards for calibration are achieved by collaboration between several national standards organizations who perform measurements on identical samples under controlled conditions (ISO 15725, 2010).

What has emerged from the foregoing discussion is that calibration has a chain-like structure in which every instrument in the chain is calibrated against a more accurate instrument immediately above it in the chain, as shown in Fig. 5.1. All of the elements in the calibration chain must be known so that the calibration of process instruments at the bottom of the chain is traceable to the fundamental measurement standards. This knowledge of the full chain of instruments involved in the calibration procedure is known as **traceability**, and is specified as a mandatory requirement in satisfying the ISO 9000 standard. Documentation must exist that shows that process instruments are calibrated by standard instruments that are linked by a chain of increasing accuracy back to national reference standards. There must be clear evidence to show that there is no break in this chain.

To illustrate a typical calibration chain, consider the calibration of micrometers (Fig. 5.2). A typical shop floor micrometer has an uncertainty (inaccuracy) of less than 1 in 10^4 . These would normally be calibrated in the instrumentation department or standards

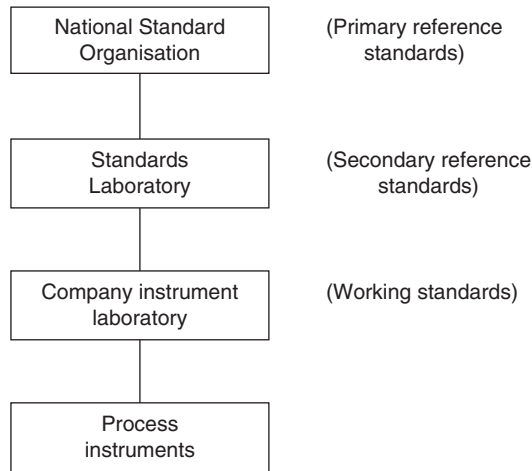


Figure 5.1
Instrument calibration chain.

laboratory of a company against laboratory standard gauge blocks with a typical uncertainty of less than 1 in 10^5 . A specialist calibration service company would provide facilities for calibrating these laboratory standard gauge blocks against reference-grade gauge blocks with a typical uncertainty of less than 1 in 10^6 . More accurate calibration

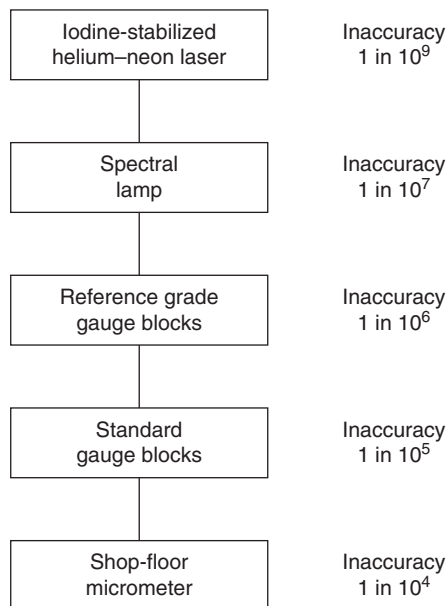


Figure 5.2
Typical calibration chain for micrometers.

equipment still is provided by national standards organizations. The National Bureau of Standards (USA) and National Physical Laboratory (UK) maintain two sets of standards for this type of calibration, a working standard and a primary standard. Spectral lamps are used to provide a working reference standard with an uncertainty of less than 1 in 10^7 . The primary standard is provided by an iodine-stabilized helium–neon laser that has a specified uncertainty of less than 1 in 10^9 . All of the links in this calibration chain must be shown in any documentation that describes the use of micrometers in making quality-related measurements.

5.5 Calibration records

An essential element in the maintenance of measurement systems and the operation of calibration procedures is the provision of full documentation. This must give a full description of the measurement requirements throughout the workplace, the instruments used, and the calibration system and procedures operated. Individual calibration records for each instrument must be included within this. This documentation is a necessary part of the quality manual, although it may physically exist as a separate volume if this is more convenient. An overriding constraint on the style in which the documentation is presented is that it should be simple and easy to read. This is often greatly facilitated by a copious use of appendices.

The starting point in the documentation must be a statement of what measurement limits have been defined for each measurement system documented. Such limits are established by balancing the costs of improved accuracy against customer requirements, and also with regard to what overall quality level has been specified in the quality manual. The technical procedures required for this, which involve assessing the type and magnitude of relevant measurement errors, are described in Chapter 3. It is customary to express the final measurement limit calculated as ± 2 standard deviations, i.e., within 95% confidence limits (see Chapter 3 for explanation of these terms).

The instruments specified for each measurement situation must be listed next. This list must be accompanied by full instructions about the proper use of the instruments concerned. These instructions will include details about any environmental control or other special precautions that must be taken to ensure that the instruments provide measurements of sufficient accuracy to meet the measurement limits defined. The proper training courses appropriate to plant personnel who will use the instruments must be specified.

Having disposed of the question about what instruments are used, the documentation must go on to cover the subject of calibration. Full calibration is not applied to every measuring

instrument used in a workplace because ISO 9000 acknowledges that formal calibration procedures are not necessary for some equipment where it is uneconomic or technically unnecessary because the accuracy of the measurement involved has an insignificant effect on the overall quality target for a product. However, any equipment that is excluded from calibration procedures in this manner must be specified as such in the documentation. Identification of equipment that is in this category is a matter of informed judgment.

For instruments that are the subject of formal calibration, the documentation must specify what standard instruments are to be used for the purpose and define a formal procedure of calibration. This procedure must include instructions for the storage and handling of standard calibration instruments and specify the required environmental conditions under which calibration is to be performed. Where a calibration procedure for a particular instrument uses published standard practices, it is sufficient to include reference to that standard procedure in the documentation rather than to reproduce the whole procedure. Whatever calibration system is established, a formal review procedure must be defined in the documentation that ensures its continued effectiveness at regular intervals. The results of each review must also be documented in a formal way.

A standard format for the recording of calibration results should be defined in the documentation. A separate record must be kept for every instrument present in the workplace, irrespective of whether the instrument is normally in use or is just kept as a spare. A form similar to that shown in [Fig. 5.3](#) should be used that includes details of the instrument's description, the required calibration frequency, the date of each calibration, and the calibration results on each occasion. Where appropriate, the documentation must also define the manner in which calibration results are to be recorded on the instruments themselves.

The documentation must specify procedures that are to be followed if an instrument is found to be outside the calibration limits. This may involve adjustment, redrawing its scale, or withdrawing an instrument, depending upon the nature of the discrepancy and the type of instrument involved. Instruments withdrawn will either be repaired or scrapped. In the case of withdrawn instruments, a formal procedure for marking them as such must be defined to prevent them being accidentally put back into use.

Two other items must also be covered by the calibration document. The traceability of the calibration system back to national reference standards must be defined and supported by calibration certificates (see [Section 5.3](#)). Training procedures must also be documented, specifying the particular training courses to be attended by various personnel and what, if any, refresher courses are required.

All aspects of these documented calibration procedures will be given consideration as part of the periodic audit of the quality control system that calibration procedures are instigated

Type of instrument:	Company serial number:	
Manufacturer's part number:	Manufacturer's serial number:	
Measurement limit:	Date introduced:	
Location:		
Instructions for use:		
Calibration frequency:	Signature of person responsible for calibration:	
<u>CALIBRATION RECORD</u>		
Calibration date:	Calibration results	Calibrated by

Figure 5.3
Typical format for instrument record sheets.

to support. Whilst the basic responsibility for choosing a suitable interval between calibration checks rests with the engineers responsible for the instruments concerned, the quality system auditor will require to see the results of tests that show that the calibration interval has been chosen correctly and that instruments are not going outside allowable measurement uncertainty limits between calibrations. Particularly important in such audits will be the existence of procedures that are instigated in response to instruments found to be out of calibration. Evidence that such procedures are effective in avoiding degradation in the quality assurance function will also be required.

5.6 Summary

Proper instrument calibration is an essential component in good measurement practice and this chapter has been dedicated to explaining the various procedures that must be followed in order to perform calibration tasks efficiently and effectively. We have learned how working instruments are calibrated against a more accurate “reference” instrument that is carefully maintained and kept just for performing calibration tasks. We considered the

importance of carefully designing and controlling the calibration environment in which calibration tasks are performed, and observed that proper training of all personnel involved in carrying out calibration tasks had similar importance. We also learned that the “first-stage” calibration of a working instrument against a reference standard is part of a chain of calibrations that provides traceability of the working instrument calibration to national and international reference standards for the quantity being measured, the latter representing the most accurate standards of measurement accuracy that are achievable. Finally, we looked at the importance of maintaining calibration records and suggested appropriate formats for these.

5.7 Problems

- 5.1 Explain the meaning of instrument calibration.
- 5.2 Explain why calibration is necessary.
- 5.3 Explain how the necessary calibration frequency is determined for a measuring instrument.
- 5.4 Explain what a calibration chain is and give a typical example of one.
- 5.5 Explain the following:
 - (a) traceability
 - (b) calibration certificate
 - (c) standards laboratory
- 5.6 Explain how the calibration procedure should be managed, particularly with regard to control of the calibration environment and choice of reference instruments.
- 5.7 Will a calibrated measuring instrument always be accurate? If not, explain why not and explain what procedures can be followed to ensure that accurate measurements are obtained when using calibrated instruments.
- 5.8 What is a quality manual relating to a measurement systems and what information should it contain?
- 5.9 Why is there no fundamental reference standard for temperature calibration? How is this difficulty overcome when temperature sensors are calibrated?
- 5.10 Discuss the necessary procedures in calibrating temperature sensors.
- 5.11 Explain the construction and working characteristics of the following three kinds of instruments that are used as a reference standard in pressure sensor calibration: dead-weight gauge, U-tube manometer, barometer.
- 5.12 Discuss the main procedures involved in calibrating pressure sensors.
- 5.13 Discuss the special equipment needed and the procedures involved in calibrating instruments that measure the volume flow rate of liquids.
- 5.14 What kind of equipment is needed for calibrating instruments that measure the volume flow rate of gases? How is this equipment used?
- 5.15 Discuss the general procedures involved in calibrating level sensors.

- 5.16 What is the main item of equipment used in calibrating mass-measuring instruments? Sketch the following instruments and briefly discuss their mode of operation: beam balance, weigh beam.
- 5.17 Discuss how the following are calibrated: translational displacement transducers, linear-motion accelerometers.
- 5.18 Explain the general procedures involved in calibrating (a) vibration sensors and (b) shock sensors.
- 5.19 Briefly discuss the procedures involved in the following: rotational displacement sensors, rotational velocity sensors, rotational acceleration sensors.
- 5.20 Explain the procedures involved in calibrating dimension-measuring instruments.

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Conversion of nonvoltage sensor outputs

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6.1 Introduction

Techniques to be described in later chapters for measuring, displaying, analyzing, or recording the output of measurement sensors all require the input to the measuring, displaying, analyzing, or recording device to be in the form of a varying voltage signal. Unfortunately, many measurement sensors do not have an output in this convenient voltage form, but rather in some other form. Such forms include translational displacements and changes in various nonvoltage electrical parameters such as resistance, inductance, capacitance, and current. In some cases, the output may alternatively take the form of variations in the phase or frequency of an alternating current (AC) electrical signal.

We therefore need to have a means of converting sensor outputs that are initially in a nonvoltage form into the more convenient voltage form. In the case of resistance, inductance, and capacitance measurement, the most commonly used tool is the electrical bridge circuit. These exist in two main forms: the direct current (DC) bridge for measuring resistance and the AC bridge for measuring inductance and capacitance. Bridge circuits provide an accurate method of measuring resistance, inductance, and capacitance values and enable the detection of small changes in these quantities about a nominal value. They are of immense importance in measurement system technology because so many transducers measuring physical quantities have an output that is expressed as a change in resistance, inductance, or capacitance.

In most cases, bridge circuits to convert the output of measurement sensors are made of discrete components in an instrumentation laboratory. Ready-built bridges are also available commercially, although these are substantially more expensive than a homemade bridge created from discrete components and a voltmeter to measure the output voltage.

6.2 Resistance measurement using a direct current bridge circuit

Resistance is measured in units of *ohms* (Ω). Devices that convert the measured quantity into a change in resistance include the resistance thermometer, thermistor, wire-coil pressure gauge, and strain gauge. Although electrical DC bridge circuits are the most common way to measure changes in the resistance of a sensor, other methods exist, as explained later in [Section 6.4](#).

In the case of resistance measurement, a DC bridge circuit is used in which the excitation of the bridge is by a DC voltage. In fact, two forms of DC bridge exist: the null type and the deflection type. The null type is more accurate but is somewhat tedious to use. The deflection type is less accurate but is easier to use. In consequence, the null type is mainly used for sensor calibration purposes and the deflection type is used in most other applications. The characteristics of each are explained in the following sections.

6.2.1 Null-type, direct current bridge (Wheatstone bridge)

A null-type bridge with DC excitation, commonly known as a Wheatstone Bridge, has the form shown in Fig. 6.1. The four arms of the bridge consist of the unknown resistance R_u , two equal value resistors R_2 and R_3 , and a variable resistor R_v (usually a decade resistance box). A DC voltage V_i is applied across points AC, and resistance R_v is varied until the voltage measured across points BD is zero. This null point is usually measured with a high-sensitivity galvanometer.

To analyze the Wheatstone bridge, define the current flowing in each arm to be $I_1 \dots I_4$, as shown in Fig. 6.1. Normally, if a high-impedance voltage-measuring instrument is used, current I_m drawn by the measuring instrument will be very small and can be approximated to zero. If this assumption is made, then, for $I_m = 0$:

$$I_1 = I_3 \text{ and } I_2 = I_4.$$

Looking at path ADC, we have a voltage V_i applied across a resistance $R_u + R_3$, and by Ohm's law:

$$I_1 = \frac{V_i}{R_u + R_3}$$

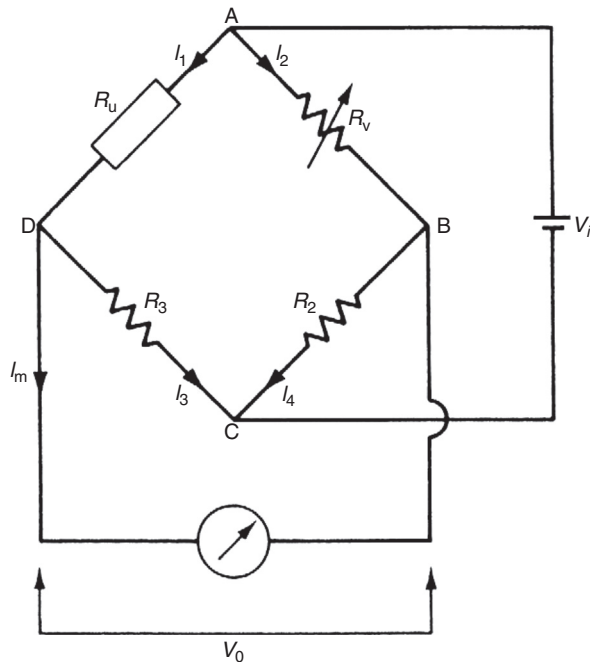


Figure 6.1
Wheatstone bridge (null-type direct current bridge).

Similarly, for path ABC:

$$I_2 = \frac{V_i}{R_v + R_2}.$$

Now, we can calculate the voltage drop across AD and AB:

$$V_{AD} = I_1 R_u = \frac{V_i R_u}{R_u + R_3} ; \quad V_{AB} = I_2 R_v = \frac{V_i R_v}{R_v + R_2}$$

By the principle of superposition, $V_o = V_{BD} = V_{BA} + V_{AD} = -V_{AB} + V_{AD}$.

Thus:

$$V_o = -\frac{V_i R_v}{R_v + R_2} + \frac{V_i R_u}{R_u + R_3} \quad (6.1)$$

At the null point $V_o = 0$, so:

$$\frac{R_u}{R_u + R_3} = \frac{R_v}{R_v + R_2}$$

Inverting both sides:

$$\frac{R_u + R_3}{R_u} = \frac{R_v + R_2}{R_v} \quad \text{i.e.} \quad \frac{R_3}{R_u} = \frac{R_2}{R_v} \quad \text{or :} \quad R_u = \frac{R_3 R_v}{R_2} \quad (6.2)$$

Thus, if $R_2 = R_3$, then $R_u = R_v$. R_v is an accurately known value because it is derived from a variable decade resistance box, which means that R_u is also accurately known.

As noted earlier, a null-type bridge is tedious to use, because careful adjustment of the variable resistance is needed to get exactly to the null point. However, it provides a highly accurate measurement of resistance, with inaccuracy values of less than $\pm 0.02\%$ being achievable with commercially available instruments. In consequence, the null-type bridge is the preferred form when sensors are being calibrated.

■ Example 6.1

A null-type Wheatstone bridge is used to accurately measure the resistance of a platinum resistance thermometer during a calibration procedure. The circuit shown in Fig. 6.1 is used, in which the known fixed resistance values are given by $R_2 = 98.3 \Omega$ and $R_3 = 102.2 \Omega$. The thermometer is inserted in the circuit as R_u and then the variable resistance box R_v is adjusted until the bridge output voltage V_o goes to zero. At this balance point, the value of R_v is 95.7Ω . Calculate the resistance of the thermometer.

■

■ Solution

At the balance point, the resistance values are related according to Eq. (6.2):

$$R_u = \frac{R_3 R_v}{R_2}$$

Substituting the resistance values into Eq. (6.2):

$$R_u = \frac{102.2 \times 95.7}{98.3} = 99.5 \ \Omega$$

Thus, the resistance of the thermometer is 99.5 Ω .

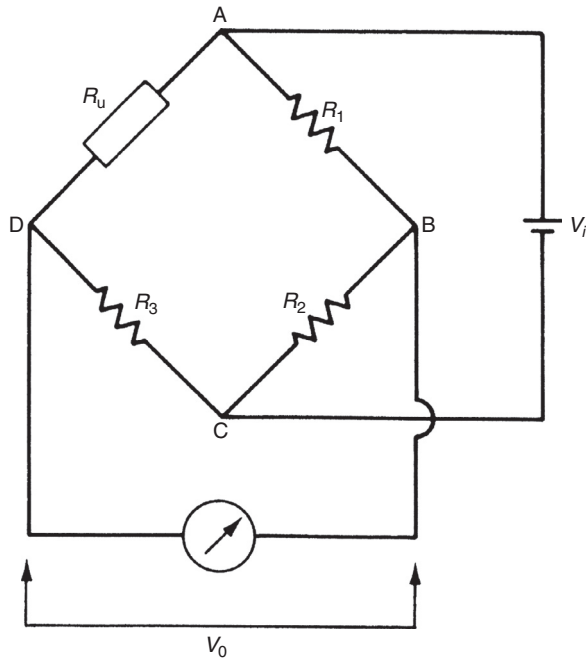


Figure 6.2

Deflection-type direct current bridge.

6.2.2 Deflection-type direct current bridge

A deflection type bridge with DC excitation is shown in Fig. 6.2. Deflection-type bridge circuits are simpler to use in practice than the null-output type, but their measurement accuracy is inferior and the nonlinear output relationship is an additional difficulty.

The deflection-type DC bridge differs from the Wheatstone bridge mainly in that the variable resistance R_v is replaced by a fixed resistance R_1 of the same value as the nominal value of the unknown resistance R_u . As the resistance R_u changes, the output voltage V_0 varies, and this relationship between V_0 and R_u must be calculated.

This relationship is simplified if we again assume that a high-impedance voltage-measuring instrument is used and the current drawn by it, I_m , can be approximated to zero. (The case when this assumption does not hold is covered later in this section.) The analysis is then exactly the same as for the preceding example of the Wheatstone bridge, except that R_v is replaced by R_1 . Thus, from Eq. (6.1), we have:

$$V_0 = V_i \left(\frac{R_u}{R_u + R_3} - \frac{R_1}{R_1 + R_2} \right) \quad (6.3)$$

When R_u is at its nominal value, i.e., for $R_u = R_1$, it is clear that $V_0 = 0$ (because $R_2 = R_3$). For other values of R_u , V_0 has negative and positive values that vary in a nonlinear way with R_u .

The deflection-type bridge is somewhat easier to use than a null-type bridge because the output measurement is given directly in the form of a voltage measurement. However, its measurement accuracy is not as good as that of a null-type bridge. Despite its inferior accuracy, ease of use means that it is the preferred form of bridge in most general measurement situations unless the greater accuracy of a null-type bridge is absolutely necessary.

■ Example 6.2

A certain type of pressure transducer, designed to measure pressures in the range 0–10 bar, consists of a diaphragm with a strain gauge cemented to it to detect diaphragm deflections. The strain gauge has a nominal resistance of 120 Ω and forms one arm of a Wheatstone bridge circuit, with the other three arms each having a resistance of 120 Ω . The bridge output is measured by an instrument whose input impedance can be assumed to be infinite. If, in order to limit heating effects, the maximum permissible gauge current is 30 mA, calculate the maximum permissible

bridge excitation voltage. If the sensitivity of the strain gauge is $338 \text{ m}\Omega/\text{bar}$ and the maximum bridge excitation voltage is used, calculate the bridge output voltage when measuring a pressure of 10 bar.

■ Solution

This is the type of bridge circuit shown in Fig. 6.2, in which the components have the following values:

$$R_1 = R_2 = R_3 = 120 \ \Omega$$

Defining I_1 to be the current flowing in path ADC of the bridge, we can write:

$$V_i = I_1(R_u + R_3)$$

At balance, $R_u = 120$ and the maximum value allowable for I_1 is 0.03 A.

Hence: $V_i = 0.03(120 + 120) = 7.2 \text{ V}$.

Thus, the maximum bridge excitation voltage allowable is 7.2 V.

For a pressure of 10 bar applied, the resistance change is $3.38 \ \Omega$; that is, R_u is then equal to $123.38 \ \Omega$.

Applying Eq. (6.3), we can write:

$$V_0 = V_i \left(\frac{R_u}{R_u + R_3} - \frac{R_1}{R_1 + R_2} \right) = 7.2 \left(\frac{123.38}{243.38} - \frac{120}{240} \right) = 50 \text{ mV}$$

Thus, if the maximum permissible bridge excitation voltage is used, the output voltage is 50 mV when a pressure of 10 bar is measured.

The nonlinear relationship between output reading and measured quantity exhibited by Eq. (6.3) is inconvenient and does not conform with the normal requirement for a linear input–output relationship. The method of coping with this nonlinearity varies according to the form of the primary transducer involved in the measurement system.

One special case is where the change in the unknown resistance R_u is very small compared with the nominal value of R_u . If we calculate the new voltage V'_0 when the resistance R_u in Eq. (6.3) changes by an amount δR_u , we have:

$$V'_0 = V_i \left(\frac{R_u + \delta R_u}{R_u + \delta R_u + R_3} - \frac{R_1}{R_1 + R_2} \right) \quad (6.4)$$

The change of voltage output is therefore given by:

$$\begin{aligned}\delta V_0 &= V'_0 - V_0 = V_i \left(\frac{R_u + \delta R_u}{R_u + \delta R_u + R_3} - \frac{R_u}{R_u + R_3} \right) \\ &= V_i \left(\frac{R_u}{R_u + \delta R_u + R_3} + \frac{\delta R_u}{R_u + \delta R_u + R_3} - \frac{R_u}{R_u + R_3} \right)\end{aligned}$$

For small changes in R_u such that $\delta R_u \ll R_u$, the denominator in the first two terms of this expression approximate to $(R_u + R_3)$.

Hence, the first and third terms inside the brackets of the expression now cancel and we can write:

$$\delta V_0 \approx \frac{V_i \delta R_u}{R_u + R_3}$$

Thus:

$$\frac{\delta V_0}{\delta R_u} \approx \frac{V_i}{R_u + R_3} \quad (6.5)$$

This expression, which is a linear relationship, describes the measurement sensitivity of the bridge. Such an approximation to make the relationship linear is valid for transducers such as strain gauges where the typical changes of resistance with strain are very small compared with the nominal gauge resistance.

However, many instruments that are inherently linear themselves at least over a limited measurement range, such as resistance thermometers, exhibit large changes in output as the input quantity changes, and the approximation of Eq. (6.5) cannot be applied. In such cases, specific action must be taken to improve linearity in the relationship between the bridge output voltage and the measured quantity. One common solution to this problem is to make the values of resistances R_2 and R_3 at least 10 times those of R_1 and R_u (nominal). The effect of this is best observed by looking at a numerical example.

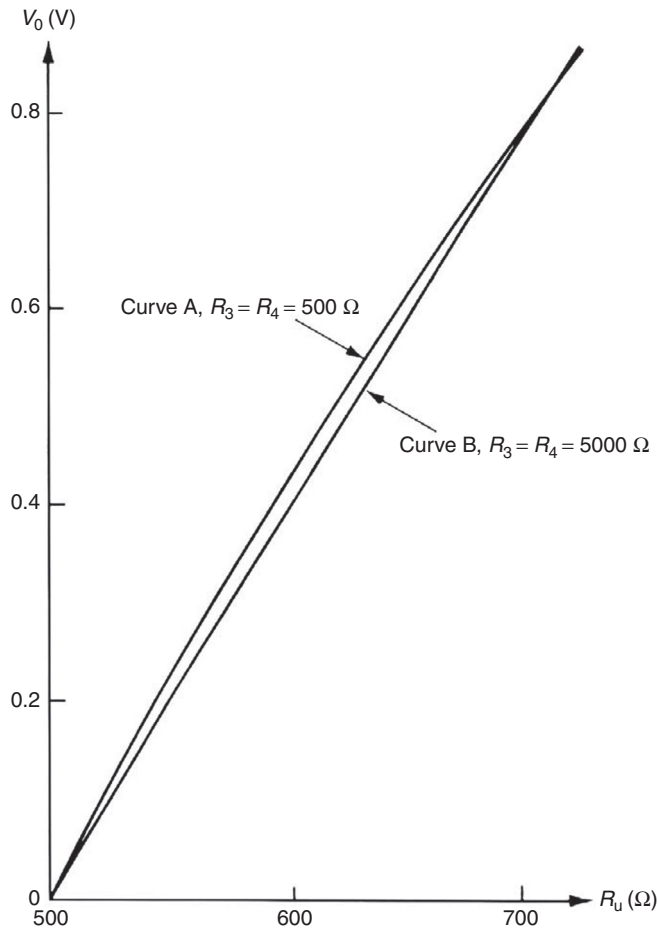
Consider a platinum resistance thermometer with a range of 0–50°C, whose resistance at 0°C is 500 Ω and whose resistance varies with temperature at the rate of 4 $\Omega/^\circ\text{C}$. Over this range of measurement, the output characteristic of the thermometer itself is nearly perfectly linear. (The subject of resistance thermometers is discussed further in Chapter 14.)

Taking first the case where $R_1 = R_2 = R_3 = 500 \Omega$ and $V_i = 10\text{V}$, and applying Eq. (6.3):

At 0°C; $V_0 = 0$

At 25°C; $R_u = 600 \Omega$ and $V_0 = 10 \left(\frac{600}{1100} - \frac{500}{1000} \right) = 0.455\text{V}$

At 50°C; $R_u = 700 \Omega$ and $V_0 = 10 \left(\frac{700}{1200} - \frac{500}{1000} \right) = 0.833\text{V}$


Figure 6.3

Linearization of bridge circuit characteristic.

This relationship between V_0 and R_u is plotted as Curve (a) in Fig. 6.3 and the nonlinearity is apparent. Inspection of the manner in which the output voltage V_0 above changes for equal steps of temperature change also clearly demonstrates the nonlinearity. For the temperature change from 0°C to 25°C , the change in V_0 is $(0.455 - 0) = 0.455$ V. For the temperature change from 25°C to 50°C , the change in V_0 is $(0.833 - 0.455) = 0.378$ V. If the relationship were linear, the change in V_0 for the $25\text{--}50^\circ\text{C}$ temperature step would also be 0.455 V, giving a value for V_0 of 0.910 V at 50°C .

Now, take the case in which $R_1 = 500\ \Omega$, but $R_2 = R_3 = 5000\ \Omega$ and let $V_i = 26.1\text{V}$:

At 0°C ; $V_0 = 0$

At 25°C ; $R_u = 600\ \Omega$ and $V_0 = 26.1 \left(\frac{600}{5600} - \frac{500}{5500} \right) = 0.424\text{V}$

At 50°C; $R_u = 700 \Omega$ and $V_0 = 26.1 \left(\frac{700}{5700} - \frac{500}{5500} \right) = 0.833\text{V}$

This relationship is shown as Curve (b) in Fig. 6.3 and a considerable improvement in linearity is achieved. This is more apparent if the differences in values for V_0 over the two temperature steps are inspected.

From 0°C to 25°C, the change in V_0 is 0.424 V.

From 25°C to 50°C, the change in V_0 is 0.409 V.

The changes in V_0 over the two temperature steps are much closer to being equal than before, demonstrating the improvement in linearity. However, in increasing the values of R_2 and R_3 , it was also necessary to increase the excitation voltage from 10 to 26.1 V to obtain the same output levels. In practical applications, V_i would normally be set at the maximum level consistent with the limitation of the effect of circuit heating in order to maximize the measurement sensitivity ($V_0/\delta R_u$ relationship). Therefore, it would not be possible to increase V_i further if R_2 and R_3 were increased, and the general effect of such an increase in R_2 and R_3 is thus a decrease in the sensitivity of the measurement system.

The importance of this inherent nonlinearity in the bridge output relationship is greatly diminished if the primary transducer and bridge circuit are incorporated as elements within an intelligent instrument. In that case, digital computation is applied to produce an output in terms of the measured quantity that automatically compensates for the nonlinearity in the bridge circuit.

Case where current drawn by measuring instrument is not negligible

For various reasons, it is not always possible to meet the condition that the impedance of the instrument measuring the bridge output voltage is sufficiently large for the current drawn by it to be negligible. Wherever the measurement current is not negligible, an alternative relationship between the bridge input and output must be derived that takes the current drawn by the measuring instrument into account.

Thévenin's theorem (see chapter 3) is again a useful tool for this purpose. Replacing voltage source V_i in Fig. 6.4a with a zero internal resistance produces the circuit shown in Fig. 6.4b, or the equivalent representation shown in Fig. 6.4c. It is apparent from Fig. 6.4c that the equivalent circuit resistance consists of a pair of parallel resistors, R_u and R_3 , in series with the parallel resistor pair R_1 and R_2 . Thus, R_{DB} is given by:

$$R_{DB} = -\frac{R_1 R_2}{R_1 + R_2} + \frac{R_u R_3}{R_u + R_3} \quad (6.6)$$

The equivalent circuit derived via Thévenin's theorem with resistance R_m of the measuring instrument connected across the output is shown in Fig. 6.4d. The open-circuit voltage across DB, E_0 , is the output voltage calculated earlier (Eq. 6.3) for the case of $R_m = \infty$:

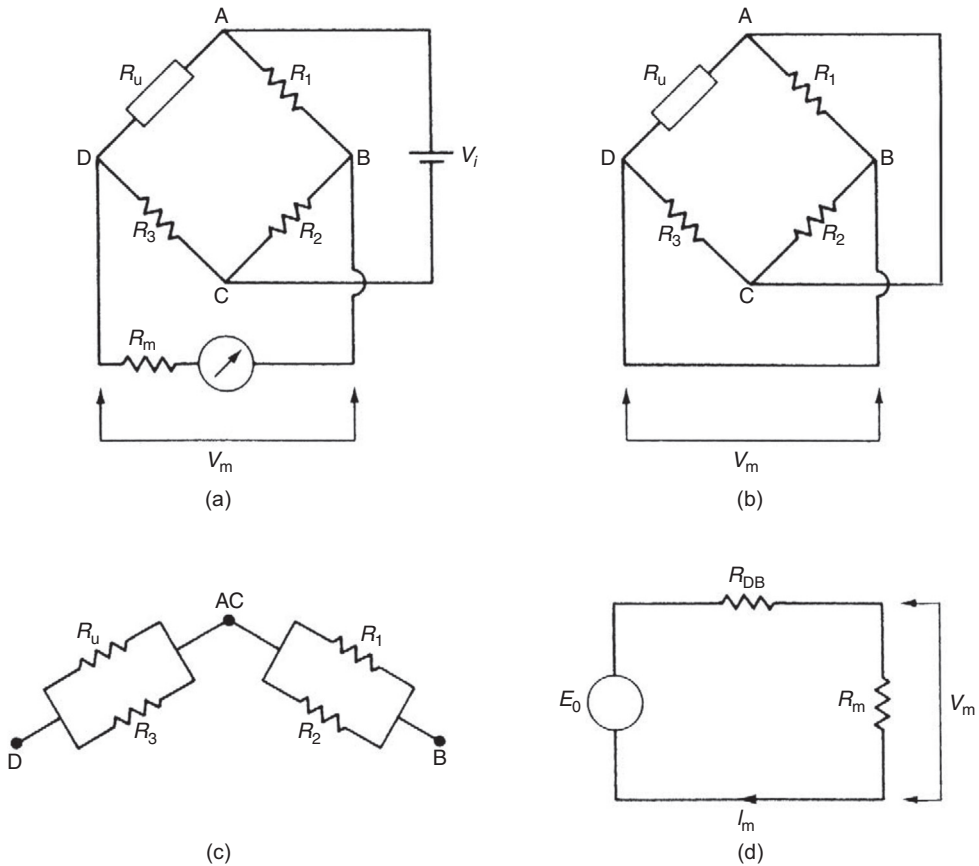


Figure 6.4
 (a) A bridge circuit. (b) Equivalent circuit by Thévenin's theorem. (c) Alternative representation.
 (d) Equivalent circuit for alternative representation.

$$E_0 = V_i \left(\frac{R_u}{R_u + R_3} - \frac{R_1}{R_1 + R_2} \right) \quad (6.7)$$

If the current flowing is I_m when the measuring instrument of resistance R_m is connected across DB, then by Ohm's law, I_m is given by:

$$I_m = \frac{E_0}{R_{DB} + R_m} \quad (6.8)$$

If V_m is the voltage measured across R_m , then again by Ohm's law:

$$V_m = I_m R_m = \frac{E_0 R_m}{R_{DB} + R_m} \quad (6.9)$$

Substituting for E_0 and R_{DB} in Eq. (6.9), using the relationships developed in Eqs. (6.6) and (6.7), we obtain:

$$V_m = \frac{V_i \left[\frac{R_u}{(R_u + R_3)} - \frac{R_1}{(R_1 + R_2)} \right] R_m}{\frac{R_1 R_2}{(R_1 + R_2)} + \frac{R_u R_3}{(R_u + R_3)} + R_m}$$

Simplifying:

$$V_m = \frac{V_i R_m (R_u R_2 - R_1 R_3)}{R_1 R_2 (R_u + R_3) + R_u R_3 (R_1 + R_2) + R_m (R_1 + R_2) (R_u + R_3)} \quad (6.10)$$

■ Example 6.3

A bridge circuit, as shown in Fig. 6.5, is used to measure the value of unknown resistance R_u of a strain gauge of nominal value 500 Ω . The output voltage measured across points DB in the bridge is measured by a voltmeter. Calculate the measurement sensitivity in volts/ohm change in R_u if:

- Resistance R_m of the measuring instrument is neglected, and
- Value of R_m is taken into account.

■ Solution

For $R_u = 500 \Omega$, $V_m = 0$.

To determine sensitivity, calculate V_m for $R_u = 501 \Omega$.

- Applying Eq. (6.3):

$$V_m = V_i \left(\frac{R_u}{R_u + R_3} - \frac{R_1}{R_1 + R_2} \right)$$

Substituting in values:

$$V_m = 10 \left(\frac{501}{5501} - \frac{500}{5500} \right) = 1.65 \text{ mV}$$

Thus, if the resistance of the measuring circuit is neglected, the measurement sensitivity is 5.00 mV per ohm change in R_u .

- Applying Eq. (6.10) and substituting in values:

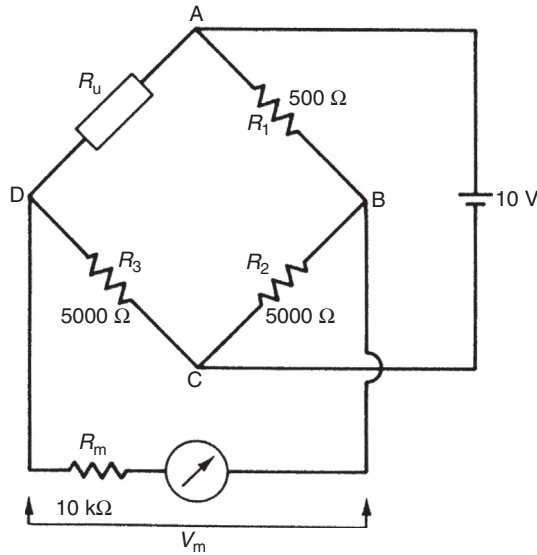


Figure 6.5
Bridge circuit for Example 6.2.

$$V_m = \frac{10 \times 10^4 \times 5000 (501 - 500)}{(500 \times 5000 \times 5501) + (501 \times 5000 \times 5500) + (10^4 \times 5500 \times 5501)}$$

$$= 1.515 \text{ mV}$$

Thus, if proper account is taken of the 10 KΩ value of the resistance of R_m , the true measurement sensitivity is shown to be 1.52 mV per ohm change in R_u .

6.2.3 Error analysis

In the application of bridge circuits, the contribution of component-value tolerances to total measurement system accuracy limits must be clearly understood. The analysis below applies to a null-type (Wheatstone) bridge, but similar principles can be applied for a deflection-type bridge. The maximum measurement error is determined by first finding the value of R_u in Eq. (6.2) with each parameter in the equation set at that limit of its tolerance that produces the maximum value of R_u . Similarly, the minimum possible value of R_u is calculated, and the required error band is then the span between these maximum and minimum values.

■ Example 6.4

In the Wheatstone bridge circuit of Fig. 6.1, R_v is a decade resistance box with a specified inaccuracy of $\pm 0.2\%$ and $R_2 = R_3 = 500 \Omega \pm 0.1\%$. If the value of R_v at the null position is 520.4Ω , determine the error band for R_u expressed as a percentage of its nominal value.

■ Solution

Applying Eq. (6.2) with $R_v = 520.4 \Omega + 0.2\% = 521.44 \Omega$,
 $R_3 = 500 \Omega + 0.1\% = 500.5 \Omega$, $R_2 = 500 \Omega - 0.1\% = 499.5 \Omega$, we get:

$$R_u = \frac{521.44 \times 500.5}{499.5} = 522.48 \Omega \quad (= +0.4\%)$$

Applying Eq. (6.2) with $R_v = 520.4 \Omega - 0.2\% = 519.36 \Omega$,
 $R_3 = 500 \Omega - 0.1\% = 499.5 \Omega$, $R_2 = 500 \Omega + 0.1\% = 500.5 \Omega$, we get:

$$R_u = \frac{519.36 \times 499.5}{500.5} = 518.32 \Omega \quad (= -0.4\%)$$

Thus, the error band for R_u is $\pm 0.4\%$.

The cumulative effect of errors in individual bridge circuit components is clearly seen. Although the maximum error in any one component is $\pm 0.2\%$, the possible error in the measured value of R_u is $\pm 0.4\%$. Such a magnitude of error is often unacceptable, and special measures are taken to overcome the introduction of error by component-value tolerances. One such practical measure is the introduction of apex balancing. This is one of many methods of bridge balancing that all produce a similar result.

Apex balancing

One form of apex balancing consists of placing an additional variable resistor R_5 at junction C between resistances R_2 and R_3 , and applying excitation voltage V_i to the wiper of this variable resistance, as shown in Fig. 6.6.

For calibration purposes, R_u and R_v are replaced by two equal resistances whose values are accurately known, and R_5 is varied until output voltage V_0 is zero. At this point, if the portions of resistance on either side of the wiper on R_5 are R_6 and R_7 (such that $R_5 = R_6 + R_7$), we can write:

$$R_3 + R_6 = R_2 + R_7$$

We have thus eliminated any source of error due to tolerance in the value of R_2 and R_3 , and the error in the measured value of R_u depends only on the accuracy of one component, the decade resistance box R_v .

■ Example 6.5

A potentiometer R_5 is put into the apex of the bridge shown in Fig. 6.6 to balance the circuit. The bridge components have the following values:

$$R_u = 500 \, \Omega, \quad R_v = 500 \, \Omega, \quad R_2 = 515 \, \Omega, \quad R_3 = 480 \, \Omega, \quad R_5 = 100 \, \Omega$$

Determine the required value of the resistances R_6 and R_7 of the parts of the potentiometer track either side of the slider in order to balance the bridge and compensate for the unequal values of R_2 and R_3 .

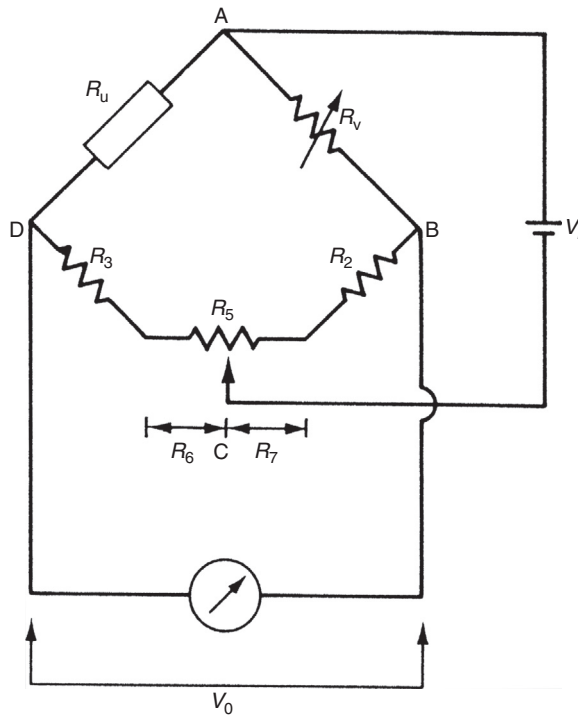


Figure 6.6
Apex balancing.

■ Solution

For balance, $R_2 + R_7 = R_3 + R_6$; hence, $515 + R_7 = 480 + R_6$.

Also, because R_6 and R_7 are the two parts of potentiometer track R_5 , whose resistance is 100Ω :

$R_6 + R_7 = 100$; Thus $515 + R_7 = 480 + (100 - R_7)$; i.e. $2R_7 = 580 - 515 = 65$

Thus, $R_7 = 32.5$. Hence, $R_6 = 100 - 32.5 = 67.5 \Omega$



6.3 Impedance measurement using alternating current bridges

The term “impedance” collectively refers to both inductance and capacitance. Inductance is measured in units of **Henry** (H) and capacitance is measured in units of **Farads** (F). The main device that has an output in the form of a change in inductance is the inductive displacement sensor. Devices that have an output in the form of a change in capacitance include the capacitive level gauge, the capacitive displacement sensor, the capacitive moisture meter, and the capacitive hygrometer. Unknown impedances in both inductive and capacitive form are commonly measured by AC bridges, although other measurement techniques exist, as explained later in [Sections 6.5 and 6.6](#).

AC bridges have an excitation in the form of an AC voltage. As with DC bridges, both null and deflection types exist, with null types being more accurate but also more tedious to use. Therefore, null types are normally reserved for use in calibration duties and any other application for which very high measurement accuracy is required. Otherwise, in all other general applications, deflection types are preferred.

6.3.1 Null-type impedance bridge

A typical null-type impedance bridge is shown in [Fig. 6.7](#). The null point can be conveniently detected by monitoring the output with a pair of headphones connected via an operational amplifier across points BD. This is a much cheaper method of null detection than the application of an expensive galvanometer that is required for a DC Wheatstone bridge.

Referring to [Fig. 6.7](#), at the null point, $I_1R_1 = I_2R_2$; $I_1Z_u = I_2Z_v$.

Thus:

$$Z_u = \frac{Z_v R_1}{R_2} \quad (6.11)$$

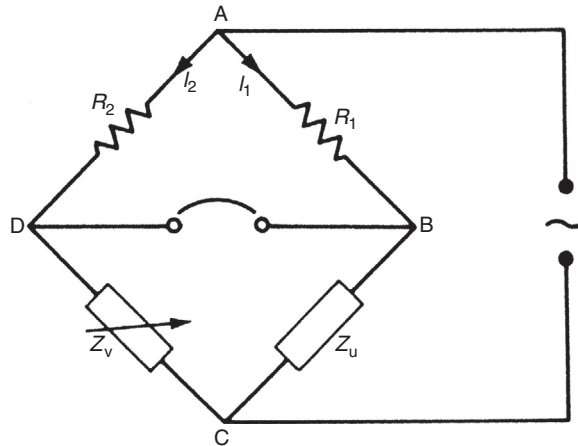


Figure 6.7

Null-type impedance bridge.

If Z_u is capacitive, i.e., $Z_u = \frac{1}{j\omega C_u}$, then Z_v must consist of a variable capacitance box, which is readily available. If Z_u is inductive, then $Z_u = R_u + j\omega L_u$.

Notice that the expression for Z_u as an inductive impedance has a resistive term in it because it is impossible to realize a pure inductor. An inductor coil always has a resistive component, although this is made as small as possible by designing the coil to have a high Q factor (Q factor is the ratio of inductance to resistance). Therefore, Z_v must consist of a variable-resistance box and a variable-inductance box. However, the latter are not readily available because it is difficult and hence expensive to manufacture a set of fixed-value inductors to make up a variable-inductance box. For this reason, an alternative kind of null-type bridge circuit, known as the **Maxwell bridge**, is commonly used to measure unknown inductances.

Example 6.6

A null-type impedance bridge is used to accurately measure the capacitance of a capacitive pressure sensor during a calibration procedure. The circuit shown in Fig. 6.7 is used, in which the known fixed resistance values are given by $R_1 = 491.7 \Omega$ and $R_2 = 483.2 \Omega$. The pressure sensor is inserted in the circuit as Z_u and an accurate variable capacitor box with capacitance C_v is used for Z_v . The capacitor box is adjusted until the bridge output voltage goes to zero. At this balance point, the value of C_v is 103.7 pF. Calculate the capacitance of the pressure sensor.

■ Solution

At the balance point, the bridge circuit components are related by Eq. (6.11),

$$Z_u = \frac{Z_v R_1}{R_2}, \text{ where } Z_u = \frac{1}{j\omega C_u} \text{ and } Z_v = \frac{1}{j\omega C_v}.$$

Substituting for Z_u and Z_v in (6.11) gives $C_u = \frac{C_v R_2}{R_1}$.

Substituting the resistance values into this equation for C_u gives:

$$C_u = \frac{103.7 \times 483.2}{491.7} = 101.4 \text{ pF}.$$

Thus, the capacitance of the pressure sensor is 101.4 pF.

6.3.2 Maxwell and Hay's bridges

The **Maxwell bridge** shown in Fig. 6.8 is an alternative form of null-type bridge. The requirement for a variable inductance box is avoided by instead introducing a second variable resistance. The circuit requires one standard fixed-value capacitor, two variable resistance boxes, and one standard fixed-value resistor, all of which are components that are readily available and inexpensive. Referring to Fig. 6.8, we have at the null-output point:

$$I_1 Z_{AD} = I_2 Z_{AB} ; I_1 Z_{DC} = I_2 Z_{BC}$$

Thus:

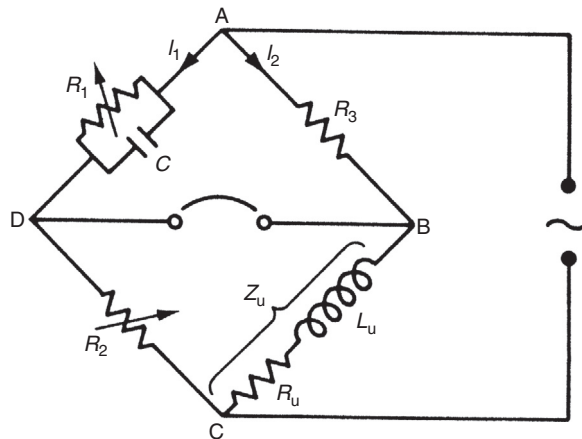


Figure 6.8
Maxwell bridge.

$$\frac{Z_{BC}}{Z_{AB}} = \frac{Z_{DC}}{Z_{AD}} \quad \text{or :} \quad Z_{BC} = \frac{Z_{DC}Z_{AB}}{Z_{AD}} \quad (6.12)$$

The quantities in Eq. (6.12) have the following values:

$$\frac{1}{Z_{AD}} = \frac{1}{R_1} + j\omega C \quad \text{or} \quad Z_{AD} = \frac{R_1}{1 + j\omega CR_1}$$

$$Z_{AB} = R_3 ; \quad Z_{BC} = R_u + j\omega L_u ; \quad Z_{DC} = R_2$$

Substituting the values into Eq. (6.12):

$$R_u + j\omega L_u = \frac{R_2 R_3 (1 + j\omega CR_1)}{R_1}$$

Taking real and imaginary parts:

$$R_u = \frac{R_2 R_3}{R_1} ; \quad L_u = R_2 R_3 C \quad (6.13)$$

This expression (6.13) can be used to calculate the quality factor (Q value) of the coil:

$$Q = \frac{\omega L_u}{R_u} = \frac{\omega R_2 R_3 C R_1}{R_2 R_3} = \omega C R_1$$

If a constant frequency ω is used: $Q \approx R_1$.

Thus, the Maxwell bridge can be used to measure the Q value of a coil directly using this relationship.

■ Example 6.7

In the Maxwell bridge shown in Fig. 6.8, let the fixed-value bridge components have the following values $R_3 = 5 \Omega$; $C = 1 \text{ mF}$. Calculate the value of the unknown impedance (L_u, R_u) if $R_1 = 159 \Omega$ and $R_2 = 10 \Omega$ at balance.

■ Solution

Substituting values into the relations developed in Eq. (6.13):

$$R_u = \frac{R_2 R_3}{R_1} = \frac{10 \times 5}{159} = 0.3145 \Omega ; \quad L_u = R_2 R_3 C = \frac{10 \times 5}{1000} = 50 \text{mH}$$

Example 6.8

Calculate the Q factor for the unknown impedance in Example 6.7 at a supply frequency of 50 Hz.

Solution

$$Q = \frac{\omega L_u}{R_u} = \frac{2\pi 50(0.05)}{0.3145} = 49.9$$

The Hay's bridge, shown in Fig. 6.9, is another alternative form of null-type bridge. It is used particularly for measuring the inductance of high- Q coils and is better than the Maxwell bridge for this purpose. Symbols R_1 and L_1 in Fig. 6.9 represent the resistance and inductance of the inductor coil. This form of bridge also avoids the need for a variable inductor box. The expressions for the values of R_1 and L_1 are found by analysis similar to that presented above for the Maxwell bridge.

6.3.3 Deflection-type alternating current bridge

A common deflection type of AC bridge circuit is shown in Fig. 6.10.

For capacitance measurement:

$$Z_u = \frac{1}{j\omega C_u} ; Z_1 = \frac{1}{j\omega C_1}$$

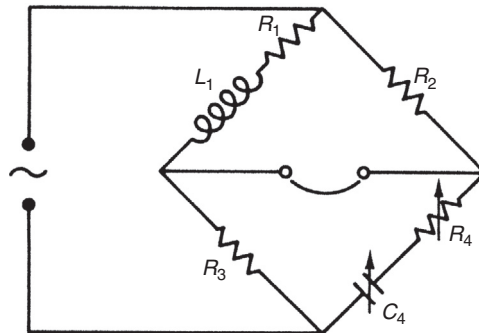
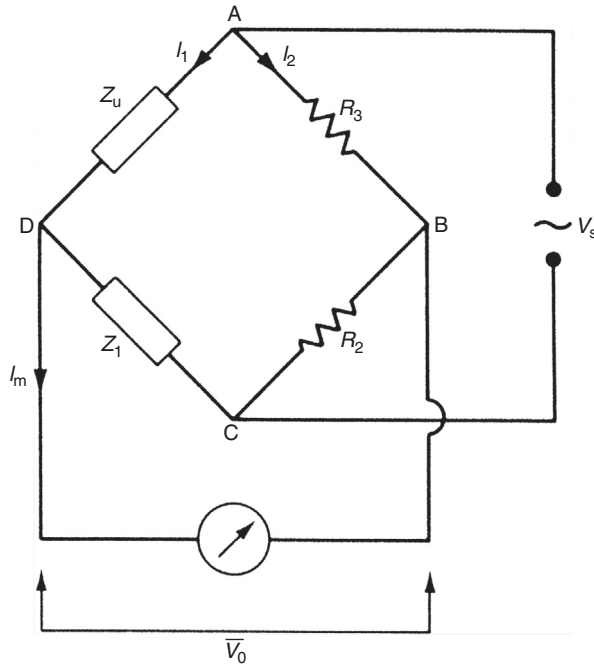


Figure 6.9
Hay's bridge.


Figure 6.10

Common deflection-type alternating current bridge.

For inductance measurement (making the simplification that the resistive component of the inductor is small and approximates zero):

$$Z_u = j\omega L_u \quad ; \quad Z_1 = j\omega L_1$$

Analysis of the circuit to find the relationship between V_0 and Z_u is greatly simplified if one assumes that I_m is negligible. This is valid provided that the instrument measuring V_0 has a high impedance. For $I_m = 0$, currents in the two branches of the bridge, as defined in Fig. 6.10, are given by:

$$I_1 = \frac{V_s}{Z_1 + Z_u} \quad ; \quad I_2 = \frac{V_s}{R_2 + R_3}$$

Also, $V_{AD} = I_1 Z_u$ and $V_{AB} = I_2 R_3$.

Hence:

$$V_0 = V_{BD} = V_{AD} - V_{AB} = V_s \left(\frac{Z_u}{Z_1 + Z_u} - \frac{R_3}{R_2 + R_3} \right)$$

Thus, for capacitances:

$$V_0 = V_s \left(\frac{\frac{1}{C_u}}{\frac{1}{C_1} + \frac{1}{C_u}} - \frac{R_3}{R_2 + R_3} \right) = V_s \left(\frac{C_1}{C_1 + C_u} - \frac{R_3}{R_2 + R_3} \right) \quad (6.14)$$

and for inductances:

$$V_0 = V_s \left(\frac{L_u}{L_1 + L_u} - \frac{R_3}{R_2 + R_3} \right) \quad (6.15)$$

This latter relationship (6.15) is in practice only approximate because inductive impedances are never pure inductances as assumed but always contain a finite resistance (i.e., $Z_u = j\omega L_u + R$). However, the approximation is valid in many circumstances.

■ Example 6.9

A deflection bridge as shown in Fig. 6.10 is used to measure an unknown capacitance, C_u . The components in the bridge have the following values:

$$V_s = 20V_{rms}, \quad C_1 = 100\mu\text{F}, \quad R_2 = 60 \Omega, \quad R_3 = 40 \Omega$$

If $C_u = 100 \mu\text{F}$, calculate the output voltage V_0 .

■ Solution

From Eq. (6.14):

$$V_0 = V_s \left(\frac{C_1}{C_1 + C_u} - \frac{R_3}{R_2 + R_3} \right) = 20(0.5 - 0.4) = 2V_{rms}$$

■ Example 6.10

An unknown inductance L_u is measured using a deflection type of bridge as shown in Fig. 6.10. The components in the bridge have the following values:

$$V_s = 10V_{rms}, \quad L_1 = 20\text{mH}, \quad R_2 = 100 \Omega, \quad R_3 = 100 \Omega$$

If the output voltage V_0 is $1V_{rms}$, calculate the value of L_u .

■ Solution

$$\text{From Eq. (6.15): } \frac{L_u}{L_1 + L_u} = \frac{V_0}{V_s} + \frac{R_3}{R_2 + R_3} = 0.1 + 0.5 = 0.6.$$

$$\text{Thus, } L_u = 0.6(L_1 + L_u) ; 0.4L_u = 0.6L_1 ; L_u = \frac{0.6L_1}{0.4} = 30\text{mH}$$

6.4 Alternative methods for measuring resistance

Apart from the DC bridge circuit, a number of other methods for measuring changes in the resistance of a sensor are available. These include the voltmeter-ammeter method, the resistance-substitution method, the digital voltmeter, and the ohmmeter. Apart from the ohmmeter, these instruments are normally used only to measure medium values of resistance in the range of 1Ω to $1 \text{ M}\Omega$, but this range is entirely adequate for all currently available sensors that convert the measured quantity into a change in resistance.

6.4.1 Voltmeter-ammeter method

The voltmeter-ammeter method consists of applying a measured DC voltage across the unknown resistance and measuring the current flowing. Two alternatives exist for connecting the two meters, as shown in Fig. 6.11. In Fig. 6.11a, the ammeter measures the current flowing in both the voltmeter and the resistance. The error due to this is minimized when the measured resistance is small relative to the voltmeter resistance. In the alternative form of connection, Fig. 6.11b, the voltmeter measures the voltage drop across the unknown resistance and the ammeter. Here, the measurement error is minimized when the unknown resistance is large with respect to the ammeter resistance. Thus, method A is best for measuring small resistances, and method B for large ones.

Having thus measured the voltage and current, the value of the resistance is then calculated simply by Ohm's law. This is a suitable method whenever it is acceptable to have the measurement inaccuracy of up to $\pm 1\%$ that it gives.

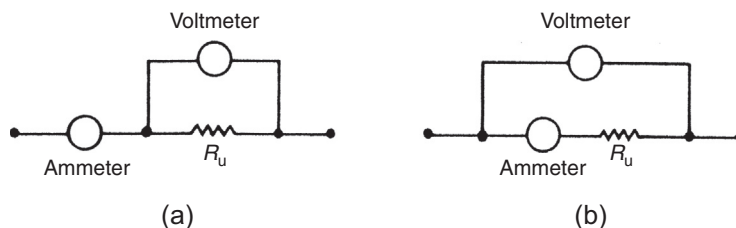


Figure 6.11

Voltmeter-ammeter method of measuring resistance.

6.4.2 Resistance-substitution method

In the voltmeter-ammeter method, either the voltmeter is measuring the voltage across the ammeter as well as across the resistance, or the ammeter is measuring the current flow through the voltmeter as well as through the resistance. The measurement error caused by this is avoided in the resistance-substitution technique. In this method, the unknown resistance in a circuit is temporarily replaced by a variable resistance. The variable resistance is adjusted until the measured circuit voltage and current are the same as those existing with the unknown resistance in place. The variable resistance at this point is equal in value to the unknown resistance.

6.4.3 Measurement using a digital voltmeter

The digital voltmeter can also be used to measure resistance if an accurate current source is included within it that passes current through the resistance. This can give a measurement inaccuracy as small as $\pm 0.1\%$.

6.4.4 Measurement using an ohmmeter

Ohmmeters are used to measure resistances over a wide range from a few milliohms up to 50 M Ω . The first generation of ohmmeters contained a battery that applied a known voltage across a combination of the unknown resistance and a known resistance in series, as shown in Fig. 6.12. Measurement of the voltage, V_m , across the known resistance, R , allows the unknown resistance, R_u , to be calculated from:

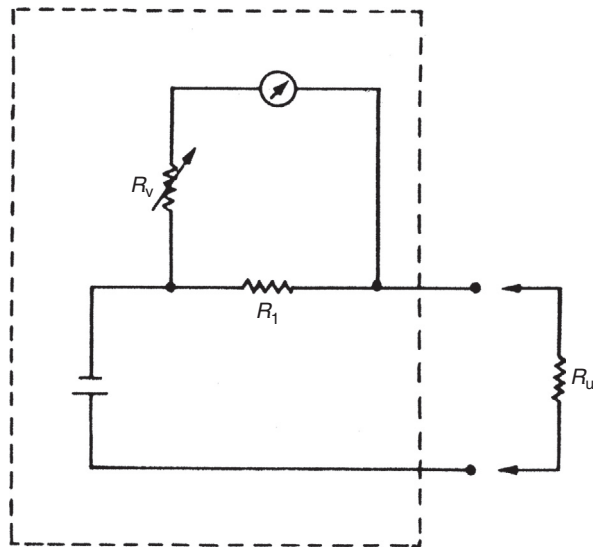


Figure 6.12
Ohmmeter.

$$R_u = \frac{R(V_b - V_m)}{V_m}$$

where V_b is the battery voltage. Unfortunately, this mode of resistance measurement gives a typical inaccuracy of $\pm 2\%$, which is acceptable only in a limited number of applications. Because of this, first-generation ohmmeters have been mostly replaced by a new type of electronic ohmmeter.

The electronic ohmmeter contains two circuits. The first circuit generates a constant current (I) that is passed through the unknown resistance. The second circuit measures the voltage (V) across the resistance. The resistance is then given by Ohm's law as: $R = V/I$. Electronic ohmmeters can achieve measurement inaccuracy as low as $\pm 0.02\%$.

Most *digital and analog multimeters* contain circuitry of the same form as in an ohmmeter, and hence can be similarly used to obtain measurements of resistance.

6.5 Alternative method for measuring inductance

The AC bridge circuit discussed earlier is the only accurate way to measure inductance, and various commercial inductance bridges are available. However, when such a commercial inductance bridge is not immediately available, the following method can be applied to give an approximate measurement of inductance.

This approximate method consists of connecting the unknown inductance in series with a variable resistance, in a circuit excited with a sinusoidal voltage, as shown in Fig. 6.13. The variable resistance is adjusted until the voltage measured across the resistance is equal to that measured across the inductance. The two impedances are then equal, and the value of inductance L can be calculated from:

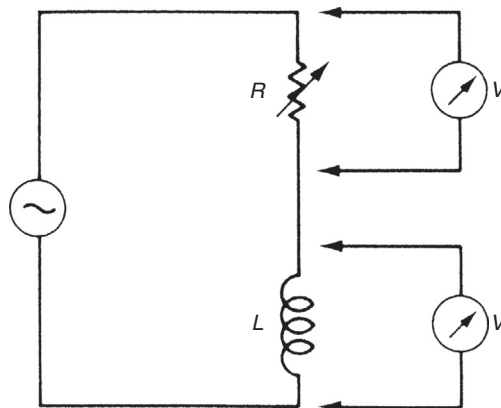


Figure 6.13

Approximate method of measuring inductance.

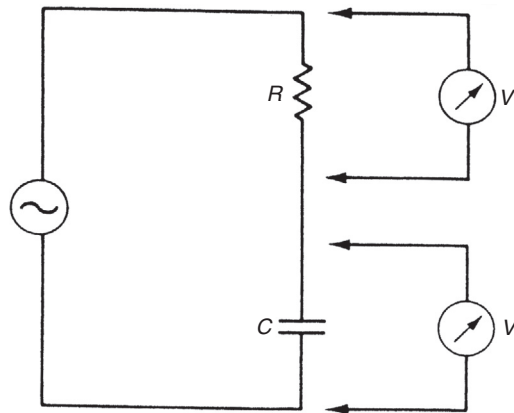


Figure 6.14
Approximate method of measuring capacitance.

$$L = \frac{\sqrt{(R^2 - r^2)}}{2\pi f}$$

where R is the value of the variable resistance, r is the value of the inductor resistance, and f is the excitation frequency.

6.6 Alternative methods to measure capacitance

Like inductance, capacitance can be measured accurately only by an AC bridge circuit, and various types of capacitance bridge are available commercially. Under circumstances in which a proper capacitance bridge is not immediately available, and if an approximate measurement of capacitance is acceptable, one of two methods can be considered.

The first of these, shown in Fig. 6.14, consists of connecting the unknown capacitor in series with a known resistance in a circuit excited at a known frequency. An AC voltmeter is used to measure the voltage drop across both the resistor and the capacitor. The capacitance value is then given by:

$$C = \frac{V_r}{2\pi f R V_c}$$

where V_r and V_c are the voltages measured across the resistance and capacitance, respectively; f is the excitation frequency; and R is the known resistance.

An alternative approximate method of measurement is to measure the time constant of the capacitor connected in a resistor–capacitor circuit.

6.7 Current measurement

Current measurement is needed for devices such as the thermocouple-gauge pressure sensor and the ionization gauge that have an output in the form of a varying electrical current. It is often also needed in signal transmission systems that convert the measured signal into a varying current. Any of the digital and analog voltmeters discussed in the last chapter can measure current if the meter is placed in series with the current-carrying circuit, and the same frequency limits apply for the measured signal as they do for voltage measurement. The upper frequency limit for AC measurement can be raised by rectifying the current into a DC form before measurement. To minimize the loading effect on the measured system, any current-measuring instrument must have a small resistance. This is opposite the case of voltage measurement, in which the instrument is required to have a high resistance for minimal circuit loading.

Besides the requirement to measure signal-level currents, many measurement applications also require higher-magnitude electrical currents to be measured. Hence, the following discussion covers the measurement of currents at both signal level and higher magnitudes.

Analog meters are useful in applications for which there is a need to display the measured value of current on a control panel. Moving coil instruments are used as panel meters to measure DC in the milliamp range up to one ampere. Moving iron meters can measure both DC and AC up to a magnitude of several hundred amps directly. To measure larger currents with electromechanical meters, it is necessary to insert a shunt resistance into the circuit and measure the voltage drop across it. Apart from the obvious disturbance of the measured system, one particular difficulty that results from this technique is the large power dissipation in the shunt. In the case of AC measurement, care must also be taken to match the resistance and reactance of the shunt to that of the measuring instrument so that frequency and waveform distortions in the measured signal are avoided.

Current transformers provide an alternative method of measuring high-magnitude currents that avoids the difficulty of designing a suitable shunt. Different versions of these exist for transforming both DC and AC. A DC transformer is shown in Fig. 6.15. The central DC

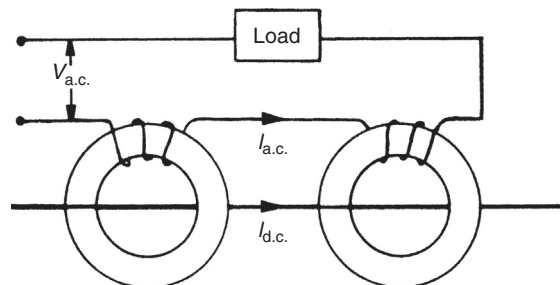


Figure 6.15

Current transformer. *a.c.*, alternating current; *d.c.*, direct current.

conductor in the instrument is threaded through two magnetic cores that carry two high-impedance windings connected in series opposition. The current flowing in the windings when excited with an AC voltage is proportional to the direct current in the central conductor. This output current is commonly rectified and then measured by a DC voltmeter.

An AC transformer typically has a primary winding consisting of only a few copper turns wound on a rectangular or ring-shaped core. The secondary winding, on the other hand, would normally have several hundred turns according to the step-down ratio required for the current. The output of the secondary winding is measured by any suitable current-measuring instrument. The design of current transformers is substantially different from that of voltage transformers. The rigidity of its mechanical construction has to be sufficient to withstand the large forces arising from short-circuit currents, and special attention has to be paid to the insulation between its windings for similar reasons. A low-loss core material is used and flux densities are kept as small as possible to reduce losses. In the case of very high-magnitude currents, the primary winding often consists of a single copper bar that behaves as a single-turn winding. The clamp-on meter described later in Chapter 10 is a good example of this.

All of the other instruments for measuring voltage discussed in Chapter 6 can be applied to current measurement by using them to measure the voltage drop across a known resistance placed in series with the current-carrying circuit. The digital voltmeter is widely applied to measure currents accurately by this method, and the oscilloscope is frequently used to obtain approximate measurements in circuit-test applications. Finally, the use of digital and analog multimeters for current measurement must also be mentioned, particularly in circuit-test applications. These instruments include a set of switchable dropping resistors and so can measure currents over a wide range. Protective circuitry within such instruments prevents damage when high currents are applied on the wrong input range.

6.8 Frequency measurement

Frequency measurement is required as part of devices that convert the measured physical quantity into a frequency change, such as the variable-reluctance velocity transducer, stroboscopes, vibrating-wire force sensor, resonant-wire pressure sensor, turbine flowmeter, Doppler-shift ultrasonic flowmeter, transit-time ultrasonic flowmeter, vibrating level sensor, quartz moisture meter, and quartz thermometer. In addition, the output relationship in some forms of AC bridge circuit used to measure inductance and capacitance requires accurate measurement of the bridge excitation frequency.

Frequency is measured in units of *Hertz* (Hz). The digital counter-timer is the most common instrument for measuring frequency. Alternatively, a phase-locked loop can be used. The oscilloscope is also commonly used, especially in circuit test and fault-diagnosis

applications. Finally, for measurements within the audio frequency range, the Wien bridge is a further instrument that is sometimes used.

6.8.1 Measurement using a digital counter-timer

A digital counter-timer is the most accurate and flexible instrument available for measuring frequency. Inaccuracy can be reduced down to 1 part in 10^8 , and all frequencies between DC and several gigahertz can be measured. The essential component within a counter-timer instrument is an oscillator that provides an accurately known and stable reference frequency, which is typically either 100 kHz or 1 MHz. This is often maintained in a temperature-regulated environment within the instrument to guarantee its accuracy. The oscillator output is transformed by a pulse-shaper circuit into a train of pulses and applied to an electronic gate, as shown in Fig. 6.16. Successive pulses at the reference frequency alternately open and close the gate. The input signal of unknown frequency is similarly transformed into a train of pulses and applied to the gate. The number of these pulses that get through the gate during the time that it is open during each gate cycle is proportional to the frequency of the unknown signal.

The accuracy of measurement obviously depends on how far the unknown frequency is above the reference frequency. Therefore, as it stands, the instrument can accurately measure only frequencies that are substantially above 1 MHz. To enable the instrument to measure much lower frequencies, a series of decade frequency-dividers are provided within it. These increase the time between the reference frequency pulses by factors of 10, and a typical instrument can have gate pulses separated in time by $1\ \mu\text{s}$ to 1 second.

Improvement in the accuracy of low-frequency measurement can be obtained by modifying the gating arrangements such that the signal of unknown frequency is made to control the opening and closing of the gate. The number of pulses at the reference frequency that pass through the gate during the open period is then a measure of the frequency of the unknown signal.

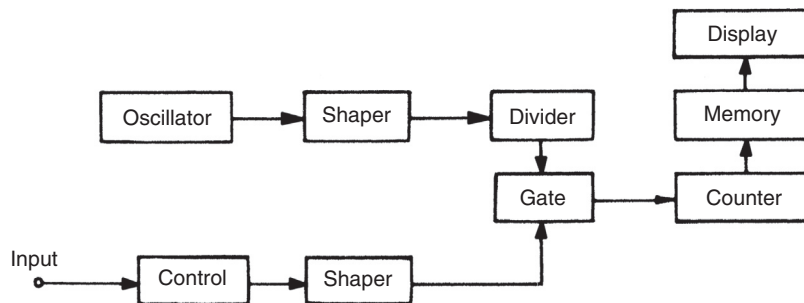


Figure 6.16
Digital counter-timer system.

6.8.2 Measurement using a phase-locked loop

A phase-locked loop is a circuit consisting of a phase-sensitive detector, a voltage-controlled oscillator (VCO), and amplifiers, connected in a closed-loop system as shown in Fig. 6.17. In a VCO, the oscillation frequency is proportional to the applied voltage. Operation of a phase-locked loop is as follows. The phase-sensitive detector compares the phase of the amplified input signal with the phase of the VCO output. Any phase difference generates an error signal, which is amplified and fed back to the VCO. This adjusts the frequency of the VCO until the error signal goes to zero, and thus the VCO becomes locked to the frequency of the input signal. The DC output from the VCO is then proportional to the input signal frequency.

6.8.3 Measurement using an oscilloscope

Many digital oscilloscopes (particularly the more expensive ones) have a push button on the front panel that causes the instrument to compute and display the frequency of the input signal automatically as a numeric value.

When this direct facility is unavailable (in some digital oscilloscopes and all analog ones), two alternative ways of using the instrument to measure frequency are available. First, the internal time base can be adjusted until the distance between two successive cycles of the measured signal can be read against the calibrated graticule on the screen. Measurement accuracy by this method is limited, but it can be optimized by measuring between points in the cycle where the slope of the waveform is steep, generally where it is crossing through from the negative to the positive part of the cycle. Calculation of the unknown frequency from this measured time interval is relatively simple. For example, suppose the distance between two cycles is 2.5 divisions when the internal time base is set at 10 ms/div. The cycle time is therefore 25 ms, and hence the frequency is $1000/25$ (i.e., 40 Hz). Measurement accuracy depends on how accurately the distance between two cycles is read, and it is difficult to reduce the error level below $\pm 5\%$ of the reading.

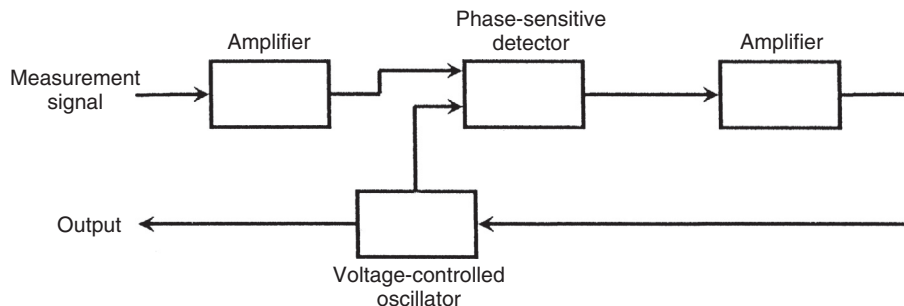


Figure 6.17
Phase-locked loop.

The alternative way of using an oscilloscope to measure frequency is to generate *Lissajous patterns*. These are produced by applying a known reference-frequency sine wave to the y input (vertical deflection plates) of the oscilloscope and the unknown-frequency sinusoidal signal to the x input (horizontal deflection plates). A pattern is produced on the screen according to the frequency ratio between the two signals, and if the numerator and denominator in the ratio of the two signals both represent an integral number of cycles, the pattern is stationary. Examples of these patterns are shown in Fig. 6.18, which also shows that phase difference between the waveforms has an effect on the shape. Frequency measurement proceeds by adjusting the reference frequency until a steady pattern is obtained on the screen and then calculating the unknown frequency according to the frequency ratio that the pattern obtained represents.

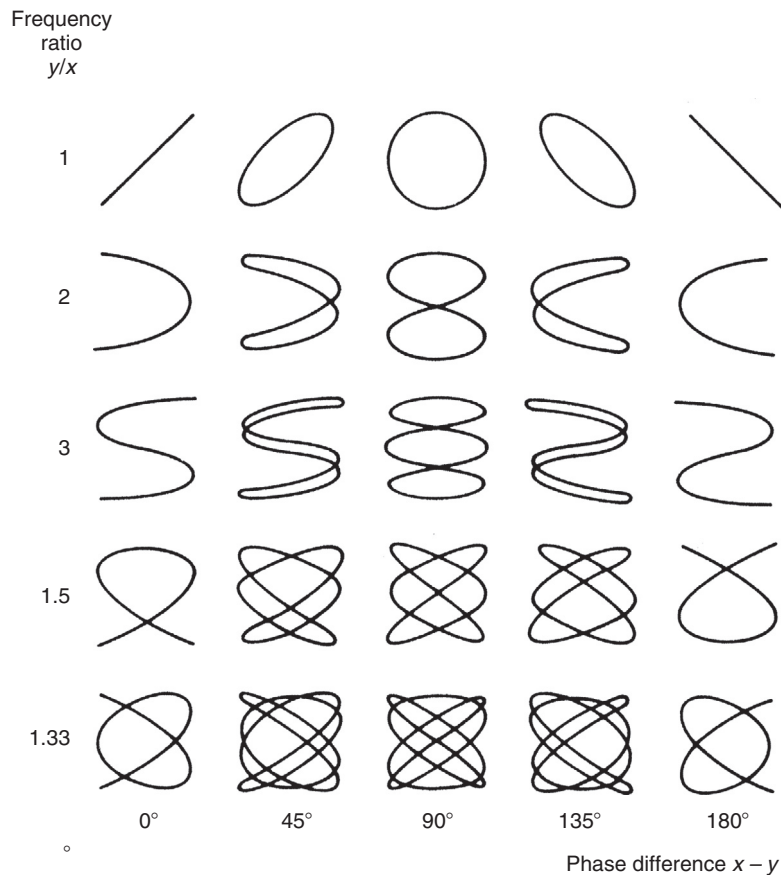


Figure 6.18
Lissajous patterns.

6.8.4 Measurement using a Wien bridge

The Wien bridge, shown in Fig. 6.19, is a special form of AC bridge circuit that can be used to measure frequencies in the audio range. An alternative use of the instrument is as a source of audio frequency signals of accurately known frequency. A simple set of headphones is often used to detect the null-output balance condition. Other suitable instruments for detecting the null point are the oscilloscope and the electronic voltmeter. At balance, the unknown frequency is calculated according to:

$$f = \frac{1}{2\pi R_3 C_3}$$

The instrument is accurate at audio frequencies, but errors due to losses in the capacitors and stray capacitance effects become significant at higher frequencies.

6.9 Phase measurement

Instruments that convert the measured variable into a phase change in a sinusoidal electrical signal include the transit-time ultrasonic flowmeter, the radar level sensor, the linear variable differential transformer, and the resolver. The most accurate instrument for measuring the phase difference between two signals is the electronic counter-timer. However, other methods exist as well. These include plotting the signals on an X-Y plotter, using an oscilloscope, and using a phase-sensitive detector.

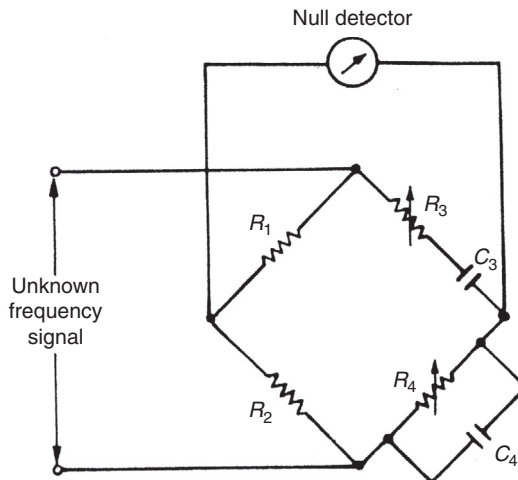


Figure 6.19
Wien bridge.

6.9.1 Measurement using an electronic counter-timer

In principle, the phase difference between two sinusoidal signals can be determined by measuring the time that elapses between the two signals crossing the time axis. However, in practice, this is inaccurate because the zero crossings are susceptible to noise contamination. The normal solution to this problem is to amplify or attenuate the two signals so that they have the same amplitude and then measure the time that elapses between the two signals crossing some nonzero threshold value.

The basis of this method of phase measurement is a digital counter-timer with a quartz-controlled oscillator providing a frequency standard that is typically 10 MHz. The crossing points of the two signals through the reference threshold voltage level are applied to a gate that starts and then stops pulses from the oscillator into an electronic counter, as shown in Fig. 6.20. The elapsed time, and hence phase difference, between the two input signals is then measured in terms of the counter display.

6.9.2 Measurement using an X–Y plotter

This is a useful technique for approximate phase measurement, but it is limited to low frequencies because of the limited bandwidth of an X–Y plotter. If two input signals of equal magnitude are applied to the X and Y inputs of a plotter, the plot obtained is an ellipse, as shown in Fig. 6.21. If the X and Y inputs have a phase difference ϕ given by:

$$V_X = V \sin(\omega t) ; V_Y = V \sin(\omega t + \phi)$$

At $t = 0$, $V_X = 0$ and $V_Y = V \sin \phi$. Thus, from Fig. 6.21, for $V_X = 0$, $V_Y = \pm h$:

$$\sin \phi = \pm h/V \quad (6.16)$$

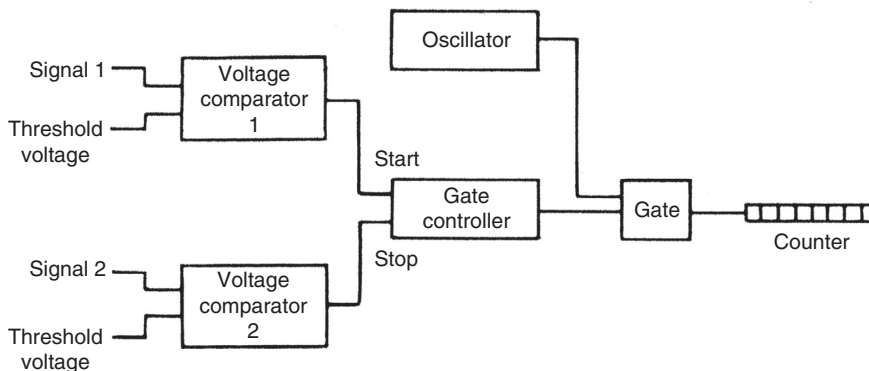


Figure 6.20
Phase measurement with digital counter-timer.

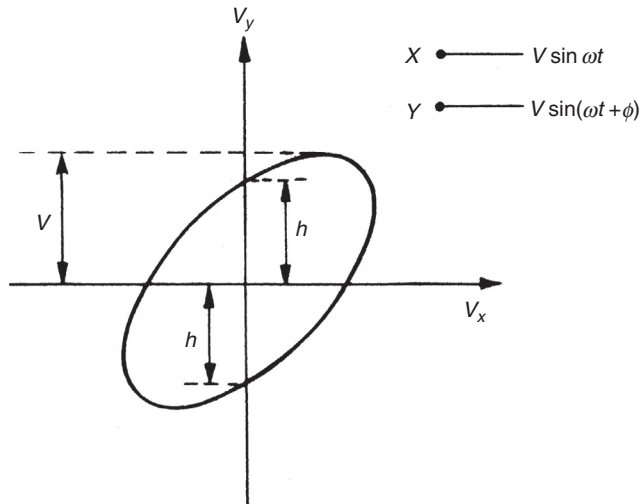


Figure 6.21
Phase measurement using X–Y plotter.

The solution of Eq. (6.4) gives four possible values for ϕ , but the ambiguity about which quadrant ϕ is in can usually be solved by observing the two signals plotted against time on a dual-beam oscilloscope.

6.9.3 Measurement using an oscilloscope

As for the case of frequency measurement, many digital oscilloscopes (particularly the more expensive ones) have a push button on the front panel that causes the instrument to automatically compute and display the phase of the input signal as a numeric value.

When this direct facility is not available (in some digital oscilloscopes and all analog ones), approximate measurement of the phase difference between signals can be made using any dual-channel oscilloscope. The two signals are applied to the two oscilloscope inputs and a suitable time base chosen such that the time between the crossing points of the two signals can be measured. The phase-difference of both low and high frequency signals can be measured by this method, in which the upper frequency limit measurable is dictated by the bandwidth of the oscilloscope (which is normally very high).

6.9.4 Measurement using a phase-sensitive detector

A phase-sensitive detector can be used to measure the phase difference between two signals that have an identical frequency. Phase sensitive detectors are also known by several alternative names, two examples of which are synchronous demodulator and synchronous detector. They can also exist physically in a number of alternative forms that

include both transformer-based and fully electronic circuits. For two signals of amplitude V_1 and V_2 , with the same frequency f , the output is given by $V_1 V_2 \cos \varphi$, where φ is the phase difference between the signals. This can be exploited in measurement devices such as the varying-phase output resolver (see Chapter 20).

6.10 Summary

This chapter has been concerned with looking at ways to deal with outputs from a measurement sensor that are not in the form of a readily measurable voltage signal. We started off by identifying the various alternative forms of output that we might have to deal with. Our list included translational-displacement-change outputs, changes in various electrical parameters such as resistance, inductance, capacitance, and current, and changes in the phase or frequency of an AC electrical signal.

This led us to realize that we needed mechanisms for converting these sensor outputs that are initially in some nonvoltage form into a more convenient voltage form. Because mechanisms for measuring translational displacements are needed for other purposes as well, we have deferred consideration of these until Chapter 19, in which the subject of translational measurement is considered in detail.

We learnt first of all in this study that bridge circuits are a particularly important in measuring resistance, inductance, and capacitance, and we therefore covered these in detail. One particularly important thing that we learned was that bridge circuits existed in two forms, null type and deflection type. Of these, null types are more tedious to use but provide better measurement accuracy, leading to these being the preferred form when sensors are being calibrated. We also noted that both DC and AC bridges exist; the former is used to interpret the output of sensors that exhibit a change in resistance, and the latter for sensors that convert the measured quantity into a change in either inductance or capacitance.

The later sections in the chapter then covered alternatives to bridge circuits to measure resistance, inductance, and capacitance. After discussing measurement of electrical current, the chapter looked at ways to interpret the output of sensors that is in the form of a change in either the frequency or the phase of an electrical signal.

6.11 Problems

- 6.1 A null-type Wheatstone bridge is used to accurately measure the resistance of a nickel resistance thermometer during a calibration procedure. The circuit shown in Fig. 6.1 is used, in which the known fixed resistance values are given by $R_2 = 123.7 \Omega$ and $R_3 = 127.4 \Omega$. The thermometer is inserted in the circuit as R_u and then the variable resistance box R_v is adjusted until the bridge output voltage V_o goes

- to zero. At this balance point, the value of R_v is 117.3Ω . Calculate the resistance of the thermometer.
- 6.2 Explain what a DC bridge circuit is and why it is so useful in measurement systems. List a few measurement sensors for which you would commonly use a DC bridge circuit to convert the sensor output into a change in output voltage of the bridge.
- 6.3 If elements in the DC bridge circuit shown in Fig. 6.2 have the values $R_u = 110 \Omega$, $R_1 = 100 \Omega$, $R_2 = 1000 \Omega$, and $R_3 = 1000 \Omega$, $V_i = 10\text{V}$, calculate the output voltage V_o if the impedance of the voltage-measuring instrument is assumed to be infinite. (Hint: Apply Eq. 6.3).
- 6.4 A null-type Wheatstone bridge is used to accurately measure the resistance of a strain gauge during a calibration procedure. The circuit shown in Fig. 6.1 is used, in which the known fixed resistance values are given by $R_2 = 331.2 \Omega$ and $R_3 = 327.5 \Omega$. The thermometer is inserted in the circuit as R_u and then the variable resistance box R_v is adjusted until the bridge output voltage V_o goes to zero. At this balance point, the value of R_v is 352.5Ω . Calculate the resistance of the strain gauge.
- 6.5 Suppose that the resistive components in the DC bridge shown in Fig. 6.2 have the following nominal values: $R_u = 3\text{K} \Omega$; $R_1 = 6\text{K} \Omega$; $R_2 = 8\text{K} \Omega$; and $R_3 = 4\text{K} \Omega$. The actual value of each resistance is related to the nominal value according to: $R_{\text{actual}} = R_{\text{nominal}} + \partial R$ where ∂R has the following values: $\partial R_u = 30 \Omega$; $\partial R_1 = -20 \Omega$; $\partial R_2 = 40 \Omega$; $\partial R_3 = -50 \Omega$. Calculate the open circuit bridge output voltage if the bridge supply voltage V_i is 50V .
- 6.6 (a) Suppose that the unknown resistance R_u in Fig. 6.2 is a resistance thermometer whose resistance at 100°C is 500Ω and whose resistance varies with temperature at the rate of $0.5 \Omega/^\circ\text{C}$ for small temperature changes around 100°C . Calculate the sensitivity of the total measurement system for small changes in temperature around 100°C , given the following resistance and voltage values measured at 15°C by instruments calibrated at 15°C : $R_1 = 500 \Omega$; $R_2 = R_3 = 5000 \Omega$; and $V_i = 10\text{V}$.
- (b) If the resistance thermometer is measuring a fluid whose true temperature is 104°C , calculate the error in the indicated temperature if the ambient temperature around the bridge circuit is 20°C instead of the calibration temperature of 15°C , given the following additional information:
- Voltage-measuring instrument zero drift coefficient = $+1.3 \text{mV}/^\circ\text{C}$.
 Voltage-measuring instrument sensitivity drift coefficient = 0 .
 Resistances R_1 , R_2 , and R_3 have a positive temperature coefficient of $+0.2\%$ of nominal value/ $^\circ\text{C}$.
 Voltage source V_i is unaffected by temperature changes.
- 6.7 Suppose that the resistive components in the DC bridge shown in Fig. 6.2 have the following nominal values: $R_u = 330 \Omega$; $R_1 = 470 \Omega$; $R_2 = 560\text{K} \Omega$; and $R_3 = 270\text{K} \Omega$. The actual value of each resistance is related to the nominal value according

to: $R_{actual} = R_{nominal} + \partial R$ where ∂R has the following values: $\partial R_u = -5 \Omega$; $\partial R_1 = +4 \Omega$; $\partial R_2 = +6 \Omega$; and $\partial R_3 = -9 \Omega$. Calculate the open circuit bridge output voltage if the bridge supply voltage V_i is 12 V.

- 6.8 Four strain gauges of resistance 120Ω each are arranged into a DC bridge configuration such that each of the four arms in the bridge has one strain gauge in it. The maximum permissible current in each strain gauge is 100 mA. What is the maximum bridge supply voltage allowable, and what power is dissipated in each strain gauge with that supply voltage?
- 6.9 (a) Suppose that the variables shown in Fig. 6.2 have the following values: $R_1 = 100 \Omega$, $R_2 = 100 \Omega$, $R_3 = 100 \Omega$; and $V_i = 12V$. R_u is a resistance thermometer with a resistance of 100Ω at $100^\circ C$ and a temperature coefficient of $+0.3 \Omega/^\circ C$ over the temperature range from $50^\circ C$ to $150^\circ C$ (i.e., the resistance increases as the temperature goes up). Draw a graph of bridge output voltage V_0 for $10^\circ C$ steps in temperature between $100^\circ C$ and $150^\circ C$ (calculating V_0 according to Eq. 6.3).
- (b) Briefly discuss whether you expect the graph that you have just drawn to be a straight line.
- (c) Draw a graph of V_0 for similar temperature values if $R_2 = R_3 = 1000 \Omega$ and all other components have the same values as given in part (a) above. Notice that the line through the data points is straighter than that drawn in part (a) but the output voltage is much less at each temperature point.
- (d) Briefly discuss the change in linearity of the graph drawn for part (c) and the change in measurement sensitivity compared with the graph drawn for part (a).
- 6.10 Four strain gauges of resistance 350Ω each are arranged into a DC bridge configuration such that each of the four arms in the bridge has one strain gauge in it. The maximum permissible current in each strain gauge is 30 mA. What is the maximum bridge supply voltage allowable, and what power is dissipated in each strain gauge with that supply voltage?
- 6.11 The unknown resistance R_u in a DC bridge circuit, connected as shown in Fig. 6.4A, is a resistance thermometer. The thermometer has a resistance of 350Ω at $50^\circ C$ and its temperature coefficient is $+1 \Omega/^\circ C$ (the resistance increases as the temperature rises). The components of the system have the following values: $R_1 = 350 \Omega$, $R_2 = R_3 = 2K \Omega$, $R_m = 20K \Omega$, and $V_i = 5V$. What is the output voltage reading when the temperature is $100^\circ C$? (Hint: use Eq. 6.10).
- 6.12 Suppose that the resistive components in the DC bridge shown in Fig. 6.2 have the following nominal values: $R_u = 270 \Omega$; $R_1 = 390 \Omega$; $R_2 = 560K \Omega$; and $R_3 = 220K \Omega$. The actual value of each resistance is related to the nominal value according to: $R_{actual} = R_{nominal} + \partial R$ where ∂R has the following values: $\partial R_u = +10 \Omega$;

$\partial R_1 = -20 \Omega$; $\partial R_2 = -15 \Omega$; and $\partial R_3 = +18 \Omega$. Calculate the open circuit bridge output voltage if the bridge supply voltage V_i is 5V.

- 6.13 The active element in a load cell is a strain gauge with a nominal resistance of 500Ω in its unstressed state. The cell has a sensitivity of $+0.5 \Omega$ per Newton of applied force and is connected in a DC bridge circuit where the other three arms of the bridge each have a resistance of 500Ω .
- (a) If the bridge excitation voltage is 20 V, what is the measurement sensitivity of the system in volts per Newton for small applied forces?
- (b) What is the bridge output voltage when measuring an applied force of 500 N?
- 6.14 Suppose that the unknown resistance R_u in Fig. 6.2 is a resistance thermometer whose resistance at 100°C is 600Ω and whose resistance varies with temperature at the rate of $+0.4 \Omega/^\circ\text{C}$ for small temperature changes around 100°C . Calculate the sensitivity of the total measurement system for small changes in temperature around 100°C , given the following resistance and voltage values:

$$R_1 = 600 \Omega ; R_2 = R_3 = 6000 \Omega ; V_i = 20\text{V}$$

Assume that the ambient temperature around the bridge circuit was the same as that at which the voltage measuring instrument and all bridge component values were calibrated.

- 6.15 The unknown resistance R_u of a resistance thermometer is measured by a deflection-type bridge circuit of the form shown in Fig. 6.2, where the parameters have the following values:

$$R_1 = 100 \Omega ; R_2 = R_3 = 1000 \Omega ; V_i = 20\text{V}$$

The thermometer has a resistance of 100Ω at 0°C and the resistance varies with temperature at the rate of $0.4 \Omega/^\circ\text{C}$ for small temperature changes around 0°C .

- (a) Calculate the bridge sensitivity in units of volts/ohm.
- (b) Calculate the sensitivity of the total measurement system in units of volts/ $^\circ\text{C}$ for small temperature changes around 0°C
- 6.16 The unknown resistance R_u of a resistance thermometer is to be measured by a bridge circuit of the form shown in Fig. 6.5, but where the bridge components and the excitation voltage are different from the values shown in Fig. 6.5, having instead the following values:

Nominal thermometer resistance at temperature of $20^\circ\text{C} = 100 \Omega$; $R_m = 10K \Omega$ (unchanged):

$$R_1 = 100 \Omega, R_2 = 1000 \Omega, R_3 = 1000 \Omega, V_i = 10\text{V}$$

- (a) Using Thévenin's theorem, derive an expression for the sensitivity of the bridge in terms of the change in output voltage V_m that occurs when there is a small change in the resistance of the thermometer.

- (b) If the resistance thermometer has a sensitivity of $400 \text{ m}\Omega/\text{C}$, calculate the temperature measurement sensitivity in bridge output volts (V_m) per $^\circ\text{C}$.
- 6.17 The unknown resistance R_u of a thermistor is to be measured by a bridge circuit of the form shown in Fig. 6.5, but where the bridge components and the excitation voltage are different from the values shown in Fig. 6.5, having instead the following values:

$$R_1 = 100 \Omega, R_2 = 1000 \Omega, R_3 = 1000 \Omega, V_i = 10\text{V}, R_m = 10\text{K} \Omega$$

The resistance (R_u) of the thermistor is related to the measured temperature (T) in degrees kelvin ($^\circ\text{K}$) according to the following expression:

$$R_u = 1000 \exp \left[3675 \left(\frac{1}{T} - 0.003354 \right) \right]$$

Draw a graph of the bridge output in volts for values of the measured temperature in steps of 5°C between 0°C and 50°C .

- 6.18 The unknown resistance R_u in a DC bridge circuit, connected as shown in Fig. 6.4A, is a resistance temperature detector (RTD). The RTD has a resistance of 100Ω at 0°C and its temperature coefficient is $+0.4 \Omega/^\circ\text{C}$ (the resistance increases as the temperature rises). The components of the system have the values: $R_1 = 100 \Omega$, $R_2 = R_3 = 1\text{K} \Omega$, $R_m = 10\text{K} \Omega$, and $V_i = 9\text{V}$. What is the output voltage reading when the temperature is 50°C ? (Hint: use Eq. 6.10).
- 6.19 In the DC bridge circuit shown in Fig. 6.22, the resistive components have the following values: $R_1 = R_2 = 120 \Omega$; $R_3 = 117 \Omega$; $R_4 = 123 \Omega$; and $R_a = R_p = 1000 \Omega$.
- (a) What are the resistance values of the parts of the potentiometer track on either side of the slider when the potentiometer is adjusted to balance the bridge?
- (b) What then is the effective resistance of each of the two left-hand arms of the bridge when the bridge is balanced?

[**Note:** This question will involve the solution of a quadratic equation. For a quadratic equation of the form: $ax^2 + bx + c = 0$, the solution is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- 6.20 The unknown resistance R_u of a strain gauge is to be measured by a bridge circuit of the form shown in Fig. 6.5, but the bridge components and the excitation voltage are different from the values shown in Fig. 6.5, having instead the values:

Nominal strain gauge resistance = 350Ω ; $R_m = 8\text{K} \Omega$:

$$R_1 = 350 \Omega, R_2 = 2000 \Omega, R_3 = 2000 \Omega, V_i = 24\text{V}$$

- (a) Using Thévenin's theorem, derive an expression for the sensitivity of the bridge in terms of the change in output voltage V_m that occurs when there is a small change in the resistance of the strain gauge.

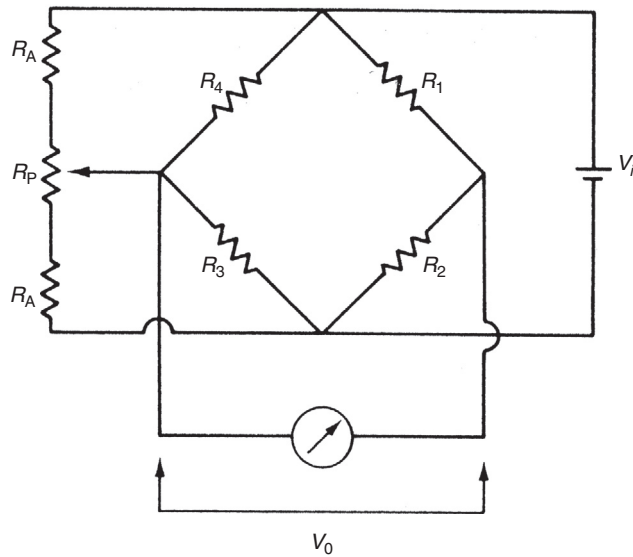


Figure 6.22

Direct current bridge with apex balancing.

- (b) If the strain gauge is part of a pressure transducer with a sensitivity of $1.2 \Omega/\text{bar}$, calculate the pressure measurement sensitivity in bridge output volts (V_m) per bar.

6.21 In the DC bridge circuit shown in Fig. 6.22, the resistive components have the values: $R_1 = R_2 = 350 \Omega$; $R_3 = 341 \Omega$; $R_4 = 359 \Omega$; and $R_a = R_p = 3000 \Omega$.

- (a) What are the resistance values of the parts of the potentiometer track on either side of the slider when the potentiometer is adjusted to balance the bridge?
 (b) What then is the effective resistance of each of the two left-hand arms of the bridge when the bridge is balanced?

[Note: This question will involve the solution of a quadratic equation. For a quadratic equation of the form: $ax^2 + bx + c = 0$, the solution is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}].$$

6.22 List a few measurement transducers and sensors for which you would commonly use an AC bridge circuit to convert the sensor output into a change in output voltage of the bridge.

6.23 A Maxwell bridge, designed to measure the unknown impedance (R_u, L_u) of a coil, is shown in Fig. 6.8.

- (a) Derive an expression for R_u and L_u under balance conditions.
 (b) If the fixed bridge component values are $R_3 = 100 \Omega$ and $C = 20 \mu\text{F}$, calculate the value of the unknown impedance if $R_1 = 3183 \Omega$ and $R_2 = 50 \Omega$ at balance.
 (c) Calculate the Q factor for the coil if the supply frequency is 50 Hz.

- 6.24 The deflection-type AC bridge shown in Fig. 6.10 is used to measure an unknown inductance L_u . The components in the bridge have the values: $V_s = 30V_{rms}$, $L_1 = 80\text{mH}$, $R_2 = 70\ \Omega$, and $R_3 = 30\ \Omega$. If $L_u = 50\text{mH}$, calculate output voltage V_0 .
- 6.25 An unknown capacitance C_u is measured using a deflection bridge, as shown in Fig. 6.10. The components of the bridge have the values: $V_s = 10V_{rms}$, $C_1 = 50\ \mu\text{F}$, $R_2 = 80\ \Omega$, and $R_3 = 20\ \Omega$. If the output voltage is $3V_{rms}$, calculate the value of C_u .
- 6.26 In the Hay's bridge shown in Fig. 6.9, the unknown inductance and resistance of the inductor coil are represented by the symbols L_1 and R_1 , respectively.
- Obtain the bridge balance conditions.
 - Show that if the Q value of the unknown inductor coil is high, the expression for the inductance value when the bridge is balanced is independent of frequency.
 - If the Q value is high, calculate the value of the inductor if the bridge component values at balance are: $R_2 = R_3 = 1000\ \Omega$; and $C = 0.02\ \mu\text{F}$.
- 6.27 A Maxwell bridge, designed to measure the unknown impedance (R_u, L_u) of a coil, is shown in Fig. 6.8.
- Explain briefly what the merits of a Maxwell bridge are compared with other forms of null-type, AC bridges.
 - Derive an expression for R_u and L_u under balanced conditions.
 - If the fixed bridge component values are $R_3 = 270\ \Omega$ and $C = 50\ \mu\text{F}$, calculate the value of the unknown impedance if $R_1 = 963.9\ \Omega$ and $R_2 = 75.4\ \Omega$ at balance.
 - Calculate the Q factor for the coil if the supply frequency is 60 Hz.
- 6.28 (a) For the deflection-type AC bridge shown in Fig. 6.10, derive expressions for the output voltage V_0 in terms of the bridge components given for the cases of (i) Z_u and Z_1 being inductances L_u and L_1 , and (ii) Z_u and Z_1 being capacitances C_u and C_1 . Assume that the impedance of the instrument measuring V_0 is very high and can be neglected.
- (b) The deflection-type bridge shown in Fig. 6.10 is used to measure an unknown inductance L_u . If the components in the bridge have the values: $V_s = 25V_{rms}$, $L_1 = 10\text{mH}$, $R_2 = 56\ \Omega$, and $R_3 = 33\ \Omega$, calculate the output voltage V_0 if $L_u = 20\text{mH}$
- 6.29 In the Hay's bridge shown in Fig. 6.9, the unknown inductance and resistance of the inductor coil are represented by the symbols L_1 and R_1 , respectively.
- Obtain the bridge balance conditions.
 - If the Q factor for the unknown inductor in the bridge has a high value, show that the expression for the inductance value when the bridge is balanced is independent of frequency.

- (d) If the Q value is high, calculate the value of the inductor if the bridge component values at balance are : $R_2 = R_3 = 500 \Omega$; and $C = 0.1 \mu\text{F}$.
- 6.30 (a) A deflection-type AC bridge of the form shown in Fig. 6.10 is used to measure the capacitance of an unknown capacitor C_u (thus, the impedances Z_u and Z_1 in the figure are replaced by capacitance values C_u and C_1). Derive an expression for the output voltage V_0 in terms of the bridge components given. Assume that the impedance of the instrument measuring V_0 is very high and can be neglected.
- (b) If the components of the bridge have the values: $V_s = 20V_{rms}$, $C_1 = 100\mu\text{F}$, $R_2 = 50 \Omega$, and $R_3 = 60 \Omega$ and the bridge output voltage measured is $5V_{rms}$, calculate the value of C_u .
- 6.31 Discuss the alternative methods of measuring the frequency of an electrical signal and indicate the likely measurement accuracy obtained with each method.
- 6.32 Using the Lissajous figure method of measuring frequency, a reference frequency signal of 1 kHz is applied to the Y channel of an oscilloscope and the unknown frequency is applied to the X channel. Determine the unknown frequency for each of the oscilloscope displays shown in Fig. 6.23. Also indicate the approximate phase difference of the unknown signal with respect to the reference signal.

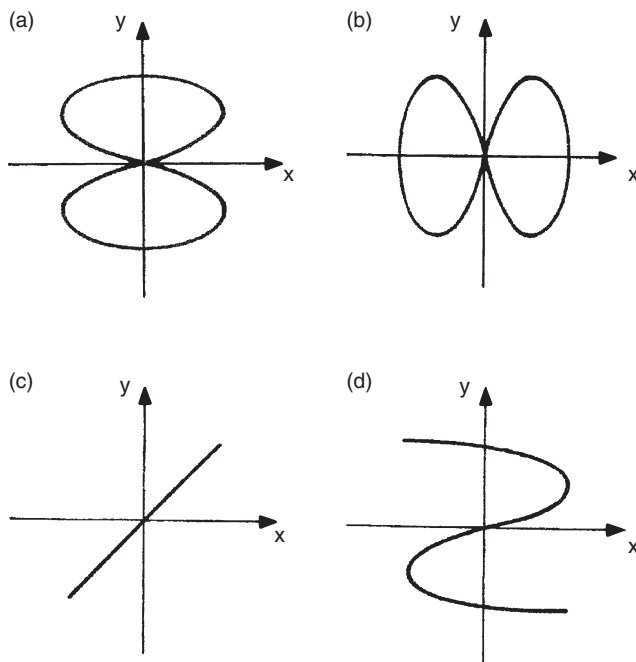


Figure 6.23
Oscilloscope displays for Problem 6.20.

- 6.33 What is the measurement range of ohmmeters? Describe two forms of ohmmeter and explain the mode of working and use of each.
- 6.34 Describe alternative ways to AC bridges for measuring inductance and capacitance.
- 6.35 What techniques are available for measuring electrical current? What measurement sensors have an output in the form of a varying electrical current?
- 6.36 What techniques are available for measuring the phase of an electrical signal? What measurement sensors have an output in the form of a varying phase?

Measurement signal transmission

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7.1 Introduction

There is often a necessity in many measurement systems to transmit measurement signals over quite large distances from the point of measurement to the place where the signals are recorded and/or used in a process control system. The need to separate the processing/recording parts of a measurement system from the point of measurement can arise for several reasons. One major reason for the separation is the environment around the point of measurement, which is often hostile toward one or more components in the rest of the measurement system. Extremes of temperature, humidity, weather, or fumes are typical examples of environments at the point of measurement that are too hostile for other measurement system components. Remoteness of the point of measurement can be another reason for transmitting measured signals to another point. We often meet this problem in environmental and weather monitoring systems. One example is water quality measurement in rivers, where sensors may actually be anchored in the river. In this sort of situation, no mains-powered electricity supply is available. Whilst it is possible to use battery or solar power for the sensors themselves, mains power is normally needed for the

necessary signal processing. Furthermore, there are usually no buildings available to protect the signal-processing elements from the environment. For these reasons, transmission of the measurements to another point is necessary.

We will therefore devote this chapter to a study of the various ways in which measurement data can be transmitted. We will see that the need to transmit measurement signals over what can sometimes be large distances creates several problems that we will investigate. We will discover in the following pages that, of the many difficulties associated with long-distance signal transmission, contamination of the measurement signal by noise is the most serious. Many sources of noise exist in industrial environments, such as radiated electromagnetic fields from electrical machinery and power cables, induced electromagnetic fields through wiring loops, and spikes (large transient voltages) that sometimes occur on the mains a.c. power supply. Our investigation into signal transmission techniques will show us that signals can be transmitted electrically, optically, by radiotelemetry, or pneumatically, in either analog or digital format. We will also discover that optical data transmission can be further divided into fiber-optic transmission and optical wireless transmission, according to whether a fiber-optic cable or just a plain air path is used as the transmission medium.

7.2 Analog transmission using copper conductors

The simplest method of electrical transmission is to transmit the measurement signal along a copper conductor as a varying analog voltage. However, this mode of transmission often causes the measurement signal to become corrupted by noise. To avoid such corruption, the signal can be transmitted as a varying current instead of a varying voltage. An alternative solution is to transmit the signal by superimposing it on an a.c. carrier system. All of these methods are discussed following.

7.2.1 Transmission as varying voltages

As most signals already exist in an electrical form as varying analog voltages, the simplest mode of transmission is to maintain the signals in the same form. This method of transmission is still widely used in industrial applications. However, because analog electrical transmission suffers problems of signal attenuation, and also exposes signals to corruption through induced noise, special measures have to be taken to overcome these problems.

Because the output signal levels from many types of measurement transducer are very low, ***signal amplification*** prior to transmission is essential if a reasonable signal-to-noise ratio is to be obtained after transmission. Amplification at the input to the transmission system is also required to compensate for the attenuation of the signal that results from the

resistance of the signal wires. The means of amplifying signals will be discussed later in chapter 8.

It is also usually necessary to provide *shielding* for the signal wires. Shielding consists of surrounding the signal wires in a cable with a metal shield that is connected to earth. This provides a high degree of noise protection, especially against capacitive-induced noise due to the proximity of signal wires to high-current power conductors. A fuller discussion on noise sources and the procedures followed to prevent the corruption of measurement voltage signals can be found in Chapter 3.

7.2.2 Current loop transmission

The signal-attenuation effect of conductor resistances can be minimized if the varying voltage signals are transmitted as varying current signals. This technique is known as current loop transmission. Despite being a very old technique, it is still widely used in industrial process control applications, especially to transmit data between sensors and proportional-integral-derivative (PID) controllers, etc. The technique provides high immunity to induced noise and uses currents in the range of 4–20 mA¹ to represent the voltage level of the analog signal. It requires a voltage-to-current converter of the form shown in Fig. 7.1, which is commonly known as a **4–20 mA current-loop interface**. Two voltage-controlled current sources are used, one providing a constant 4 mA output that is used as the power supply current and the other providing a variable 0–16 mA output that is scaled and proportional to the input voltage level. The net output current therefore varies between 4 and 20 mA, corresponding to analog signal levels between zero and the maximum value. The use of a positive, nonzero current level to represent a zero value of

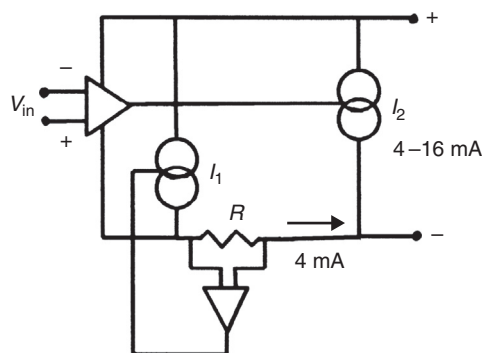


Figure 7.1

Voltage-to-current convertor (current loop interface).

¹ The 4–20 mA standard was agreed in 1972, prior to which a variety of different current ranges were used for signal transmission.

the transmitted signal enables transmission faults to be readily identified. If the transmitted current is zero, this automatically indicates the presence of a transmission fault, since the minimum value of current that represents a proper signal is 4 mA.

Current to voltage conversion is usually required at the termination of the transmission line to change the transmitted currents back to voltages. An operational amplifier, connected as shown in Fig. 7.2, is suitable for this purpose. The output voltage V is simply related to the input current I by $V = IR$.

The advent of intelligent devices led to the development of a modified current loop interface known as the *extended 4–20 mA current interface protocol*. This provides for the transmission of command/status information and the device power supply in analog form on the signal wires. In this extended protocol, signals in the range of 3.8–20.5 mA are regarded as “normal” measurement signals, thus allowing for under and over range from the 4–20 mA measurement signal standard. The current bands immediately outside this in the range of 3.6–3.8 mA and 20.5–21.0 mA are used for the conveyance of commands to the sensor/transmitter and the receipt of status information from it. This means that, if the signal wires are also used to carry the power supply to the sensor/transmitter, the power supply current must be limited to 3.5 mA or less to avoid the possibility of it being interpreted as a measurement signal or fault indicator. Signals greater than 21 mA (and less than 3.6 mA if the signal wires are not carrying a power supply) are normally taken to indicate either a short circuit or open circuit in the signal wiring.

Whilst this extended protocol in which the status and command information is transmitted in analog form is still used, it is now more common to find this status/command information transmitted as a digital channel superimposed on the analog current loop signal. The digital protocol used to achieve this is known as hybrid-mode HART, and is discussed more fully in Chapter 11.

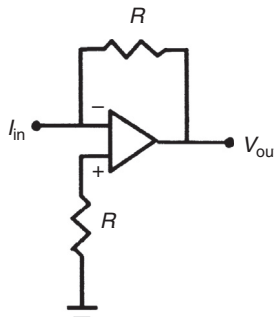


Figure 7.2
Current-to-voltage converter.

7.2.3 Transmission using an A.C. carrier

Another solution to the problem of noise corruption in low level d.c. voltage signals is to transfer the signal onto an a.c. carrier system before transmission and extract it from the carrier at the end of the transmission line. Both amplitude modulation (AM) and frequency modulation (FM) can be used for this.

AM consists of translating the varying voltage signal into variations in the amplitude of a carrier sine wave at a frequency of several kHz. An a.c. bridge circuit is commonly used for this, as part of the system for transducing the outputs of sensor that have a varying resistance (R), capacitance (C), or inductance (L) form of output. Referring back to Eqs. 6.14 and 6.15 in Chapter 6, for a sinusoidal bridge excitation voltage of $V_s = V \sin(\omega t)$, the output can be represented by $V_o = FV \sin(\omega t)$. V_o is a sinusoidal voltage at the same frequency as the bridge excitation frequency and its amplitude FV represents the magnitude of the sensor input (R, C, or L) to the bridge. For example, in the case of Eq. (6.15):

$$FV = \left(\frac{L_u}{L_1 + L_u} - \frac{R_3}{R_2 + R_3} \right) V$$

After shifting the d.c. signal onto a high frequency a.c. carrier, a high-pass filter can be applied to the AM signal. This successfully rejects noise in the form of low-frequency drift voltages and mains interference. At the end of the transmission line, demodulation is carried out to extract the measurement signal from the carrier.

FM achieves even better noise rejection than AM and involves translating variations in an analog voltage signal into frequency variations in a high-frequency carrier signal. A suitable voltage-to-frequency conversion circuit is shown in Fig. 7.3, in which the analog voltage signal input is integrated and applied to the input of a comparator that is preset to a certain threshold voltage level. When this threshold level is reached, the comparator generates an output pulse that resets the integrator and is also applied to a monostable. This causes the frequency f of the output pulse train to be proportional to the amplitude of the input analog voltage.

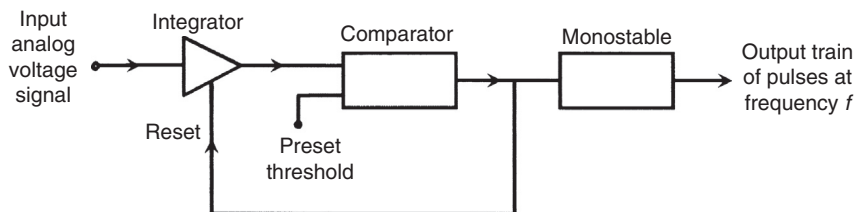


Figure 7.3
Voltage-to-frequency convertor.

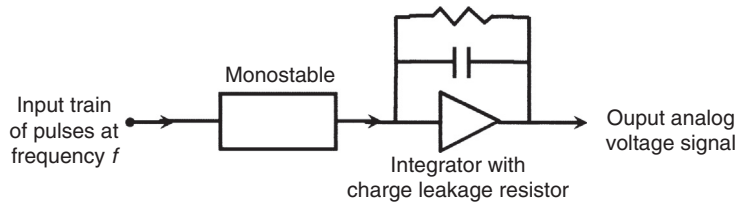


Figure 7.4
Frequency-to-voltage convertor.

At the end of the transmission line, the FM signal is usually converted back to an analog voltage by a frequency-to-voltage converter. A suitable conversion circuit is shown in Fig. 7.4, in which the input pulse train is applied to an integrator that charges up for a specified time. The charge on the integrator decays through a leakage resistor, and a balance voltage is established between the input charge on the integrator and the decaying charge at the output. This output balance voltage is proportional to the input pulse train at frequency f .

7.3 Digital transmission using copper conductors

Digital transmission has very significant advantages compared with analog transmission because the possibility of signal corruption during transmission is greatly reduced. Many different protocols exist for digital signal transmission. However, the protocol that is normally used for the transmission of data from a measurement sensor or circuit is asynchronous serial transmission, with other forms of transmission being reserved for use in instrumentation and computer networks. Asynchronous transmission involves converting an analog voltage signal into a binary digital equivalent, using an analog-to-digital converter. This is then transmitted as a sequence of voltage pulses of equal width that represent binary “1” and “0” digits. Commonly, a voltage level of either +5V or +6V is used to represent binary “1” and 0V represents binary “0.” Thus, the transmitted signal takes the form of a sequence of 6V pulses separated by 0V pulses. This is often known by the name of *pulse code modulation*. Such transmission in digital format provides very high immunity to noise because noise is typically much smaller than the amplitude of a pulse representing binary 1. At the receiving end of a transmitted signal, any pulse level between 0 and 3V can be interpreted as a binary “0” and anything greater than 3V can be interpreted as a binary “1.” A further advantage of digital transmission is that other information, such as the status of industrial equipment, can be conveyed as well as parameter values. However, consideration must be given to the potential problems of aliasing and quantization, which requires that the sampling frequency is chosen carefully, as discussed later in Chapter 8.

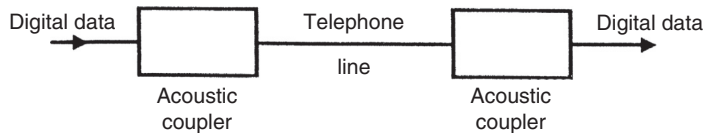


Figure 7.5

Digital transmission along a telephone line.

Many different mediums can be used to transmit digital signals electrically. Electrical cable, in the form of a twisted pair or coaxial cable, is commonly used as the transmission path. However, in some industrial environments, the noise levels are so high that even digital data becomes corrupted when transmitted as electrical pulses. In such cases, alternative transmission mechanisms have to be used.

One alternative is to modulate the pulses onto a high-frequency carrier, with positive and zero pulses being represented as two distinct frequencies either side of a center carrier frequency. Once in such a frequency modulated format, a normal mains electricity supply cable operating at mains frequency is often used to carry the data signal. The large frequency difference between the signal carrier and the mains frequency prevents any corruption of the data transmitted, and simple filtering and demodulation is able to extract the measurement signal after transmission. The public switched telephone network can also be used to transmit frequency-modulated data at speeds up to 1200 bits/s, using acoustic couplers as shown in Fig. 7.5. The transmitting coupler converts each binary “1” into a tone at 1.4 kHz and each binary “0” into a tone at 2.1 kHz, whilst the receiving coupler converts the tones back into binary digits.

Another solution is to apply the signal to a digital-to-current converter unit and then use current loop transmission (see Section 7.2), with 4 mA representing binary “0” and 20 mA representing binary “1.” This permits baud rates up to 9600 bit/s at transmission distances up to 3 km. Fiber-optic links and radio telemetry are also widely used to transmit digital data.

7.4 Fiber-optic transmission

Light has a number of advantages over electricity as a medium for transmitting information. For example, it is intrinsically safe, and noise corruption of signals by neighboring electromagnetic fields is almost eliminated. The most common form of optical transmission consists of transmitting light along a fiber-optic cable, although wireless transmission also exists as described in Section 7.5.

Apart from noise reduction, optical signal attenuation along a fiber-optic link is much less than electric signal attenuation along an equivalent length of metal conductor. However,

there is an associated cost penalty because of the higher cost of a fiber-optic system compared with the cost of metal conductors. In short fiber-optic links, cost is dominated by the terminating transducers that are needed to transform electrical signals into optical ones and vice versa. However, as the length of the link increases, the cost of the fiber-optic cable itself becomes more significant.

Fiber-optic cables are used for signal transmission in three distinct ways. Firstly, relatively short fiber-optic cables are used as part of various instruments to transmit light from conventional sensors to a more convenient location for processing, often in situations where space is very short at the point of measurement. Secondly, longer fiber-optic cables are used to connect remote instruments to controllers in instrumentation networks. Thirdly, even longer links are used for data transmission systems in telephone and computer networks. These three application classes have different requirements and tend to use different types of fiber-optic cable.

Signals are normally transmitted along a fiber-optic cable in digital format, although analog transmission is sometimes used. If there is a requirement to transmit more than one signal, it is more economical to multiplex the signals onto a single cable rather than transmit the signals separately on multiple cables. **Multiplexing** involves switching the analog signals in turn, in a synchronized sequential manner, into an analog-to-digital converter that outputs onto the transmission line. At the other end of the transmission line, a digital-to-analog converter transforms the digital signal back into analog form, and it is then switched in turn onto separate analog signal lines.

7.4.1 Principles of fiber optics

The central part of a fiber-optic system is a light transmitting cable containing at least one, but more often a bundle, of glass or plastic fibers. This is terminated at each end by a transducer, as shown in Fig. 7.6. At the input end, the transducer converts the signal from the electrical form in which most signals originate into light. At the output end, the transducer converts the transmitted light back into an electrical form suitable for use by data recording, manipulation, and display systems. These two transducers are often known as the transmitter and receiver, respectively.

Fiber-optic cable consists of an inner cylindrical core surrounded by a cylindrical cladding and a protective jacket, as shown in Fig. 7.7. The jacket consists of several layers of

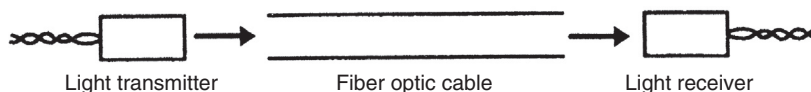


Figure 7.6
Fiber-optic cables and transducers.

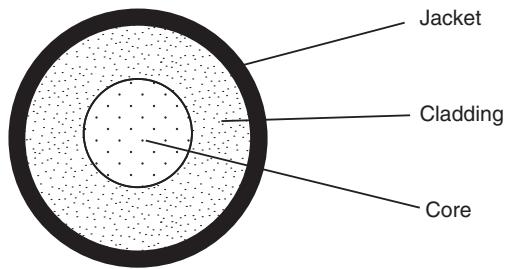


Figure 7.7

Cross section through fiber-optic cable.

polymer material and serves to protect the core from mechanical shocks that might affect its optical or physical properties. The refractive index of the inner core is greater than that of the surrounding cladding material, and the relationship between the two refractive indices affects the transmission characteristics of light along the cable. The amount of attenuation of light as it travels along the cable varies with the wavelength of the light transmitted. This characteristic is very nonlinear, and a graph of attenuation against wavelength shows a number of peaks and troughs. The position of these peaks and troughs varies according to the material used for the fibers. It should be noted that fiber manufacturers rarely mention these nonlinear attenuation characteristics and quote the value of attenuation that occurs at the most favorable wavelength.

Two forms of cable exist, known as single mode and multimode. Single-mode cables (sometimes known as monomode cables) have a small diameter core, typically $6\ \mu\text{m}$, whereas multimode cables have a much larger core, typically between 50 and $200\ \mu\text{m}$ in diameter. Both glass and plastic in different combinations are used in various forms of cable. One option is to use different types of glass fiber for both the core and the cladding. A second, and cheaper, option is to have a glass fiber core and a plastic cladding. This has the additional advantage of being less brittle than the all-glass version. Finally, all-plastic cables also exist, where two types of plastic fiber with different refractive indices are used. This is the cheapest form of all but it has the disadvantage of having high-attenuation characteristics, making it unsuitable for transmission of light over medium-to-large distances.

Protection is normally given to the cable by enclosing it in the same types of insulating and armoring materials that are used for copper cables. This protects the cable against various hostile-operating environments and also against mechanical damage. When suitably protected, fiber-optic cables can even withstand being engulfed in flames.

The *fiber-optic transmitter* is responsible for converting the electrical signal from a measurement sensor into light and transferring this into the fiber-optic cable. It is theoretically possible to encode the measurement signal by modulating either the intensity, frequency, phase, or polarization of the light injected into the cable, but light intensity

modulation has now achieved a dominant position in most fiber-optic transmission systems. Either laser diodes, light-emitting diodes (LEDs), or vertical cavity surface emitting laser (VCSEL) devices can be used as the light source in the transmitter. Laser diodes generate coherent light of a higher power than the incoherent light produced by light-emitting diodes, but they are more complex, more expensive, and less reliable than light-emitting diodes and their input current/light output relationship is less linear. These differences resulted in LEDs being preferred in most applications over laser diodes. However, this situation has now changed with the emergence of VCSEL devices. VCSEL devices are faster and have improved power and spectral properties compared with LEDs, and are thus now used more often than LEDs.

The characteristics of the light source chosen for the transmitter must closely match the attenuation characteristics of the light path through the cable and the spectral response of the receiving transducer. This is because the proportion of the power from the light source that is coupled into the fiber-optic cable is more important than the absolute output power of the emitted light. This proportion is maximized by making purpose-designed transmitters that have a spherical lens incorporated into the chip during manufacture. This produces an approximately parallel beam of light into the cable with a typical diameter of 400 μm .

The proportion of light entering the fiber-optic cable is also governed by the quality of the end face of the cable and the way it is bonded to the transmitter. A good end face can be produced by either polishing or cleaving. Polishing involves grinding the fiber end down with progressively finer polishing compounds until a surface of the required quality is obtained. Attachment to the transmitter is then normally achieved by gluing. This is a time-consuming process but uses cheap materials. Cleaving makes use of special kits that nick the fiber, break it very cleanly by applying mechanical force, and then attach it to the transmitter by crimping. This is a much faster method, but cleaving kits are quite expensive. Both methods produce good results.

A further factor that affects the proportion of light transmitted into the optic fibers in the cable is the transmitter alignment. It is very important to achieve proper alignment of the transmitter with the center of the cable. The effect of misalignment depends on the relative diameters of the beam and the core of the cable. [Fig. 7.8](#) shows the effect on the proportion of power transmitted into the cable for the cases of (a) cable core diameter $>$ beam diameter, (b) cable core diameter $=$ beam diameter, and (c) cable core diameter $<$ beam diameter. This shows that some degree of misalignment can be tolerated except where the beam and cable core diameters are equal. The cost of producing exact alignment of the transmitter and cable is very high, as it requires the light source to be exactly aligned in its housing, the fiber to be exactly aligned in its connector, and the housing to be exactly aligned with the connector. Therefore, great cost savings can be achieved wherever some misalignment can be tolerated in the specification for the cable.

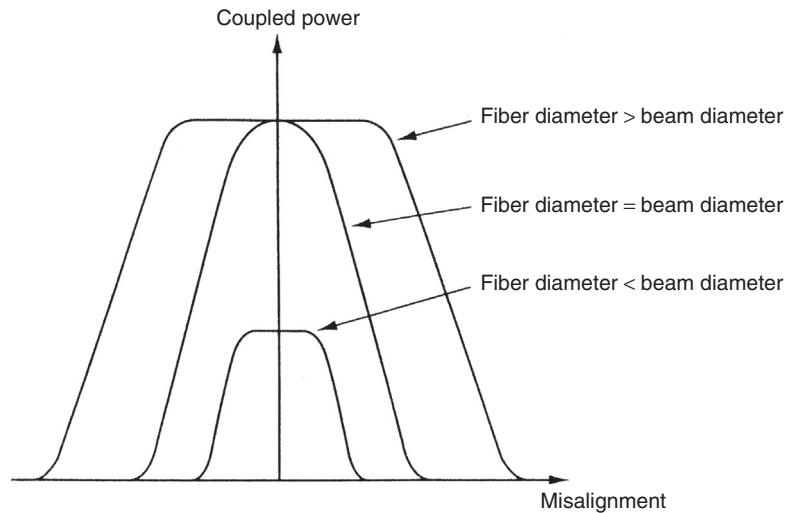


Figure 7.8
Effect of transmitter alignment on light power transmitted.

The *fiber-optic receiver* is the device that converts the optical signal back into electrical form. It is usually either a photodiode or a phototransistor. Phototransistors have good sensitivity but only have a low bandwidth. On the other hand, PIN diodes have a much higher bandwidth but a lower sensitivity. If both high bandwidth and high sensitivity are required, then special avalanche photodiodes are used, but at a severe cost penalty. The same considerations about losses at the interface between the cable and receiver apply as for the transmitter, and both polishing and cleaving are used to prepare the fiber ends.

The output voltages from the receiver are very small and amplification is always necessary. The system is very prone to noise corruption at this point. However, the development of receivers that incorporate an amplifier are finding great success in reducing the scale of this noise problem.

7.4.2 Transmission characteristics

Single-mode cables have very simple transmission characteristics because the core has a very small diameter and light can only travel in a straight line down it. On the other hand, multimode cables have quite complicated transmission characteristics because of the relatively large diameter of the core.

Whilst the transmitter is designed to maximize the amount of light that enters the cable in a direction that is parallel to its length, some light will inevitably enter multimode cables at other angles. Light that enters a multimode cable at any angle other than normal to the end face will be refracted in the core. It will then travel in a straight line until it meets the

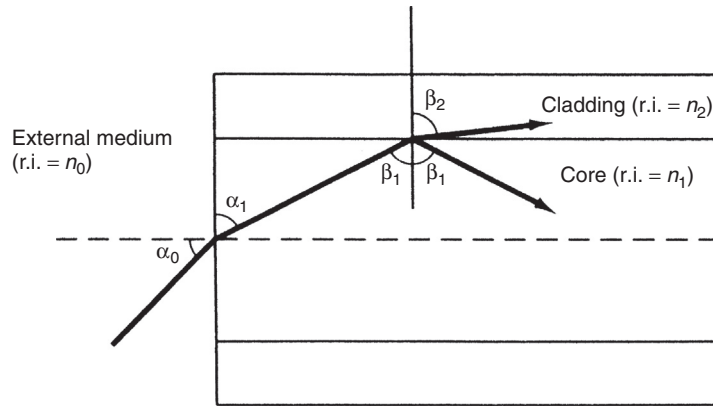


Figure 7.9

Transmission of light through cable.

boundary between the core and cladding materials. At this boundary, some of the light will be reflected back into the core and some will be refracted in the cladding.

For materials of refractive indices n_1 and n_2 as shown in Fig. 7.9, light entering from the external medium with refractive index n_0 at an angle α_0 will be refracted at an angle α_1 in the core and, when it meets the core-cladding boundary, part will be reflected at an angle β_1 back into the core and part will be refracted at an angle β_2 in the cladding. α_1 and α_0 are related by Snell's law according to:

$$n_0 \sin \alpha_0 = n_1 \sin \alpha_1 \quad (7.1)$$

Similarly, β_1 and β_2 are related by:

$$n_1 \sin \beta_1 = n_2 \sin \beta_2 \quad (7.2)$$

Light that enters the cladding is lost and contributes to the attenuation of the transmitted signal in the cable. However, observation of Eq. (7.1) shows how this loss can be prevented. If $\beta_2 = 90^\circ$, then the refracted ray will travel along the boundary between the core and cladding, and if $\beta_2 > 90^\circ$, all of the beam will be reflected back into the core. The case where $\beta_2 = 90^\circ$, corresponding to incident light at an angle α_c , is therefore the critical angle for total internal reflection to occur at the core/cladding boundary. The condition for this is that $\sin \beta_2 = 1$.

Setting $\sin \beta_2 = 1$ in Eq. (7.1):

$$\frac{n_1 \sin \beta_1}{n_2} = 1.$$

Thus:

$$\sin \beta_1 = n_2/n_1$$

Inspection of Fig. 7.9 shows that $\cos \alpha_1 = \sin \beta_1$.

Hence:

$$\sin \alpha_1 = \sqrt{1 - \cos^2 \alpha_1} = \sqrt{1 - \sin^2 \beta_1} = \sqrt{1 - (n_2/n_1)^2}$$

From Eq. (7.1):

$$\sin \alpha_c = \sin \alpha_0 = \frac{n_1}{n_0} \sin \alpha_1.$$

Thus:

$$\sin \alpha_c = \frac{n_1}{n_0} \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

Therefore, provided that the angle of incidence of the light into the cable is greater than the critical angle given by $\theta = \sin^{-1} \alpha_c$, all of the light will be internally reflected at the core/cladding boundary. Further reflections will occur as the light passes down the fibers, and it will thus travel in a zigzag fashion to the end of the cable.

Whilst attenuation has been minimized, there is a remaining problem that the transmission time of the parts of the beam that travel in this zigzag manner will be greater than light that enters the fiber at 90° to the face and so travels in a straight line to the other end. In practice, the incident light rays to the cable will be spread over the range given by $\sin^{-1} \alpha_c < \theta < 90^\circ$ and so the transmission times of these separate parts of the beam will be distributed over a corresponding range. These differential delay characteristics of the light beam are known as modal dispersion. The practical effect is that a step change in light intensity at the input end of the cable will be received over a finite period of time at the output. Multimode cables where this happens are known as **step-index** cables.

It is possible to largely overcome this latter problem in multimode cables by using cables made solely from glass fibers in which the refractive index changes gradually over the cross-section of the core rather than abruptly at the core/cladding interface as in the step index cable discussed so far. This special type of cable is known as a **graded index** cable, and it progressively bends light incident at less than 90° to its end face rather than reflecting it off the core/cladding boundary. Although the parts of the beam away from the center of the cable travel further, they also travel faster than the beam passing straight down the center of the cable because the refractive index is lower away from the center. Hence, all parts of the beam are subject to approximately the same propagation delay. In consequence, a step change in light intensity at the input produces an approximately step change of light intensity at the output. The alternative solution is to use a single-mode cable. This usually propagates light in a single mode only, which means that time dispersion of the signal is almost eliminated.

7.4.3 Multiplexing schemes

Various types of multiplexing schemes are available. It is outside the scope of this text to discuss these in detail, and interested readers are recommended to consult a specialist text in fiber-optic transmission. It is sufficient to note here that wavelength division multiplexing is used predominantly in fiber-optic transmission systems. This uses a multiplexer in the transmitter to merge the different input signals together, and a demultiplexer in the receiver to separate out the separate signals again. A different modulated frequency is used to transmit each signal. Since a single optic fiber is capable of propagating in excess of 100 different wavelengths without cross-interference, multiplexing allows more than 100 separate distributed sensors to be addressed. Wavelength division multiplexing systems normally use single-mode cable of 9 μm diameter, although there are also examples of usage of 50 or 62.5 μm diameter multimode cable.

7.5 Optical wireless telemetry (open air path transmission)

Wireless telemetry allows signal transmission to take place without laying down a physical link in the form of electrical or fiber-optic cable. This can be achieved using either radio or light waves to carry the transmitted signal across a plain air path between a transmitter and a receiver.

Optical wireless transmission was first developed in the early 1980s. It consists of a light source (usually infrared) transmitting encoded data information across an open, unprotected air path to a light detector. Three distinct modes of optical telemetry are possible, known as point-to-point, directed, and diffuse:

- **Point-to-point telemetry** uses a narrowly focused, fine beam of light, which is commonly used for transmission between adjacent buildings. A data transmission speed of 5 Mbit/s is possible at the maximum transmission distance of 1000m. However, if the transmission distance is limited to 200 m, a transmission speed of 20 Mbit/s is possible. Point-to-point telemetry is commonly used to connect electrical or fiber-optic Ethernet networks in adjacent buildings.
- **Directed telemetry** transmits a slightly divergent beam of light that is directed toward reflective surfaces, such as the walls and ceilings in a room. This produces a wide area of coverage and means that the transmitted signal can be received at a number of points. However, the maximum transmission rate possible is only 1 Mbit/s at the maximum transmission distance of 70 m. If the transmission distance is limited to 20 m, a transmission speed of 10 Mbit/s is possible.
- **Diffuse telemetry** is similar to directed telemetry but the beam is even more divergent. This increases the area of coverage but reduced transmission speed and range. At the maximum range of 20 m, the maximum speed of transmission is 500 kbit/s, though this increases to 2 Mbit/s at a reduced range of 10 m.

In practice, implementations of optical wireless telemetry are relatively uncommon because the transmission of data across an open, unprotected air path is susceptible to random interruption. In cases where immunity to electromagnetic noise is particularly important, open-path optical transmission is sometimes used because it provides immunity of the transmitted signal to electromagnetic noise at low cost. However, other ways of providing immunity to the transmitted signal against electromagnetic noise are often preferred despite their higher cost.

The usual alternative to open-air-path optical transmission for solving electromagnetic noise problems is to use fiber-optic transmission. In cases where laying a physical fiber-optic cable link is difficult, radio transmission is commonly used. This is preferred over optical transmission because it is much less prone to interference than optical transmission over an open air path, since radio waves can pass through most materials. However, there are some situations where radio transmission is subject to interference from neighboring radio-frequency systems operating at a similar wavelength and, in such circumstances, open-air-path optical transmission is sometimes a better option.

7.6 Radio telemetry (radio wireless transmission)

Radio telemetry is normally used over transmission distances up to 400 miles, though this can be extended by special techniques to provide communication through space over millions of miles. However, radio telemetry is also commonly used over quite short distances to transmit signals where physical electrical or fiber-optic links are difficult to install or maintain. This occurs particularly when the source of the signals is mobile. The great advantage that radio telemetry has over optical wireless transmission through an air medium is that radio waves are attenuated much less by most types of obstacle between the energy transmitter and receiver. Hence, as noted previously, radio telemetry usually performs better than optical wireless telemetry and is therefore used much more commonly.

In radio telemetry, data are usually transmitted in an FM format. A typical scheme is shown in Fig. 7.10, although other arrangements also exist. In this particular scheme shown, 18 data channels are provided over the frequency range from 0.4 kHz to 70 kHz, as given in the table below. Each channel is known as a subcarrier frequency and can be used to transmit data for a different physical variable. Thus, the system can transmit information from 18 different sensors simultaneously.

Band	1	2	3	4	5	6	7	8	9
Center frequency (kHz)	0.4	0.56	0.73	0.96	1.3	1.7	2.3	3.0	3.9
Band	10	11	12	13	14	15	16	17	18
Center frequency (kHz)	5.4	7.35	10.5	14.5	22.0	30.0	40.0	52.5	70.0

Maximum frequency deviation allowed is $\pm 7.5\%$

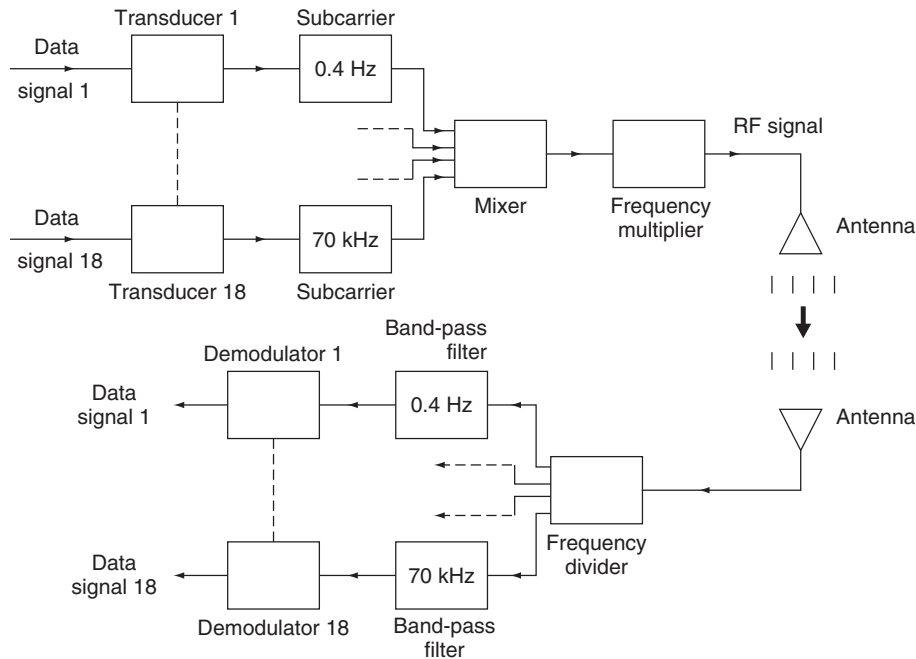


Figure 7.10
Radio transmission using FM/FM system.

A voltage-to-frequency converter is used in the first FM stage to convert each analog voltage signal into a varying frequency around the center frequency of the subcarrier assigned for that channel. The 18 channels are then mixed into a single signal spanning the frequency range 0.4 kHz to 70 kHz. For transmission, the length of the antenna has to be one-quarter or one-half of the wavelength. At 10 kHz, which is a typical subcarrier frequency in an 18-channel system, the wavelength is 30 km. Hence, an antenna for transmission at this frequency is totally impractical. In consequence, a second FM stage is used to translate the 0.4 kHz to 70 kHz signal into the radio frequency range as modulations on a typical carrier frequency of 217.5 MHz². At this frequency, the wavelength is 1.38 m, and so a transmission antenna of length 0.69 m or 0.345 m would be suitable. The signal is received by an antenna of identical length some distance away. A frequency converter is then used to change the signal back into the 0.4 kHz to 70 kHz subcarrier frequency spectrum, following which a series of band-pass filters are applied to extract the 18 separate frequency bands containing the measurement data. Finally, a demodulator is applied to each channel to return each signal into varying voltage form.

² Particular frequencies are allocated for industrial telemetry. These are subject to national agreements and vary in different countries.

The inaccuracy of radio telemetry is typically $\pm 1\%$. Thus, measurement uncertainty in transmitting a temperature measurement signal with a range of $0\text{--}100^\circ\text{C}$ over one channel would be $\pm 1\%$, i.e., $\pm 1^\circ\text{C}$. However, if there are unused transmission channels available, the signal could be divided into two ranges ($0\text{--}50^\circ\text{C}$ and $50\text{--}100^\circ\text{C}$) and transmitted over two channels, reducing the measurement uncertainty to $\pm 0.5^\circ\text{C}$. By using 10 channels for one variable, a maximum measurement uncertainty of $\pm 0.1^\circ\text{C}$ could be achieved.

In theory, radio telemetry is very reliable because, although the radio frequency waveband is relatively crowded, specific frequencies within it are allocated to specific usages under national agreements that are normally backed by legislation. Interference is avoided by licensing each frequency to only one user in a particular area, and limiting the transmission range through limits on the power level of transmitted signals, such that there is no interference to other licensed users of the same frequency in other areas.

Unfortunately, interference can still occur in practice, due both to adverse atmospheric conditions extending the transmission range beyond that expected into adjoining areas, and also due to unauthorized transmissions by other parties at the wavelengths licensed to registered users. There is a legal solution to this latter problem, although some time may elapse before the offending transmission is successfully stopped.

7.7 Pneumatic transmission

In recent years, pneumatic transmission tends to have been replaced by other alternatives in most new implementations of instrumentation systems. However, the technology is certainly not dead, and many examples can still be found in operation in the process industries, especially for control applications in flammable or explosive environments.

Pneumatic transmission consists of transmitting analog signals as a varying pneumatic pressure level that is usually in the range of $3\text{--}15$ p.s.i. (Imperial units are still commonly used in process industries, though the equivalent range in SI units is $207\text{--}1034$ mbar, which is often rounded to $200\text{--}1000$ mbar in metric systems). A few systems also use alternative ranges of $3\text{--}27$ p.s.i. or $6\text{--}48$ p.s.i. Frequently, the initial signal is in the form of a varying voltage level that is converted into a corresponding pneumatic pressure. However, in some examples of pneumatic transmission, the signal is in varying current form to start with, and a current-to-pressure converter is used to convert the $4\text{--}20$ mA current signals into pneumatic signals prior to transmission.

Pneumatic transmission has the advantage of being intrinsically safe, and it provides similar levels of noise immunity to current loop transmission. However, one disadvantage of using air as the transmission medium is that transmission speed is much slower than electrical or optical transmission. A further potential source of error would arise if there were a pressure gradient along the transmission tube. This would introduce a measurement error because air pressure changes with temperature.

Pneumatic transmission is found particularly in pneumatic control systems where sensors or actuators or both are pneumatic. Typical pneumatic sensors are the pressure thermometer (see Chapter 14) and the motion-sensing nozzle-flapper (see Chapter 19), and a typical actuator is a pneumatic cylinder that converts pressure into linear motion. A pneumatic amplifier is often used to amplify the pneumatic signal to a suitable level for transmission.

7.8 Summary

This chapter on techniques for transmitting measurement signals over what can, in some circumstances, be quite large distances, completes the discussion on the different components that exist in a measurement system. However, as for some other measurement systems components such as those used to convert sensor outputs that are not in voltage form, a mechanism for signal transmission is not needed in all measurement systems.

We started our discussion by observing that signals can be transmitted from the sensor at the point of measurement to the rest of the measurement system in a number of alternative ways. As well as the alternatives of electrical, optical, radio, and pneumatic transmission, both analog and digital forms exist as alternative transmission formats to carry the transmitted measurement signal.

We started off by noting that electrical transmission in an analog form along a copper conductor is the simplest way to convey measurement signals over some distance, but we noted that this has associated problems of attenuation of the measured signal and also a tendency for the measured signal to be corrupted by noise. We therefore went on to look at solutions to these problems. The first solution covered in the case of electrical transmission along a copper conductor was to amplify the measurement signal to compensate for attenuation during transmission and to shield the transmission wires to minimize noise corruption. We went on to look at the alternative electrical transmission method known as current loop transmission, whereby the measurement signal is transmitted as a varying current rather than a varying voltage in order to better protect the measurement signal from induced noise. Finally, we looked at a solution that involved transmitting the measurement signal on a carrier wave using either amplitude modulation or frequency modulation.

Next, we looked at electrical transmission along a copper conductor using digital protocols. Digital transmission has very significant advantages compared with analog transmission because the possibility of signal corruption during transmission is greatly reduced. Whilst many different protocols exist for digital signal transmission, we noted that the most common method for the transmission of data from a measurement sensor or circuit is asynchronous serial transmission. Having looked at how this works, we finished off by looking at the main two alternative means of transmitting the digital data, along a “twisted pair” or coaxial electrical cable, and as modulated pulses on a high-frequency carrier.

Our next subject was fiber-optic transmission, where we noted that fiber optics provided both intrinsically safe operation and gave the transmitted signal immunity to noise corruption by neighboring electromagnetic fields. Attenuation of the transmitted signal along a fiber-optic cable is also much less than for varying-voltage transmission along an equivalent length of electric cable.

As well as optical transmission of measurement data along a fiber-optic cable, we noted that it was also possible to transmit data optically across an air space rather than along a cable. We observed that this type of transmission existed in three forms, known as point-to-point telemetry, directed telemetry, and diffuse telemetry. Unfortunately, all of these alternatives suffer from the common problem of unreliability in data transmission because the transmission path is susceptible to random interruption when data is transmitted across an open, unprotected air path. In consequence, use of a fiber-optic cable is usually preferred for transmission of the data, even though this is much more expensive than transmitting the data across an open air path.

We then went on to look at radio transmission. We noted that this is normally used over transmission distances up to 400 miles, but special techniques can allow communication through space over millions of miles. In addition, radio telemetry is also commonly used over quite short distances to transmit signals where physical electrical or fiber-optic links are difficult to install or maintain, particularly when the measurement signal source signals is mobile. Although obstacles between the energy transmitter and receiver can cause some attenuation of the transmitted measurement data, this problem is far less than that which occurs when attempts are made to transmit data optically over an open air path. Whilst radio telemetry is generally reliable, two problems can occur, which are both related to the transmission frequency. Normally, licensing arrangements give each radio transmission system a unique transmission frequency within a given geographical area. Unfortunately, adverse atmospheric conditions can extend the range of transmission systems into adjoining areas and cause contamination of transmitted signals. A similar problem can occur when there are unauthorized transmissions by other parties at the wavelengths licensed to registered users.

To conclude the chapter, we finally looked at pneumatic transmission. We noted that this had the disadvantage of only transmitting measurement signals at relatively slow speeds

and, in consequence, was now much less used than it has been in the past. However, we observed that pneumatic transmission is still used in three specific circumstances. Firstly, since it is an intrinsically safe method of transmission, it is still used in some applications where intrinsic safety is required. Secondly, it provides an alternative to current loop transmission when a high level of noise immunity is required. Finally, it is convenient to use pneumatic transmission in pneumatic control systems where the sensors or actuators or both are pneumatic.

7.9 Problems

- 7.1 Discuss some reasons why it is necessary in many measurement situations to transmit signals from the point of measurement to some other point.
- 7.2 Discuss the main features of electrical and pneumatic signal transmission. Give examples of the sort of situations where you would use each of these transmission methods.
- 7.3 Discuss the main features of fiber-optic and radio signal transmission. Give examples of the sort of situations where you would use each of these transmission methods.
- 7.4 Discuss the different forms of electrical signal transmission. What are the merits and demerits of each alternative form?
- 7.5 What is a current loop interface and how does it work? Discuss some measurement situations where this would be the preferred form of signal transmission.
- 7.6 Pneumatic transmission is now rarely used in transmission systems. Why has this form of transmission fallen out of favor? What sort of conditions might cause a system designer to still specify a pneumatic system in a new measurement system.
- 7.7 Discuss the main features of fiber-optic transmission.
- 7.8 What are the principal advantages of fiber-optic transmission? Given its significant advantages, why is fiber-optic transmission not used more widely?
- 7.9 Discuss the three different ways in which fiber-optic cables are used for signal transmission.
- 7.10 A fiber-optic transmitter converts an electrical measurement signal into light and then injects this into the fiber-optic cable. Discuss the important design features that maximize the proportion of the light produced in the transmitter that passes into the cable.
- 7.11 Discuss the main features of single-mode and multimode fiber-optic transmission.
- 7.12 Discuss the main features of digital signal transmission using copper conductors and its advantages over analog transmission.

Principles of data acquisition and signal processing

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8.1 Introduction

This chapter focuses on the *fundamental principles* that underlie data acquisition and signal processing. Although it is possible to use physical instruments, namely oscilloscopes or multimeters, to monitor sensed variables, it is usually preferable to use a

computer to view and record the sensor data through the use of a data acquisition (DAQ) device (typically a specialized card within the computer or a universal serial bus–based device external to it). One particular advantage of using computers in this respect is that data can easily be stored and converted to a format that can be used by specialized software packages such as MATLAB¹ for more extensive analysis. Another advantage is that significant *digital processing* of the data can be performed in real time via the same computing platform that is used to acquire the data (for instance LabVIEW²). This can significantly improve the process of performing an experiment by making the real-time data interpretable and useful for subsequent actions (e.g., decisions that have to be made on the basis of the acquired data)

It is a prerequisite of data acquisition that the measurement system is designed to maximize the quality of sensor outputs and to minimize the occurrence of systematic errors, as discussed in earlier chapters. However, it is usually impossible to eliminate all errors, and it is often difficult to achieve sensor outputs of a suitable amplitude. Thus, in general, signals acquired via sensors must be amplified and/or conditioned to make them suitable for subsequent analysis or for integration within active control loops. Because signals acquired using common sensors (temperature, pressure, strain, etc.) are often *analog* (i.e., continuous in time and in range of values), some level of analog signal processing is generally required. This is to (1) amplify weak sensor signals, (2) filter the noise that is usually mixed in with the original signal, and (3) to eliminate high-frequency components in the signal that would cause *aliasing* (i.e., appearance of dubious counterparts of the original signal when sampled below a certain sampling rate commonly, referred to as the *Nyquist rate*). Signal processing is also usually necessary to deal with the induced noise that arises when signals are transmitted from sensors to other parts of the measurement system (as discussed in earlier chapters), even when efforts have been made to minimize the magnitude of this induced noise as much as possible.

Analog signal processing is therefore essential to the process of data acquisition even if one is largely reliant on digital computers for a significant part of the overall data acquisition process. With this in mind, in this chapter, we shall initially consider the essential aspects of *analog* signal processing with a view toward techniques that are meant to address these concerns (amplification, filtering, and antialiasing). We will subsequently consider how *digital* signal processing techniques, in their abstract form as well as via computer-based tools such as MATLAB/Simulink and Microsoft Excel can be used to implement more sophisticated filtering and data analysis techniques. Clearly, for real-time implementation of digital signal processing techniques, additional software tools are necessary, as for instance the MATLAB Real-time Workshop, in conjunction with digital signal processing hardware

¹ MATLAB is a trademark of The Mathworks, Natick, MA 01760.

² LabVIEW is a trademark of National Instruments, Austin, TX 78759.

such as those manufactured by dSpace, Inc. and National Instruments, among others. These implementation issues are not as much discussed in this chapter, however, because they require additional expertise beyond that which can be reasonably covered in an introductory course. In the next chapter, however, we will consider how real-time implementation of digital signal processing techniques can be accomplished via LabVIEW, which is among the standard tools for computer-based data acquisition systems.

With this in mind, the following basic concepts are discussed in the current chapter:

- Data acquisition process and its components
- Definition of frequency spectrum of an analog signal
- Discrete time representation of analog signals: sampling and quantization
- Aliasing phenomenon: Nyquist frequency and Nyquist rate
- Analog filters and their implementation in passive and active form
- Digital filters and their implementation in MATLAB and Excel

In completing this chapter, the reader should be able to understand the basic processes that are required to transform a sensor signal from its raw form into a form suitable for subsequent analysis in a digital computer.

8.2 Preliminary definitions

As stated earlier, common sensor (or transducer) signals are often *analog* (i.e., continuous in time and in range of values as schematically depicted in Fig. 8.1).

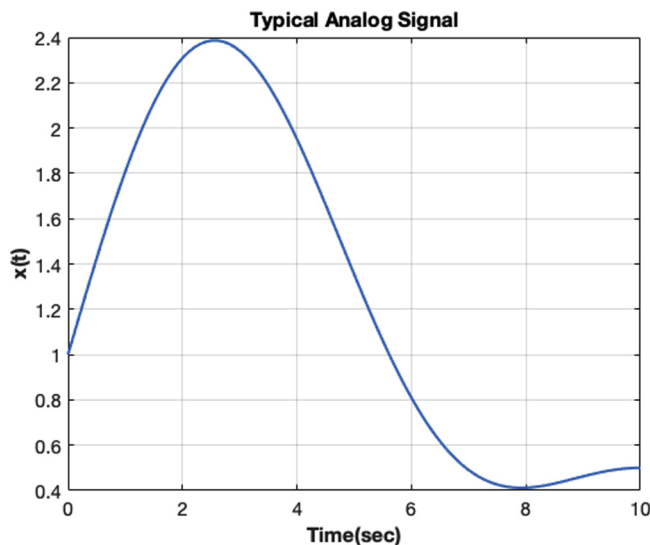


Figure 8.1
Typical analog signal.

The signal show in the figure is given by:

$$x(t) = 1 + \sin(0.15\pi t) + 0.5\sin(0.25\pi t) \quad (8.1)$$

where t represents time in seconds and x denotes the amplitude of the signal. This particular signal may be associated with a sensor reading with virtually no noise present. It has a constant (or direct current [DC]) offset of 1 unit and is composed of two sinusoidal components of frequencies $\omega_1 = 0.15\pi$ radians per second (rad/sec) and $\omega_2 = 0.25\pi$ rad/sec. These frequencies are more commonly stated as $f_1 = \frac{\omega_1}{2\pi} = 0.075\text{Hz}$ Hz (cycles/sec) and $f_2 = \frac{\omega_2}{2\pi} = 0.125\text{Hz}$. In other words, $x(t)$ is a simple two-component signal with a maximum frequency of $f_2 = 0.125\text{Hz}$. Note that ω is typically used to denote the frequency of a given signal in radians per second (rad/sec) whereas f is used to denote the frequency in cycles per second or Hertz.

We consider the signal presented earlier; i.e. $x(t)$, to be *band-limited* with a *bandwidth* of 0.125 Hz, in which the term *band* (or *bandwidth*) signifies the spread of the signal over a given frequency range. In addition, the term *spectrum*, defined shortly, refers to the frequency content of the signal over the given range. In this case, the *spectrum* of the signal is *discrete* (or made of individual components), as shown in Fig. 8.2, where it is evident that the amplitude of the signal is zero at all frequencies except at 0 Hz, where the DC offset occurs, 0.075 and 0.125 Hz.

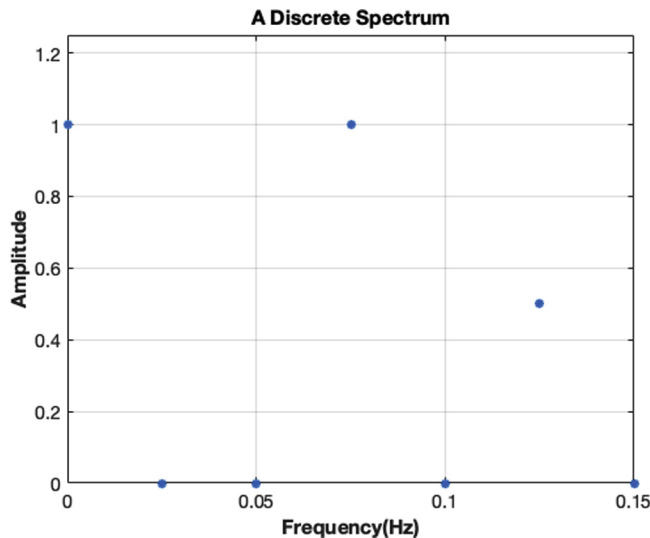


Figure 8.2

Discrete spectrum of a simple signal.

In realistic situations, sensor signals are more complex. If a signal is truly periodic, it can be represented by a (possibly infinite) sum of sinusoidal functions of different amplitudes (its *Fourier series*) as, for instance, given by:

$$x(t) = X_1 \sin(\omega t + \phi_1) + X_2 \sin(2\omega t + \phi_2) + X_3 \sin(3\omega t + \phi_3) + \dots \quad (8.2)$$

where X_i , and $i = 1, 2, \dots$ are the *amplitudes* of the components of the signal at frequencies that are multiples of the *fundamental frequency* (the lowest frequency or the overall periodic rate) of the original signal. The spectrum of the signal would be discrete as in this case, but with components at each of the so-called *harmonics* (multiples of the fundamental frequency). A typical signal is depicted in Fig. 8.3.

The signal in this case is given by:

$$x(t) = 1 + \sin(0.125\pi t) + 0.5 \sin(0.25\pi t) + 0.25 \sin(0.5\pi t) + 0.125 \sin(\pi t) \quad (8.3)$$

which is the sum of four sinusoids. The fundamental frequency of the signal is $\omega = 0.125 \pi$ rad/sec or $f = \frac{\omega}{2\pi} = \frac{1}{16}$ Hz. The harmonics at 0.25π rad/sec or $\frac{1}{8}$ Hz, 0.5π rad/sec or $\frac{1}{4}$ Hz, and finally at π rad/sec or $\frac{1}{2}$ Hz are also present in the signal.³ As complex as this signal looks, it is still periodic with a period of $T = \frac{2\pi}{\omega} = \frac{2\pi}{0.125\pi} = 16$ sec and remains

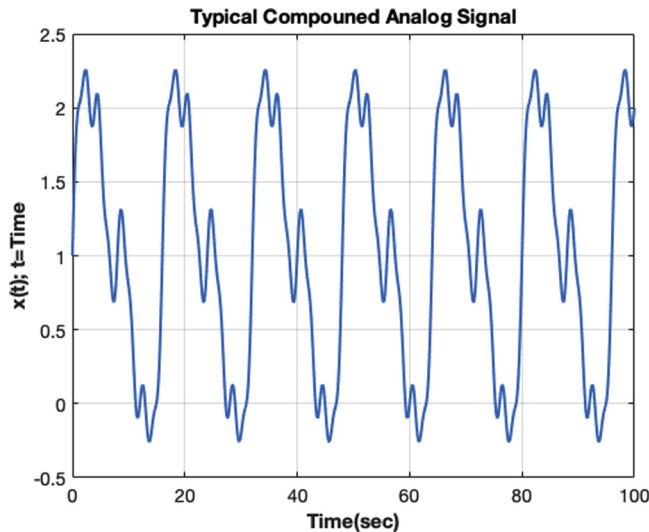


Figure 8.3

Typical compound sinusoidal signal.

³ In this case, we happen to have only the *even* harmonics but in principle, both even and odd harmonics can appear.

band-limited at $\frac{1}{2}$ Hz. In general, a given signal may not even be periodic nor band-limited, in which case it must be represented by a *Fourier transform*, which is an *integral* as opposed to a series sum, as we have it in previous example. In this case, the spectrum of the signal will be continuous, albeit with certain peaks at significant frequencies. We shall generally assume that the given signal is at least practically band-limited with a frequency spectrum that is capped at a certain maximum value, called the *bandwidth* of the signal.

8.3 Sensor signal characteristics

In general, signals acquired from sensors are either weak and small in magnitude or are contaminated with noise, or both. For instance, the original voltage reading generated by a thermocouple is in the range of microvolts, making it difficult to process via a data acquisition system without appropriate amplification and other types of preconditioning. Similarly, the signal generated by an accelerometer is generally noisy, as depicted in Fig. 8.4. In the figure, a three-axis accelerometer attached to a stationary object shows variations that are quite consistent with the nature of this type of measurement. The expected signal is constant, and all variations in this case result from sensor noise (although this is not purely due to electrical noise as in many other cases.) The so-called *spectrum* of this type of noise is often *wide*, necessitating preconditioning the signal using an analog filter before acquisition with a digital data acquisition system. This has to do with the so-called *aliasing* effect that can seriously undermine any digital signal

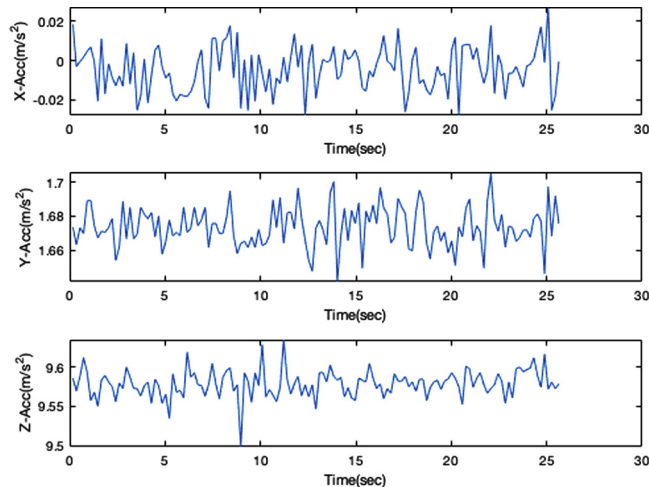


Figure 8.4
Sample accelerometer data.

processing system if the given signal is not properly prefiltered. In general, aliasing occurs if the bandwidth of the analog signal that is being sampled by the system exceeds the so-called *Nyquist frequency* (half the sampling rate) of the sampling system (or digitizer [i.e., the DAQ card used to acquire the signal]). This issue is discussed next at more length.

8.4 Aliasing

When a high-frequency signal is sampled by a computer data acquisition system, only a limited amount of information is captured (once per sample). If the sampling rate is sufficiently high, the loss of fidelity is not significant. However, as the sampling rate is reduced, the signal looks less and less like its original analog. This is schematically depicted in Fig. 8.5, in which a sinusoidal function of 1 Hz frequency is sampled initially at 50 samples/sec, later at 5 samples/sec, and finally at 2 samples/sec. Clearly, the discretized representation of the original analog signal is very realistic at high sampling rates and degrades perceptibly as the sampling rate is reduced. Below the final limit of 2 samples/sec, which in this case corresponds to the so-called *Nyquist rate* of the signal (twice its bandwidth), the sampled signal can no longer be guaranteed to represent its original analog version. Formally, the Nyquist rate of a band-limited signal is twice the maximum frequency of that signal. Alternatively, one can view the situation from the

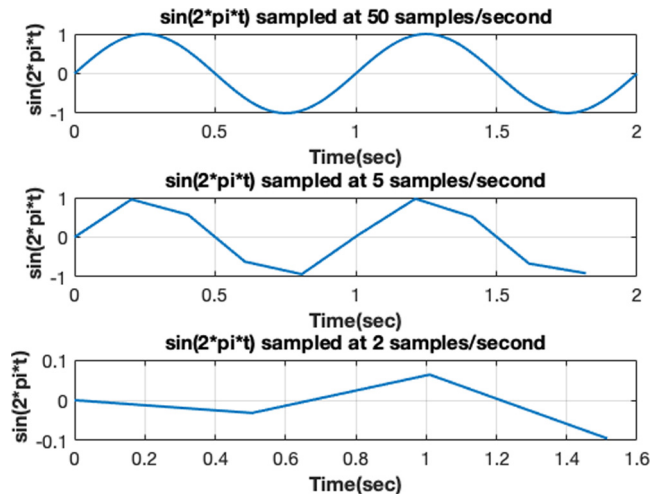


Figure 8.5

Sampling a signal at different rates.

perspective of the data acquisition system and consider the *Nyquist frequency*, which is half the sampling rate used to sample the signal and which serves as the upper limit for the maximum frequency of a band-limited analog signal that can be accurately⁴ captured by a discrete-time sampling process.

In more specific terms, a signal that is sampled at a rate below its Nyquist rate will appear to include low-frequency *aliases* of the higher-frequency components of the original signal that exceed the Nyquist frequency of the sampling system. This can be made more precise by considering the so-called *digital frequency* of a sampled signal. Let us assume that we are dealing with a continuous time domain signal of the form

$$x(t) = \sin(2\pi t + \phi). \quad (8.4)$$

In discrete time (or sampled data form) form, this signal is stated as:

$$x[k] = \sin(2\pi Fk + \phi). \quad (8.5)$$

where $k = 1, 2, \dots$ is the sample count; effectively $t = kT_s$, where T_s is the sampling interval (say, 0.001 s, 0.01 s, etc.). The relationship between the original analog frequency, f , and the digital frequency, F , is obtained by a comparison of these equations while noting that $f_s = \frac{1}{T_s}$ is the sampling frequency of the system. Starting with the fact that

$$Fk = ft \quad (\text{by definition}) \quad (8.6)$$

we have:

$$F = \frac{ft}{k} = \frac{fkT_s}{k} = fT_s = \frac{f}{f_s} \quad (8.7)$$

In other words, the digital frequency, F , is effectively the analog frequency, f , divided by (or normalized by) the sampling frequency, f_s . Now, given the property of the $\sin()$ function that adding any multiples of 2π (say, $2\pi m$, where m is any integer) to its argument does not change the value of the function, we let $m = Nk$, where N is a fixed integer and k , also an integer, varies from zero to infinity as a sample-time index. We then have the following result for the given sampled signal $x[k]$:

$$x[k] = \sin(2\pi Fk + \phi) = \sin(2\pi Fk + 2\pi Nk + \phi) = \sin(2\pi(F + N)k + \phi). \quad (8.8)$$

This implies that a signal of frequency $F + N$ is captured as if it were of (lower) frequency F . In other words, a high-frequency signal is *aliased* at a lower frequency. In general, the digital frequency, F , is less than $\frac{1}{2}$ because the analog frequency, f , must be less than $\frac{1}{2}f_s$ for the signal to be accurately represented in discrete-time form. For reasons not discussed at length here, the more complete technical assumption is that

⁴Accuracy in this context is theoretical. In practice, a sampling rate of higher than the Nyquist rate is necessary to capture the signal effectively.

$$-\frac{1}{2} \leq F \leq \frac{1}{2} \quad (8.9)$$

This means that F can be negative, although this fact has little bearing on the discussion at hand. The range between $-\frac{1}{2}$ and $\frac{1}{2}$ is called the *principal range* for F . If an analog frequency, f , is such that its digital equivalent, F , falls *outside* this principal range, it would have aliases *within* the principal range. For instance, if an 8-Hz signal ($f = 8$) is sampled at 6 Hz ($f_s = 6$) (i.e., below the required Nyquist rate of 16 Hz), we will have

$$F = \frac{f}{f_s} = \frac{8}{6} = \frac{4}{3} > \frac{1}{2}. \quad (8.10)$$

In other words, because F falls outside the principal range, it will be aliased at a (lower) frequency that would fall within the principal range. The specific nature of this alias is determined as follows. To place the alias within the principal range, we will need to subtract a whole integer,⁵ $N = 1$, from $4/3$, which produces $F' = \frac{1}{3}$ (within the principal range.) The resulting frequency is in effect a lower frequency alias of the original signal (i.e., its equivalent analog frequency is $f' = F'f_s = \frac{1}{3}6 = 2\text{Hz}$). In other words, the 8-Hz signal appears as a 2-Hz signal in its discrete-time representation and is thus considered an *alias* in the digital realm. One can illustrate this point via a MATLAB/Simulink model and its response, as depicted in Fig. 8.6.

In this case, two signals, one low-frequency (2 Hz) and the other, high-frequency (8 Hz), are discretized at the same rate (6 Hz), and once sampled at this rate, both produce the same discrete-time signal as illustrated. The original analog signals (actually discretized at high sample rates so they can be illustrated here) are shown on the left. The same signals sampled

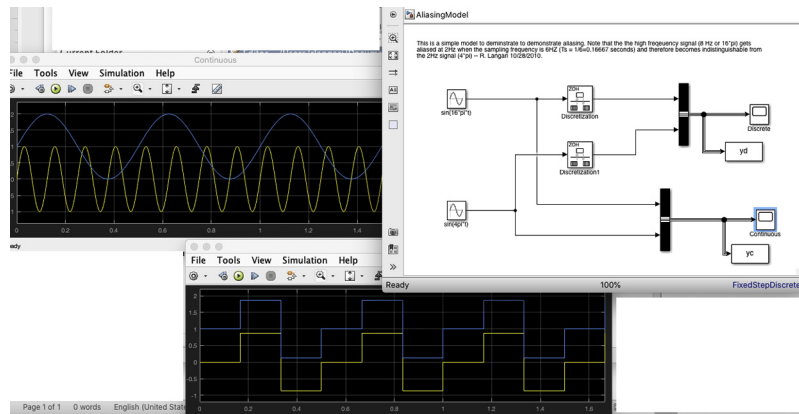


Figure 8.6
Illustration of aliasing effect via MATLAB.

⁵ N may be larger than 1, but it is always a whole number (i.e., an integer).

at 6 Hz are shown at the bottom of the figure. The relative offset is intentionally introduced to distinguish the two signals, whereas the frequencies and amplitudes are clearly the same.

8.5 Quantization

Another impact of the discretization of analog signals is *quantization*, in which a given analog signal is represented as a finite resolution digital signal. This is schematically depicted in Fig. 8.7, where an analog signal, $x(t)$, shown as a dashed line, is quantized as $x[k]$ as a solid line that changes stepwise. The resolution or size of the steps is Δ , which depends on the resolution of the A/D conversion process. In particular, if a 12-bit A/D converter has a range of ± 10 V, then its resolution or *quantization level* is given by

$$\Delta = \frac{20}{2^{12}} = 0.005V \quad (8.11)$$

or 5 mV. In other words, signal values that fall between the levels defined by this quantization are rounded up or down to the next level, which leads to the schematic graph shown in Fig. 8.7. As shown, the analog signal is rounded up or down to the nearest multiple of the resolution of the A/D converter (i.e., Δ).

8.6 Analog signal processing

Although we normally use discretized or digital signals and work with them extensively, certain tasks such as antialiasing *must* be performed in the analog domain. There are other instances in which it is simply more efficient to perform filtering on the analog signal as opposed to performing it in the digital realm. To this end, the concepts of frequency spectrum and bandwidth (which have been alluded to earlier) are essential in understanding the nature of sensor signals and devising analog appropriate filters. For instance, to eliminate noise, which has a high-frequency spectrum, a *low-pass* filter (one that passes through low-frequency components of the given signal) is used. There are also instances, such as in processing linear variable differential transformer signals, in which a *high-pass* filter must be used. Finally, there are instances in which the desired spectrum is constrained within a certain range, in which case a *band-pass* filter must be used. Finally,

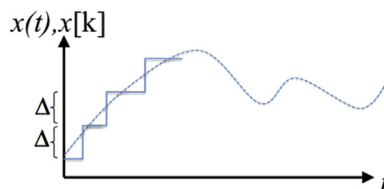


Figure 8.7
Quantization effect.

there are cases in which a specific undesirable frequency such as the 60-Hz power line frequency must be prefiltered (eliminated), in which case a *notch* filter is used.

In the sequel, we will discuss the design of both *passive* and *active* filters. Passive filters, as the name implies, are made of only passive components such as resistors and capacitors. They are simple to design and implement. However, they can excessively load the source circuit and therefore undermine the signal processing process. In addition, passive filters do not provide for a *robust* design, because their frequency response characteristics (their functional bandwidth) varies with the characteristics of the circuitry in which they are integrated.

Active circuits (filters and amplifiers), on the other hand, incorporate operational amplifiers or transistors in addition to resistors and capacitors, and are more robust and effective as signal processing devices. They require a power source, which may or may not present a problem, depending on the circumstances. Active circuits are also generally more complex to design and implement. Nevertheless, in most practical signal processing applications, active filters (used as prefilters or as amplifier/filters) are preferred to passive ones for the reasons noted earlier.

8.7 Passive filters

Passive filters can be implemented with a few simple electronic components (resistors and capacitors.) The basic *low-pass* filter, depicted in Fig. 8.8, can be used to attenuate high-frequency noise in the original signal, in this case denoted by v_i . The underlying assumption here is that any function of time can be viewed as a combination of sinusoidal functions, as alluded to earlier. More precisely any *periodic* function of time is approximated by an infinite series of sinusoids at frequencies that are multiples of the so-called fundamental frequency of the original signal. These frequencies form the spectrum of the signal as also alluded to earlier. (Nonperiodic functions can be viewed in essentially the same way, but we must allow for a continuous spectrum.) For example, the original signal, v_i , may be approximated by:

$$v_i = V_{i,1} \sin(\omega t + \theta_1) + V_{i,2} \sin(2\omega t + \theta_2) + V_{i,3} \sin(3\omega t + \theta_3) + \dots \quad (8.12)$$

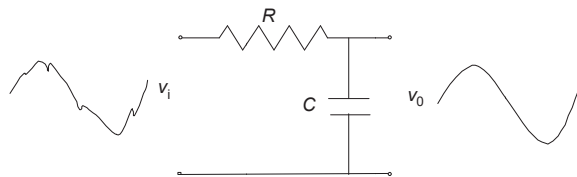


Figure 8.8
Passive low-pass filter.

where $V_{i,1}$, $V_{i,2}$, $V_{i,3}$, ... are the amplitudes of the sequentially higher-frequency components or (super) *harmonics* of the original signal.

These higher-frequency components may represent the natural fluctuations of the signal or possible electrical noise that we may wish to attenuate to prevent aliasing and generally to present a clean signal to the computer-based DAQ system. For instance, the filter in Fig. 8.8 produces an output, v_o , which has the *same set* of components (in terms of the respective frequencies) as the original signal v_i but at reduced amplitude:

$$v_o = V_{o,1} \sin(\omega t + \theta_1 + \phi_1) + V_{o,2} \sin(2\omega t + \theta_2 + \phi_2) + V_{o,3} \sin(3\omega t + \theta_3 + \phi_3) + \dots \quad (8.13)$$

where $V_{o,1}$, $V_{o,2}$, $V_{o,3}$, ... are the amplitudes of the sinusoidal components of v_o and ϕ_1 , ϕ_2 , ϕ_3 , ... are the respective phase shifts (generally negative) that reflect the temporal distortion of the signal as it passes through the filter.

The extent of attenuation of these components depends on the values of R and C in the filter shown in Fig. 8.8. For instance, $V_{o,1}$ may be close to $V_{i,1}$, say 98% of this value, but $V_{o,2}$ may be about 76% of $V_{i,2}$, and so forth. The precise amount of attenuation can be found from the so-called frequency response graph of the filter (commonly referred to as its *bode plot*) as is depicted in Fig. 8.9. The figure reflects the fact a low-pass filter would pass through frequencies of up to a certain limit (called the corner frequency) and gradually attenuates those at higher frequencies. There is a corresponding phase shift that distorts the signal that passes through the filter, as is also depicted in the figure.

8.7.1 Filter transfer function

The specific equation for the filter depicted in Fig. 8.8 from the application of Kirchhoff's current law at the output node (assuming an open circuit or no load at the output) is given by:

$$\frac{v_i - v_o}{R} = C \frac{dv_o}{dt} \quad (8.14)$$

This can be simplified into:

$$RC \frac{dv_o}{dt} + v_o = v_i \quad (8.15)$$

As a first-order differential equation, this equation can be transformed from the time domain into the frequency domain (i.e., into *transfer function* form), using the so-called Laplace transform, as:

$$\frac{\widehat{v}_o}{\widehat{v}_i} = \frac{1}{\tau s + 1} \quad (8.16)$$

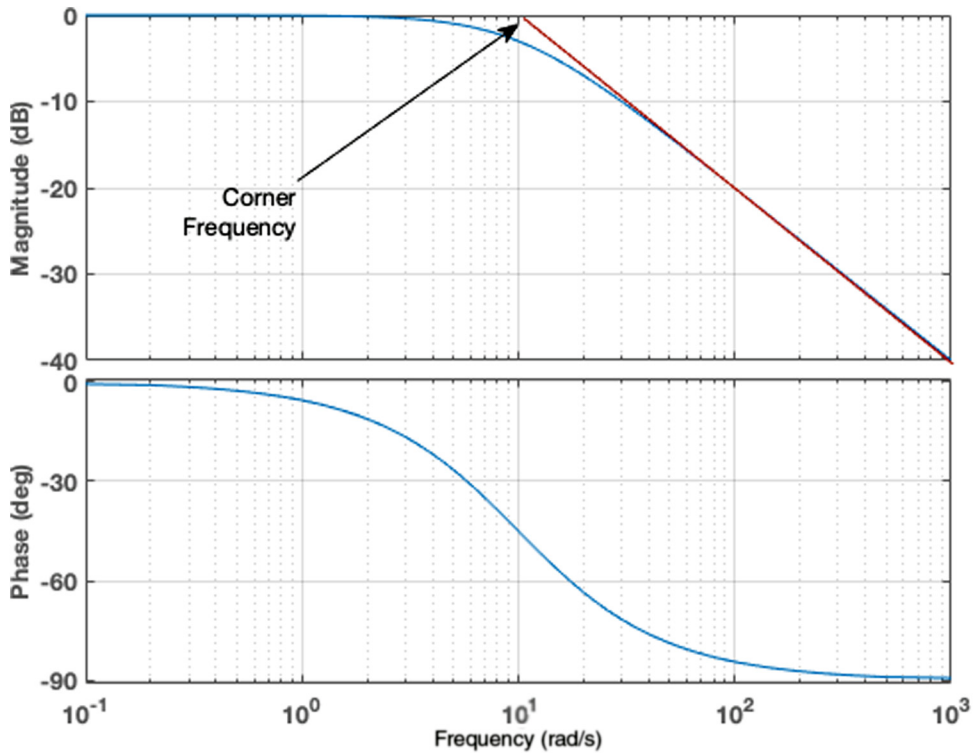


Figure 8.9

Bode plot of the passive low-pass filter.

where $\tau = RC$ is the time-constant of the filter. The inverse of τ (i.e., $\omega_c = \frac{1}{\tau}$) is known as the *corner* frequency of the filter (i.e., the frequency above which the filter starts to attenuate its input). In Fig. 8.9, which depicts the bode plot of this transfer function (and formally derived shortly next), ω_c is set to 10 rad/sec (or approximately 1.6 Hz). This example is largely for illustration purposes. In practice, this may be too low a corner frequency for practical signal processing applications, for which we may have corner frequencies of 10 Hz, 100 Hz, 1 kHz, and above, because the range of measured signals can be several Hertz, tens of Hertz, or even hundreds of Hertz and above. Accompanying the bode magnitude plot shown in the figure is the phase shift, which reflects the temporal distortion of the signal passing through the filter.

8.7.2 Low-pass filter bode plot

To develop the *bode plot* (or frequency response plot) of the filter formally, we need to consider its response to generic sinusoidal signals. Let us assume the input takes the form of sinusoidal function of the simple form of a single frequency sinusoid with zero baseline phase (i.e., $v_i = V_i \sin(\omega t)$), whose Laplace transform is given by:

$$\hat{v}_i = \frac{\omega V_i}{s^2 + \omega^2} \quad (8.17)$$

Substituting in (Eq. 8.17) and taking partial fractions and performing the necessary simplifications (including dropping the transient response term as detailed in the Appendix) the steady-state output for a given sinusoid would be $v_o = V_o \sin(\omega t + \phi)$, where

$$\frac{V_o}{V_i} = \frac{1}{\sqrt{(\tau\omega)^2 + 1}} \quad \text{and} \quad \phi = -\tan^{-1}(\tau\omega) \quad (8.18)$$

In more standard form, we have

$$\left. \frac{V_o}{V_i} \right|_{dB} = 20 \log \frac{1}{\sqrt{(\tau\omega)^2 + 1}} = 20 \log 1 - 10 \log((\tau\omega)^2 + 1) \quad (8.19)$$


$$\left. \frac{V_o}{V_i} \right|_{dB} = \begin{cases} 0 & \tau\omega \ll 1 \\ -20 \log(\tau\omega) & 1 \ll \tau\omega \end{cases} \quad (8.20)$$

The graph of the $\left. \frac{V_o}{V_i} \right|_{dB}$ is precisely what was depicted earlier in Fig. 8.9, where it is evident that the higher-frequency components, particularly those higher than the corner frequency $\frac{1}{\tau} = 10$ rad/sec, are attenuated at an increasing rate. In the figure, for instance, signals at frequencies of 100 rad/sec are attenuated by a factor of 10 (−20 dBs).

The Matlab implementation of Fig. 8.9 is given as below⁶:

```
>> s = tf('s');           % Create a basic transfer function to simplify the
                           % definition of more complex transfer functions
>> g = 1/(0.1*s+1);       % Define a first order transfer function g(s) = k/(τs+1)
                           % with k = 1 and τ = 0.1.
>> bode(g);               % Create a bode phase and magnitude plot for g.
>> grid on;               % Define the grid in the graph for clarity
```

As noted above, the corner frequency of this low-pass filter is also its bandwidth, which is 10 rad/sec (~1.6 Hz). In practice, the bandwidth of actual filters we use in data acquisition process may be in the range 10 Hz to 1 kHz, depending on the application domain. The filter structure itself may be more complex as well, but this example illustrates the very basic idea of a low-pass passive filter.

⁶You can invoke MATLAB generally by double clicking on its icon () on a computer on which it is installed. MATLAB sets up its workspace and immediately produces its command prompt (>>), at which point the commands can be entered as noted in the text.

8.7.3 Passive high-pass filter

High-pass filters can also be developed using a simple resistor–capacitor pair, as show previously in Fig. 8.8, by switching the positions of the resistor and capacitor. The resulting filter would then be given by:

$$\frac{\hat{v}_o}{\hat{v}_i} = \frac{\tau s}{\tau s + 1} \quad (8.21)$$

As is evident in the figure, the filter attenuates low-frequency signals and passes through the high-frequency components of the given signal. This filter is defined by the following Matlab script and is depicted in Fig. 8.10:

```
>> s = tf('s');           % Create a basic transfer function to allow for
                           % definition % of more complex one.
>> g = 0.1*s/(0.1*s+1);   % Define a first order transfer function  $g(s) = \tau s / (\tau s + 1)$ 
                           % with  $\tau = 0.1$ .
>> bode(g);               % Create a bode phase and magnitude plot for  $g$ .
>> grid on;               % Define the grid in the graph for clarity
```

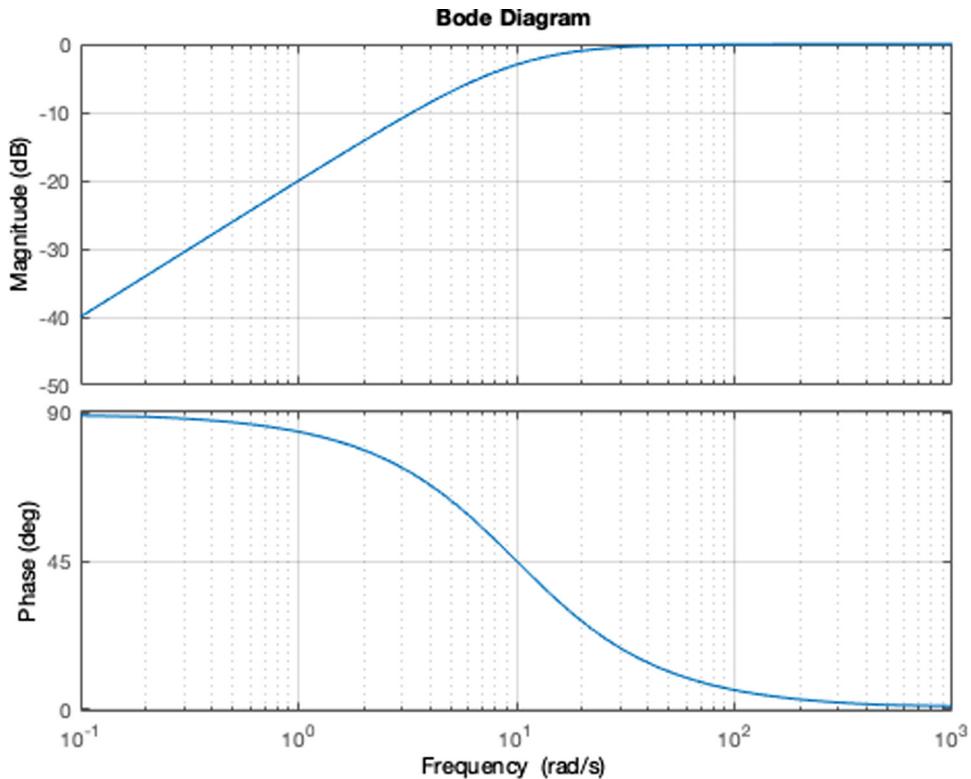


Figure 8.10

High-pass filter response.

The corner frequency of this filter is once again 10 rad/sec (~ 1.6 Hz). Signals at frequencies of 1 rad/sec (~ 0.16 Hz) are attenuated by 20 dB or a factor of 10 whereas signals of frequencies 0.1 rad/sec (0.016 Hz) are attenuated by 40 dB or a factor of 100, and so forth. Clearly, true DC signals are fully attenuated, which makes this filter useful in eliminating the DC offset of some types of signals.

8.8 Active filters

The passive filters discussed above are simple and can in principle be used to filter out undesirable components of a given signal. However, because they are made entirely of passive components (resistors and capacitors), they draw current from their input sources and will also load the circuits that are connected to their outputs. Operational amplifiers (op-amps) can eliminate this problem because the current that is drawn from the input stage is small (because op-amps have large input resistances or impedances of the order of 10 M Ω .) Likewise, as active devices, op-amps supply current to drive their outputs, and hence minimize the impact of the given filter on any output circuit such as the DAQ card,⁷ thereby less drastically affecting the reading of acquired signal. With this in mind, op-amps are often used in conjunction with resistors and capacitors to create an active filter.

8.8.1 Active low-pass filter

As a case study in active filters, we consider a low-pass filter. The configuration for an *active* low-pass filter is shown in Fig. 8.11A.

To be able to use this circuit, we must determine the relation between input v_i and output v_o . Referring to Fig. 8.11B, the summation of currents at the inverting input, v_- , is given by:

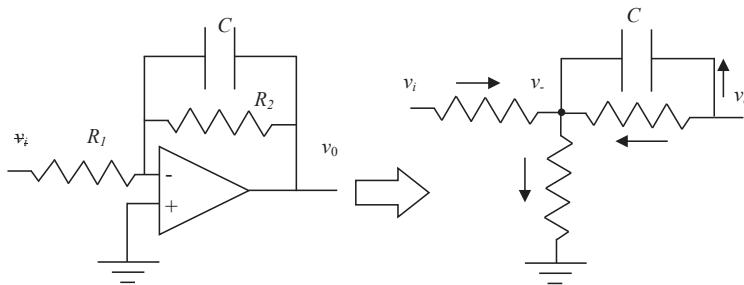


Figure 8.11
Active low-pass filter.

⁷ A typical, well-designed DAQ card has high-input impedance input ports.

$$\frac{v_i - v_-}{R_1} + \frac{v_o - v_-}{R_2} + C \frac{d(v_o - v_-)}{dt} = 0 \quad (8.22)$$

Now, in a negative feedback configuration as shown, $v_- = v_+ = 0$ (because the input impedance of the amplifier [i.e., R_i] is high), and we can simplify this to:

$$\frac{v_i}{R_1} + \frac{v_o}{R_2} + C \frac{d(v_o)}{dt} = 0 \quad (8.23)$$

We can rewrite this as:

$$R_2 C \frac{dv_o}{dt} + v_o = -\frac{R_2}{R_1} v_i \quad (8.24)$$

This is similar to a passive filter equation, with the exception of the gain on v_i . The negative sign means that the filtered signal will lag at least 180 degrees behind the unfiltered one. (This issue can be resolved via an inverting filter of gain 1.) We can call this gain $k = \frac{R_2}{R_1}$. In addition, we see that the time constant is given by $\tau = R_2 C$. So, we have two degrees of freedom in our filter configuration. We may adjust the gain k to amplify low-frequency signals. However, this will also raise the value of the crossover frequency and may allow noise to have a higher amplification. In addition, we can change the corner frequency ($\omega_c = \frac{1}{\tau}$) by adjusting the relevant parameters. This will have effects on the gain as well that must be considered. These can be better understood by examining the system in the frequency domain. In the frequency domain, the ratio of the output to the input voltage is given by:

$$\left. \frac{V_o}{V_i} \right|_{dB} = 20 \log \frac{k}{\sqrt{(\tau\omega)^2 + 1}} = 20 \log k - 10 \log \left((\tau\omega)^2 + 1 \right) \quad (8.25)$$

$$\left. \frac{V_o}{V_i} \right|_{dB} = \begin{cases} 20 \log k & \tau\omega \ll 1 \\ -20 \log(\tau\omega) & 1 \ll \tau\omega \end{cases} \quad (8.26)$$

We can understand better how the system works by examining the bode plot of the filter, which is schematically depicted in [Fig. 8.12](#). (No actual numerical values are shown for various parameters.) As discussed earlier, the graph is meant to illustrate how the filter passes through signals up to its corner frequency and gradually attenuates those beyond this frequency, as depicted in the figure. The gain k (or $20 \log k$ in decibels) appears as an offset relative to the 0-dB line (corresponding to unity gain, or $k = 1$). There is an accompanying phase shift as well that is not shown but is similar to that in [Fig. 8.9](#).

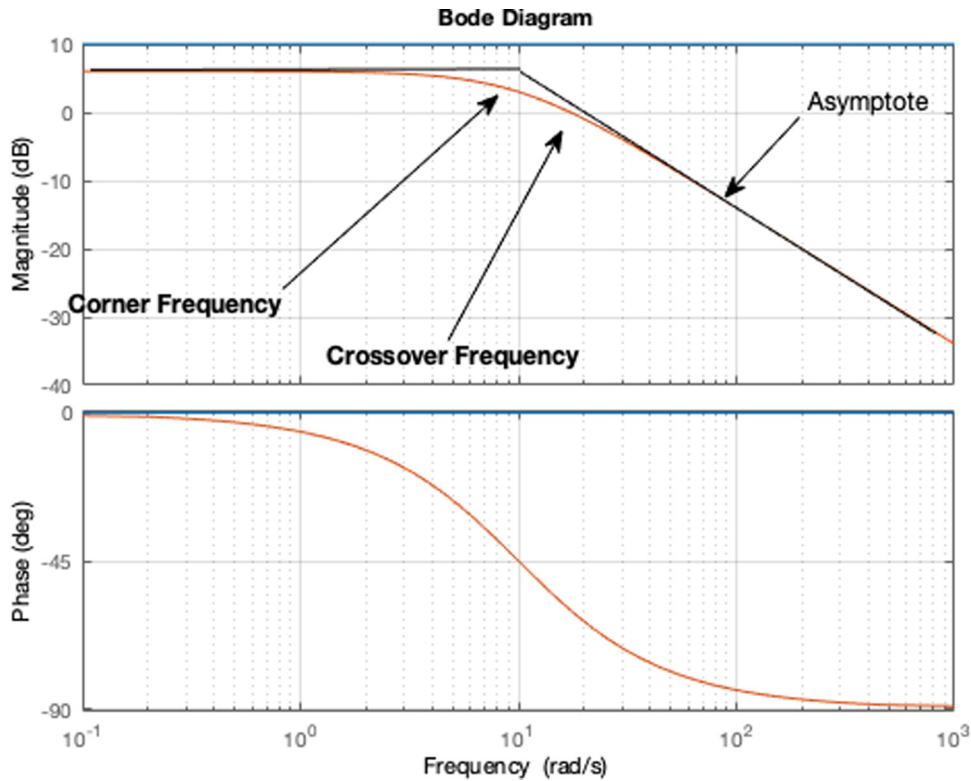


Figure 8.12

Bode diagram of low-pass filter.

8.8.2 Signal amplification

Signal amplification is carried out when the typical signal output level of a sensor is considered to be too low (as, for instance, in the case of a weak thermocouple signal, which is on the order of microvolts.) A sample configuration is given in Fig. 8.13:

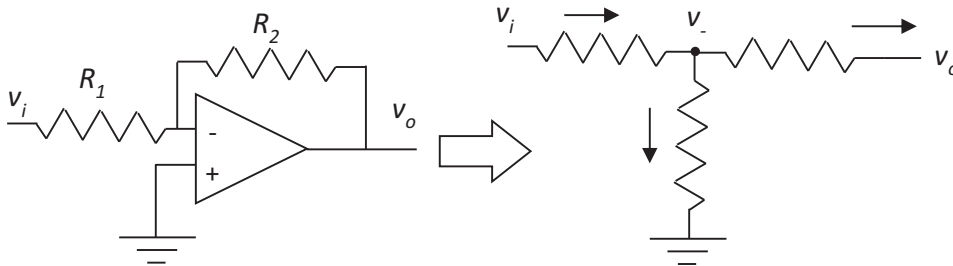


Figure 8.13

Active inverting amplifier.

To be able to use this circuit, we must determine the relation between the input, v_i , and the output, v_o . The summation of currents at the inverting input (v_-) is given by:

$$\frac{v_i - v_-}{R_1} = \frac{v_- - v_o}{R_2} \quad (8.27)$$

Once again, in a negative feedback configuration as shown earlier, $v_- \approx v_+ = 0$, we can simplify this to:

$$v_o = -\frac{R_2}{R_1}v_i \quad (8.28)$$

This implies that the above circuit acts as an inverting amplifier with a gain of $\frac{R_2}{R_1}$. In other words, the circuit can amplify (and simultaneously invert) its input signal. The amplification ratio is determined by the designer through the selection of the respective resistance values, R_1 and R_2 . Typically, these values are chosen with reference to the internal resistance of the amplifier. If the op-amp input impedance is on the order of $10^8 \Omega$, then R_1 and R_2 will need to be on the order of 10^4 – $10^6 \Omega$, offering a factor of up to 100 amplification. It is typically not sensible to use these simple op-amp circuits for amplifications beyond this range, particularly because noise can be magnified, but also because the internal circuitry of the op-amp is not set up for high amplification. In addition, the op-amp saturates quickly at high amplification ratios, rendering it useless beyond a small range.

As is evident in this circuit, the simple inverting amplifier is a subset of the low-pass filter in Fig. 8.11. This itself suggests that the active low-pass filter can be tailored to act as a combined filter/amplifier (albeit with an appropriate choice of resistance and capacitance values). Moreover, the inverting feature of both circuits presents a challenge in certain situations in which the sign of the original signal must be preserved (although if the signal is digitally acquired, this may not be an issue because it is always possible to perform the sign inversion inside the PC). In such cases, one must either tag a second inverting amplifier (with a gain of 1) onto the first stage filter/amplifier or devise a single noninverting amplifier (or low-pass filter) as necessary.

8.8.3 Noninverting amplifier

A simple noninverting amplifier is shown in Fig. 8.14.

To be able to use this circuit, we must determine the relation between input v_i and output v_o . Because of the high input impedance of the op-amp, $v_- \approx v_+ = 0$. Subsequently, we have

$$\frac{v_i}{R_1} + \frac{v_i - v_o}{R_2} = 0 \quad (8.29)$$

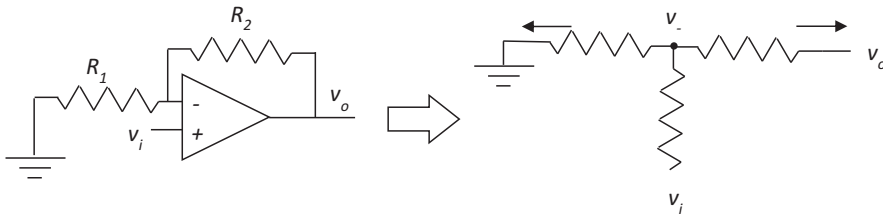


Figure 8.14
Active noninverting amplifier.

We can simplify this to:

$$\frac{v_o}{R_2} = v_i \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (8.30)$$

$$\frac{v_o}{v_i} = \left(1 + \frac{R_2}{R_1} \right) \quad (8.31)$$

This implies that the circuit acts as a noninverting amplifier with a gain of $\frac{v_o}{v_i} = 1 + \frac{R_2}{R_1}$, and so the (nonunique) choice of the resistance values, R_1 and R_2 , determines the amplification factor.

8.8.4 Differential amplification

Fig. 8.15 shows a common amplifier configuration used to amplify the small difference that may exist between two voltage signals, V_A and V_B .

These may represent the pressures on either side of an obstruction device placed in a pipe to measure the flow rate of fluid flowing. The output voltage V_o is determined as follows:

$$\frac{V_A - V_1}{R_1} = \frac{V_1 - V_o}{R_3} \quad (8.32)$$

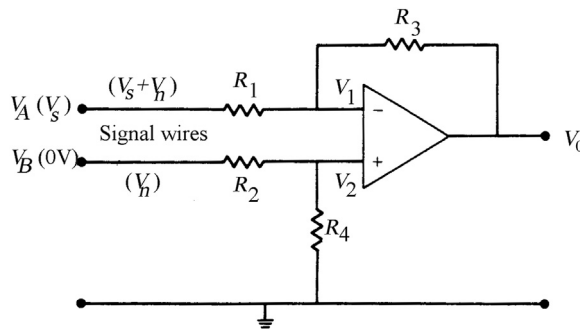


Figure 8.15
Active differential amplifier.

$$V_2 = \frac{R_4}{R_2 + R_4} V_B \quad (8.33)$$

$$V_2 \approx V_1 \quad (8.34)$$

$$\frac{V_0}{R_3} = V_1 \left(\frac{1}{R_1} + \frac{1}{R_3} \right) - \frac{V_A}{R_1} \quad (8.35)$$

$$V_0 = V_1 \left(\frac{R_1 + R_3}{R_1} \right) - V_A \frac{R_3}{R_1} \quad (8.36)$$

$$V_0 = \frac{R_4}{R_2 + R_4} \left(\frac{R_1 + R_3}{R_1} \right) V_B - \frac{R_3}{R_1} V_A \quad (8.37)$$

Now, for $R_3 = R_1$ and $R_4 = R_2$, these equations resolve into:

$$V_0 = V_B - V_A \quad (8.38)$$

This is effectively a differential amplifier, which is also useful for removing *common mode noise* voltages. Suppose V_A and V_B in Fig. 8.15 are set up such that $V_B = V_s + V_n$ and $V_A = V_n$, where V_s is the sensor signal and V_n is noise. In that case, the differential amplifier produces $V_0 = V_s + V_n - V_n = V_s$, effectively eliminating the common mode noise.

8.8.5 Instrumentation amplifier

For applications requiring the amplification of very low-level signals, a special type of amplifier known as an *instrumentation amplifier* is used. This consists of a circuit containing three standard operational amplifiers, as shown in Fig. 8.16. The advantage of the instrumentation amplifier compared with a standard operational amplifier is that its differential input impedance is much higher. In consequence, its common mode rejection capability is much better. This means that if a twisted wire pair is used to connect a transducer to the differential inputs of the amplifier, any induced noise will contaminate each wire equally and will be rejected by the common mode rejection capacity of the amplifier.

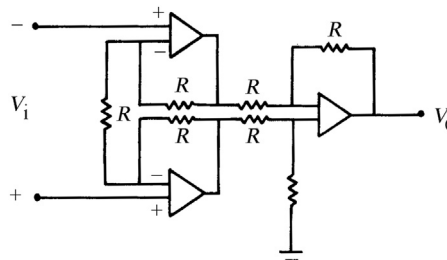


Figure 8.16
Instrumentation amplifier.

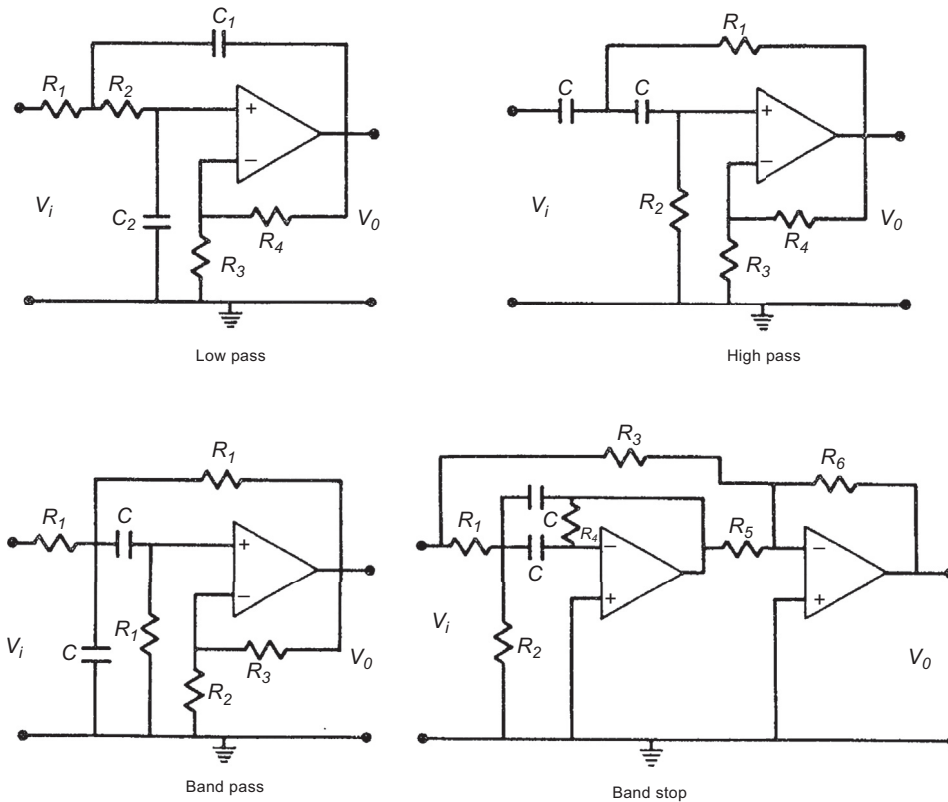


Figure 8.17
Other amplifiers and filters.

8.8.6 Other op-amp based filters and amplifiers

Numerous other variants of low-pass filters and other amplifiers can be implemented using op-amps and simple resistance and capacitor elements. Several examples are depicted in (Fig. 8.17).

8.9 Digital filters

Digital filters are generally implemented via a microprocessor, a digital signal processor, or microcontroller. The simplest digital filter takes the form of

$$y_k = \alpha u_k + (1 - \alpha)u_{k-1} \quad (8.39)$$

We can write $\alpha_0 \triangleq \alpha$ and $\alpha_1 \triangleq 1 - \alpha$ and hence write this more generally as

$$y_k = \alpha_0 u_k + \alpha_1 u_{k-1} \quad (8.40)$$

and thus further extend this formulation to more complex filters such as

$$y_k = \alpha_0 u_k + \alpha_1 u_{k-1} + \alpha_2 u_{k-2} + \dots + \alpha_n u_{k-n} \quad (8.41)$$

This general form is called a *moving average* or finite impulse response filter, because in effect, it averages current and past values of the input signal, each with its respective weight. Selection of these weights is often an issue and can be formalized, although in the current context we will deal with simpler cases in which an intuitive approach to selecting these values can be used.

8.9.1 Filter with memory

In the filter with memory, previously filtered values will be used to adjust the new output. This filter will take the form

$$y_k = \alpha u_k + \alpha_1 y_{k-1} \quad (8.42)$$

in which α is the weight on the current value of the unfiltered signal, u_k . The remainder is from the previous value of the filtered signal, y_{k-1} . Varying α will change the extent to which the input signal is filtered. In particular, a relatively large α weighs in the current value of the input signal whereas a small α weighs in the past (filtered) signal, y_k . Normally $\alpha \leq 1$. This is evident in the next example.

8.9.2 Example

A set of data points is measured from a continuous signal, as given in [Table 8.1](#).

A simple input averaging filter with α values of 0.25, 0.5, 0.75, and 1.0 is used to filter these values, as depicted in [Fig. 8.18](#). The filters produce a range of response patterns indicating that no single value of α is necessarily the best. However, one can argue, based on proximity to the general pattern of the input signal, that $\alpha = 0.5$ may be a reasonable

Table 8.1: Data for digital filtering example.

k	u_k
0	0.10
1	1.05
2	1.92
3	3.90
4	4.02
5	4.94

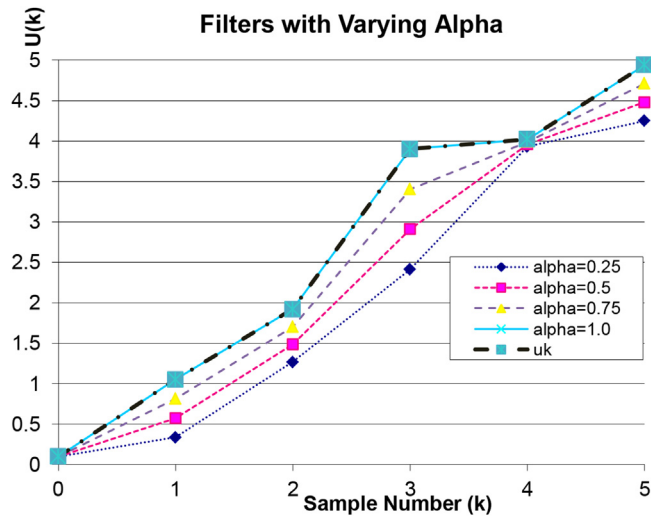


Figure 8.18

A simple digital filter response.

choice in preserving the general trend of the data, although some fidelity is lost relative to $\alpha = 0.75$, for instance. In practice, one has to fine-tune α (or similar parameters of a given filter) to fit the application in mind. As we shall see, there are formal methods (based on digital signal processing) that allow the user to select the filter parameter to affect the frequency response of the filter (similar to the analog case). The mathematical technique underlying these tools are beyond the scope of this book, although their application will be discussed subsequently.

8.9.3 ARMA and IIR filters

Although the simple filter in the previous section works reasonably well, one can build more effective filters using a combination of autoregressive terms and moving average terms, via an ARMA (AutoRegressive Moving Average) model, also referred to as an infinite impulse response (IIR) filter. The general equation for such a filter is given as:

$$y_k = -a_1 y_{k-1} - a_2 y_{k-2} - \cdots - a_n y_{k-n} + b_0 u_k + b_1 u_{k-1} + b_2 u_{k-2} + \cdots + b_n u_{k-n} \quad (8.43)$$

a_i and b_i must be properly chosen for stable and effective performance. The essential idea is to place the so-called poles and zeros (the roots of the denominator and numerator of the corresponding discrete time transfer functions) in reasonable locations in the complex plane. These are not readily clear from the brief discussion in this chapter, and we do not plan to delve into the underlying concepts in these few short pages. It is possible, however, to produce the filter coefficients in MATLAB or a similar tool and use a similar technique as in the previous section to implement the given filter.

For this reason, there are many other digital filtering approaches. Many revolve around designing an analog filter and approximating it as a digital filter. There are also well-known design patterns for digital filters. This is the case for the so-called *Butterworth* filter. Luckily, this design process can be automated using MATLAB. MATLAB uses a command called `butter()` to generate the coefficients for a filter with a certain order and cutoff frequency. A sample command is given below.

```

              order      2fc/fs
              ↙         ↘
>> [b,a]=butter(3,0.1)

b = 0.0029    0.0087    0.0087    0.0029
   ↙         ↙         ↙         ↙
  b0      b1      b2      b3

a = 1.0000   -2.3741    1.9294   -0.5321
   ↙         ↙         ↙         ↙
  a0      a1      a2      a3

```

The command given above generates a third-order Butterworth filter. The first term in the command (3 in this example) is the order of the filter. By setting this term, you can control how many past data points the filter uses. The second term in the argument is the ratio of the cutoff (or corner) frequency to the Nyquist frequency. The Nyquist frequency, as noted earlier, is one-half the sampling rate, so this ratio, r , is given by:

$$r = \frac{f_c}{f_s/2} \quad (8.44)$$

These frequencies are both given in Hertz, which makes r a unitless quantity. Digital filters only filter frequencies relative to the sampling frequency. If we increase the cutoff frequency and increase the sampling frequency by the same amount, we will achieve the same result. In addition, the maximum value of r is 1.0. However, this would correspond to an unfiltered response.

Finally, from an implementation standpoint, the equation for the filter is given by:

$$y_k = \frac{1}{a_0} (-a_1 y_{k-1} - a_2 y_{k-2} - a_3 y_{k-3} + b_0 u_k + b_1 u_{k-1} + b_2 u_{k-2} + b_3 u_{k-3}) \quad (8.45)$$

The response of this filter to the compound analog signal we discussed earlier is shown next. The solid line is the original signal and the dotted line represents the filtered signal, but they overlap. The sampling frequency of the system is set at 100 Hz, so the Nyquist frequency is 50 Hz. The filter's corner frequency is 0.1, which means the corner frequency of the filter is 5 Hz, well beyond the bandwidth of the signal (0.5 Hz). This means the signal is not noticeably attenuated or otherwise affected by the filter, which is why the solid and dashed lines are not distinguishable in [Fig. 8.19](#).

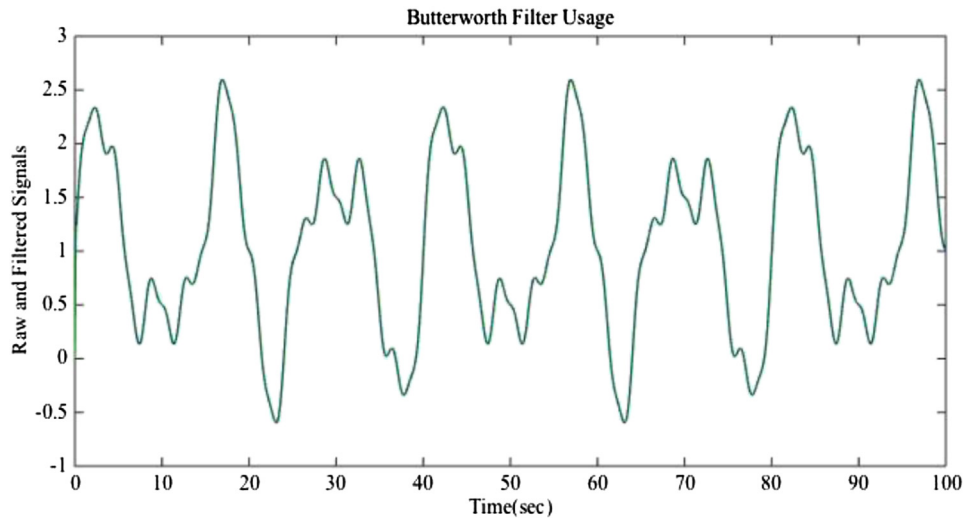


Figure 8.19
Butterworth filter response.

However, we can redesign the filter as:

```
>> [b,a] = butter(3,0.005)
b = 1.0e-05 * [ 0.0477  0.1431  0.1431  0.0477]
a = 1.0000   -2.9686  2.9377  -0.9691
```

The filter's cutoff frequency is now 0.25 Hz, and the results are shown below where the dotted line shows that the filtered signal is smoother (some higher frequency components are attenuated). In addition, the filtered signal is not drastically different from the original signal and does not lag behind significantly, either, which is a positive aspect of this filter design (Fig. 8.20).

A further redesign of the filter is:

```
>> [b,a] = butter(3,0.001)
b = 1.0e-07 * [0.0386  0.1159  0.1159  0.0386]
a = 1.0000   -2.9937  2.9875  -0.9937
```

The filter's cutoff frequency is now 0.05 Hz. The dotted line shows that the filtered signal is missing the bulk of information in the original signal. This means that filtering must be done judiciously (as, for instance, in the previous case); otherwise, meaningful information may be missing from the filtered signal (Fig. 8.21).

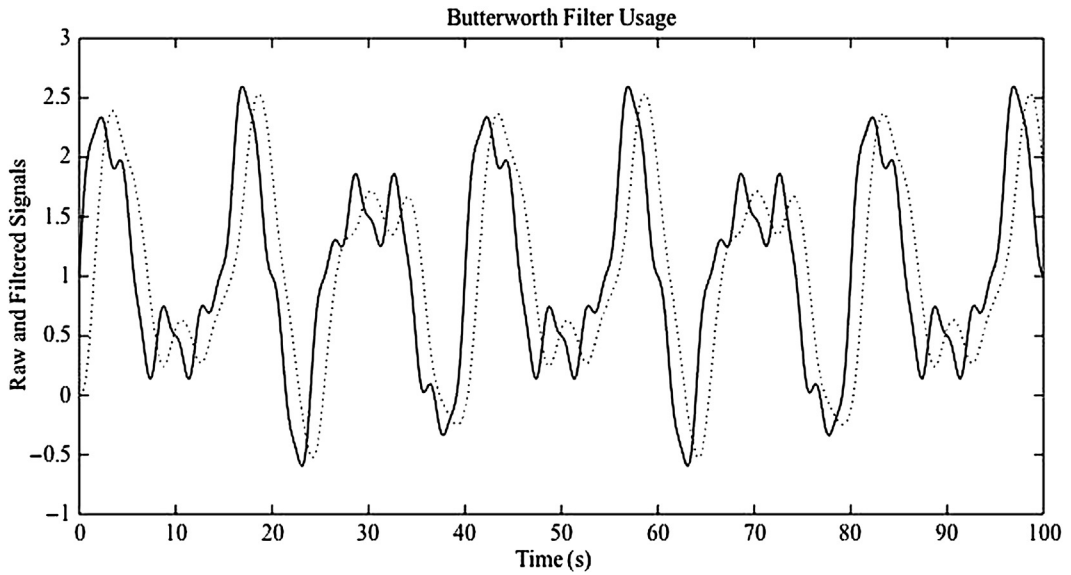


Figure 8.20
Butterworth filter with low-pass effect.

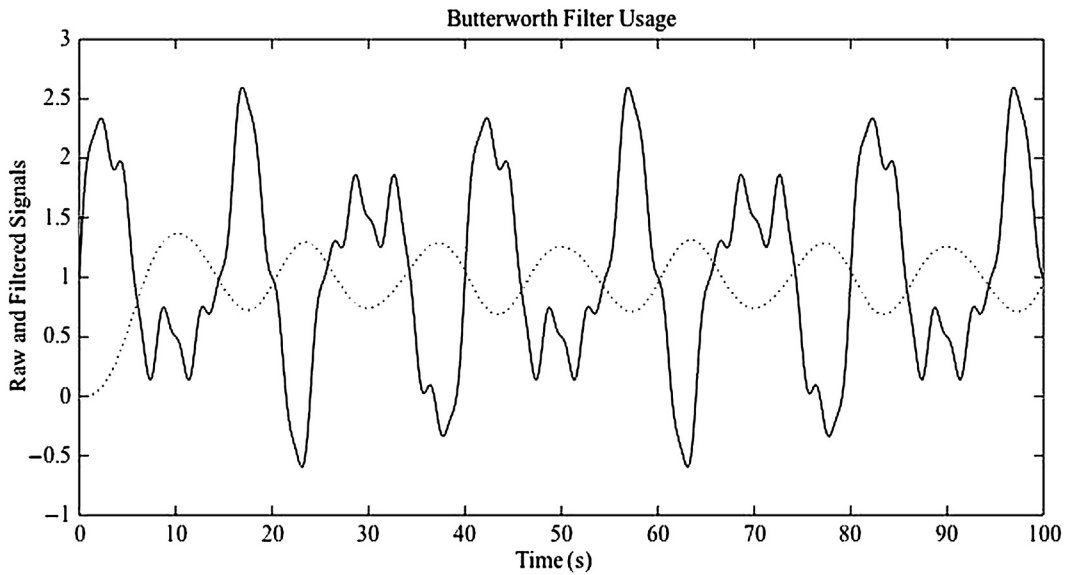


Figure 8.21
Butterworth filter with increased low-pass effect.

8.10 Summary

This chapter discussed signal processing techniques commonly needed to process sensor data acquired from mechanical experiments. These basic signal processing techniques include analog filtering using passive and active components as well as digital filtering. We also pointed out the need to be mindful of aliasing (i.e., the presence of spurious signals if the original signal is not properly sampled). The techniques discussed in this chapter are the basic tools needed in processing sensor signals. Clearly, both analog and digital signal processing are rich topics from which the reader can benefit by consulting more exhaustive references.⁸

8.11 Exercises

- 8.1 A signal with a bandwidth of 750 Hz must be sampled.
 - a. What is the Nyquist rate of the signal?
 - b. What is the minimum sampling frequency needed to avoid aliasing?
 - c. Unbeknownst to us, the signal is contaminated with noise at a 15-kHz rate. What happens to this noise when sampled at the frequency specified in part b?
- 8.2 A 1-kHz sine wave is sampled at 5 kHz.
 - a. What is the Nyquist frequency of the sampling system in this case?
 - b. What is the Nyquist rate of the signal?
 - c. Is the signal properly sampled (i.e., will there be aliases in the digitally recorded signal)?
 - d. What if the original analog signal included both even and odd harmonics of its primary frequency (i.e., 2, 3, 5 kHz, etc.)? What would be the highest harmonic that will be faithfully represented in the discrete-time version of the analog signal?
 - e. Following on with the previous step, what is the lowest harmonic that will be aliased and what would be resulting alias frequency?
- 8.3 In this chapter, we proved that aliasing occurs when an analog signal is discretized. Can aliasing occur with purely analog signals? In other words, suppose we have a purely analog circuit (as opposed to a digital computer; you could call such a circuit an analog computer). Is it possible for aliasing to occur in this case?
- 8.4 What is the maximum quantization error for an A/D (Analog to Digital) converter with 14 bits of resolution assuming a voltage range of ± 10 V?

⁸ Proakis, John G., and Dimitris G. Manolakis. *Introduction to digital signal processing*. Prentice Hall Professional Technical Reference, 1988.

- 8.5 Suppose we have a signal in the range of ± 10 V. What is the minimum number of bits needed in an A/D converter such that the signal is sampled with at least 0.005 V resolution?
- 8.6 Design a simple passive low-pass filter with a corner frequency of 50 Hz. You may assume that a 0.01 μF capacitor is available. Assume standard resistor sizes are available.⁹ What would be the resistor value that best approximates this filter? With your selection of this resistor value, recalculate the corner frequency and determine the percent difference from the original specification. In addition, draw the bode plot of the filter using MATLAB or a similar tool.
- 8.7 Design a simple passive high-pass filter with a corner frequency of 100 Hz. You may assume that a 0.01- μF capacitor is available. What would be the resistor value that best approximates this filter? With your selection of the resistor value, recalculate the corner frequency and determine the percent difference from the original specification. In addition, draw the bode plot of the filter using MATLAB.
- 8.8 Explain why the maximum value of r in the MATLAB command `butter()` is 1.
- 8.9 Extend the data in Table 8.1 by replicating the data in an up-and-down manner as:
0.1, 1.05, 1.92, 3.9, 4.02, 4.94, 4.02, 3.9, 1.92, 1.05, 0.1, 1.05, 1.92, 3.9, 4.02, 4.94, 4.02, 3.9, 1.92, 1.05, 0.1.
- Apply the simple moving average filter discussed in the chapter with alpha values, 0.25, 0.5, 0.75, and 1. In your view, what is the most optimal choice?
- 8.10 Assume the sampling rate is 1000 Hz. Design a digital low-pass Butterworth filter with MATLAB with a cutoff frequency of 50 Hz.
- 8.11 Assume a sampling rate of 100 Hz. Filter the signal given by
- $$x(t) = 1 + \sin(0.125\pi t) + 0.5 \sin(0.25\pi t) + 0.25 \sin(0.5\pi t) + .125 \sin(\pi t)$$
- by filters of progressively lower cutoff frequencies of 50, 5, 0.5, and 0.05 Hz.
- 8.12 Assume a sampling rate of 1000 Hz. Filter the signal given by
- $$x(t) = 1 + \sin(100t) + \sin(400t) + \sin(500t)$$
- by filters of progressively lower cutoff frequencies of 300, 100, 30, and 10 Hz, respectively.

⁹ A variety of sources on the Internet provide the list of standard resistor values (see, for instance, <http://www.resistorguide.com/resistor-values/>).

Appendix

Simple filter solution

We start with

$$\frac{\widehat{v}_o}{\widehat{v}_i} = \frac{1}{\tau s + 1}. \quad (8.46)$$

Assume sinusoidal functions of the form $v_i = V_i(\omega t)$, or $\widehat{v}_i = \frac{\omega V_i}{s^2 + \omega^2}$. We have the partial fraction expansion:

$$\widehat{v}_o = \frac{1}{\tau s + 1} \frac{\omega V_i}{s^2 + \omega^2} = \frac{A}{s + 1/\tau} + \frac{B}{s + j\omega} + \frac{C}{s - j\omega}. \quad (8.47)$$

where

$$A = \lim_{s \rightarrow -1/\tau} (s + 1/\tau) \widehat{v}_o(s) = \frac{\omega V_i}{(-1/\tau)^2 + \omega^2} = \frac{\omega \tau^2 V_i}{1 + \tau^2 \omega^2} \quad (8.48)$$

$$B = \lim_{s \rightarrow -j\omega} (s + j\omega) \widehat{v}_o(s) = \frac{\omega V_i}{(-j\tau\omega + 1)(-2j\omega)} \quad (8.49)$$

$$C = \lim_{s \rightarrow j\omega} (s - j\omega) \widehat{v}_o(s) = \frac{\omega V_i}{(j\tau\omega + 1)(2j\omega)} \quad (8.50)$$

Now, noting that in time domain, Eq. (8.47) resolves into

$$v_o(t) = Ae^{-t/\tau} + Be^{-j\omega t} + Ce^{j\omega t}. \quad (8.51)$$

and that the first term dies out with time, we look at the steady-state value of $v_o(t)$ as:

$$v_o(t) = \frac{\omega V_i}{(-j\tau\omega + 1)(-2j\omega)} e^{-j\omega t} + \frac{\omega V_i}{(j\tau\omega + 1)(2j\omega)} e^{j\omega t} \quad (8.52)$$

and further into:

$$v_o(t) = \frac{(j\tau\omega + 1)V_i}{(\tau^2\omega^2 + 1)(-2j)} e^{-j\omega t} + \frac{(-j\tau\omega + 1)}{(\tau^2\omega^2 + 1)(-2j)} e^{j\omega t} \quad (8.53)$$

and yet further into:

$$v_o(t) = \frac{V_i}{(\tau^2\omega^2 + 1)(-2j)} ((j\tau\omega + 1)e^{-j\omega t} + (-j\tau\omega + 1)e^{j\omega t}) \quad (8.54)$$

and:

$$v_o(t) = \frac{V_i}{(\tau^2\omega^2 + 1)(-2j)} (-j\tau\omega(e^{j\omega t} - e^{-j\omega t}) + (e^{j\omega t} + e^{-j\omega t})) \quad (8.55)$$

and:

$$v_0 = \frac{V_i}{(\tau^2\omega^2 + 1)(-2j)} (2\tau\omega \sin(\omega t) + 2 \cos(\omega t)) \quad (8.56)$$

and:

$$v_0(t) = \frac{V_i}{\tau^2\omega^2 + 1} (\tau\omega \sin(\omega t) + \cos(\omega t)) \quad (8.57)$$

and:

$$v_0(t) = V_i(\cos(\phi)\sin(\omega t) + \sin(\phi)\cos(\omega t)) \quad (8.58)$$

We thus have, using a simple trigonometric identity:

$$v_0(t) = V_i \sin(\omega t + \varphi) \quad (8.59)$$

Use of LabVIEW in data acquisition and postprocessing of signals

Chapter Outline

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9.1 Introduction

This chapter is designed to introduce the reader to the concept of computer-based data acquisition and to LabVIEW, a software package developed by National Instruments that is used extensively in laboratory environments. The main reason for focusing on LabVIEW is its prevalence in both academic and industry settings. To be sure there are other software tools that support laboratory data acquisition made by a range of vendors. These are not discussed due to limitations of space. We should also point out that MATLAB and other software tools used to model and simulate dynamic systems and to perform extensive numerical computations are at times used in laboratory environments although their use is often limited to specialized applications such as real-time control as opposed to data acquisition. For this reason, we do not discuss these tools in the present chapter.

LabVIEW itself is as an extensive programming platform. It includes a multitude of functionalities ranging from basic algebraic operators to advanced signal processing components that can be integrated into sophisticated and complex programs for use in laboratory, and even industrial, settings. For pedagogical reasons and given the context of this presentation, we will only introduce the main ideas from LabVIEW that are necessary for functioning in a typical undergraduate engineering laboratory environment. Advanced programming skills can be developed over time as the reader gains comfort with the basic functioning of LabVIEW and its external interfaces.

Specific topics discussed in this chapter and the associated learning objectives are as follows:

- Structure of computer-based data acquisition (DAQ) systems, the purpose of DAQ cards, and the role of LabVIEW in this context
- Development of simple virtual instruments (VIs) using basic functionalities of LabVIEW, namely arithmetic and logic operations
- Construction of functionally enhanced VIs using LabVIEW program flow control operations such as the *while* loop and the *case* structure
- Development of VIs that allow for interaction with external hardware, for instance, acquisition of external signals via DAQ input channels and generation of functions using DAQ card output channels

9.2 Computer-based data acquisition

In studying mechanical systems, it is often necessary to use sensors to measure certain variables such as temperature (using thermocouples or resistance temperature devices [RTDs]), pressure (using piezoelectric transducers), strain (using strain gauges), and so forth. Although it is possible to use oscilloscopes (as for instance depicted in [Fig. 9.1](#)) or voltmeters to monitor these variables, it is often preferable to use a computer to view and record the data through the use of a data acquisition (DAQ) card. One particular advantage of using computers in this respect is that data can be easily stored and converted to a format that can be used by software packages such as LabVIEW itself or Excel or MATLAB for more extensive analysis. Another advantage is that significant digital processing of data can be performed in *real time* via the same platform that is used to acquire the data. This can significantly improve the process of performing an experiment by making the real-time data more useful for further processing.



Figure 9.1
A typical laboratory oscilloscope.

9.3 Acquisition of data

One important step in the data acquisition process is conversion of analog signals from typically analog sensors to digital representations that can be processed by a computer.¹ Since data must be stored in the computer's memory in the form of individual data points represented by binary numbers, incoming analog data must be *sampled* at discrete time intervals and *quantized* (rounded off) to one of a set of predefined values. In most cases, this is accomplished using an analog to digital (A/D) conversion component on the DAQ card inside the computer or interconnected to it via a universal serial bus (USB) port. Note that both options are used commonly. However, laptop computers and/or low-profile personal computers (PCs) generally require the use of USB-based DAQ devices (such as MyDAQ produced by National Instruments).

9.4 National instruments LabVIEW

LabVIEW is a software package that provides control and a user interface for the data acquisition process. Fig. 9.2 depicts a schematic of data flow in the data acquisition process. Note that the physical system maybe a mechanical system such as a beam subjected to stress, a chemical process such as a distillation column, a DC motor with both

¹ Here we make note of the fact that certain sensors such as micro-electro-mechanical-systems (MEMS) accelerometers are equipped with basic signal conversion from analog into digital. Many common sensors such as strain gauges produce an analog signal, however, that must be preprocessed before digitization.

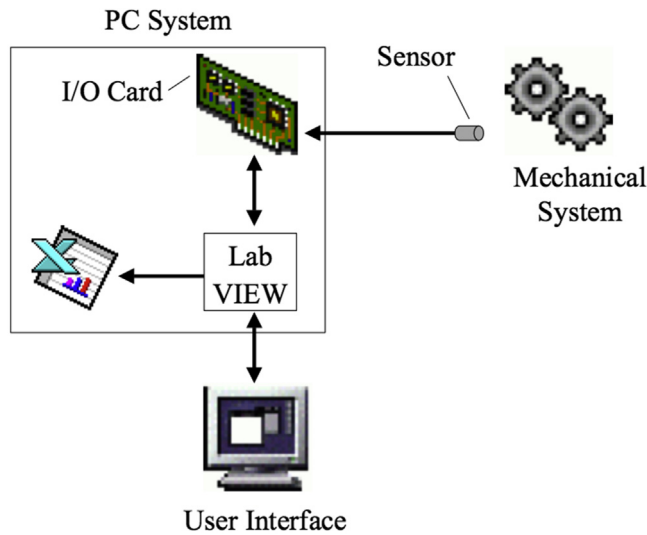


Figure 9.2
Schematic of the data acquisition process.

mechanical and electrical components, and so forth. The key issue here is that certain measurements are taken from the given physical system and are acquired and processed by the PC-based data acquisition system.

LabVIEW plays a pivotal role in the data acquisition process. Through the use of VIs, LabVIEW directs the real-time sampling of sensor data through the DAQ card (also known as the I/O card) and is capable of storing, processing, and displaying the collected data. In most cases, one or more sensors transmit analog readings to the DAQ card in the computer. This analog data is then converted to individual digital values by the DAQ card and is made available to LabVIEW, at which point it can be displayed to the user. Although LabVIEW is capable of some data analysis functions, it is often preferable to export the data to a format that is readable by MATLAB or spreadsheet programs.

Virtual instruments

A VI is a program created in the LabVIEW programming environment that simulates physical or hard instruments such as oscilloscopes and function generators. A simple VI used to produce a waveform is depicted in Fig. 9.3. The front panel of the VI (shown in the figure) acts as the *user interface* while the data acquisition (in this case the data generation) process is performed by the combination of the PC and the DAQ card. Much like the front panel of a real instrument (such as the oscilloscope in Fig. 9.1), the front panel of the VI contains *controls* (i.e., knobs, switches, etc.) that allow the user to modify

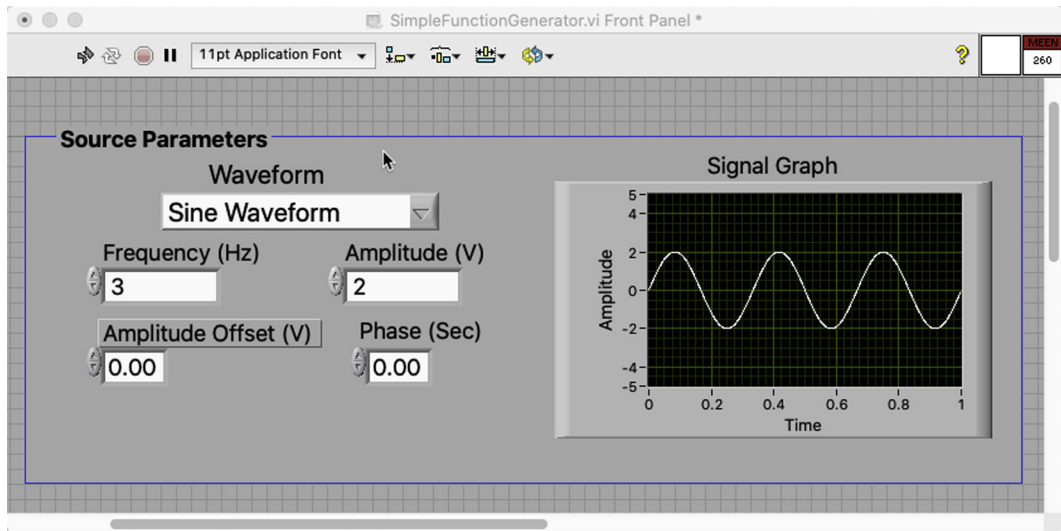


Figure 9.3

A simple function generator virtual instrument (VI).

certain parameters during the experiment. These include a selector to choose the type of waveform, numerical controls to choose the frequency and amplitude of the generated waveform as well as its phase and amplitude offset.

The front panel of a VI also typically contains *indicators* that display the acquired or generated data or other important information related to the experiment. In this case a *graph* is used to depict the waveform. The block diagram (not shown but further discussed below) is analogous to the wiring and internal components of a real instrument. The configuration of the VI's block diagram determines how the front panel controls and indicators are related. It also incorporates functions such as communicating with the DAQ card and exporting the data to permanent files in its native, text, or in spreadsheet format.

9.5 Introduction to graphical programming in LabVIEW

In this section we illustrate how LabVIEW is used to build a simple VI. To begin with we must start LabVIEW, which requires double clicking on the LabVIEW icon or starting it from a command line. Either way, the LabVIEW opening screen appears as shown in Fig. 9.4. This screen or dialog box allows the user to open an existing VI or create one by starting a project. Selecting "Create Project" opens a dialog box in which one can select a VI as the starting point for the subsequent part of this presentation. This results in a blank VI front panel and block diagram and a palette of controls or functions to appear as shown in Fig. 9.5 depending on whether the front panel or the block diagram is in focus.

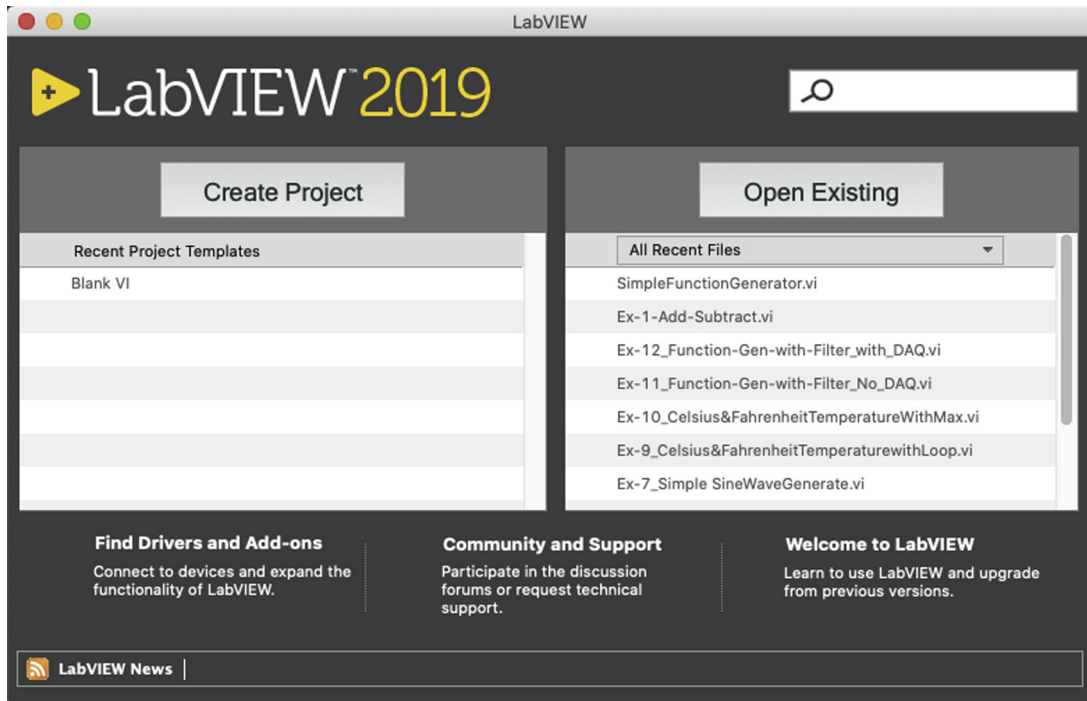


Figure 9.4
LabVIEW startup screen.

LabVIEW makes use of a graphical programming language that determines how each VI will work. In this section we discuss the inner workings of a simple LabVIEW VI that is used to add and subtract two numbers.² While this VI is not particularly useful in the laboratory setting, it illustrates how basic LabVIEW components can be used to construct a simple VI and hence helps the reader move toward developing more sophisticated VIs. Fig. 9.6 shows the front panel and block diagram of the completed VI, which accepts two numbers from the user (X and Y) via two simple *numeric controls*, and produces the sum and difference ($X + Y$ and $X - Y$) of the numbers that are displaced on the front panel via two simple *numeric indicators*. The terms *control* and *indicator* are used in LabVIEW to mean elements that control how LabVIEW functions or displays an outcome produced by LabVIEW.

The block diagram of the VI is a graphical, or more accurately a *data flow*, program that defines how the controls and indicators on the front panel are interrelated. The controls on the front panel of the VI show values that can be modified by the user while the VI is operating. The indicators display values that are output by the VI. Each control and

² This VI is the subject of Exercise 1 later in this chapter.

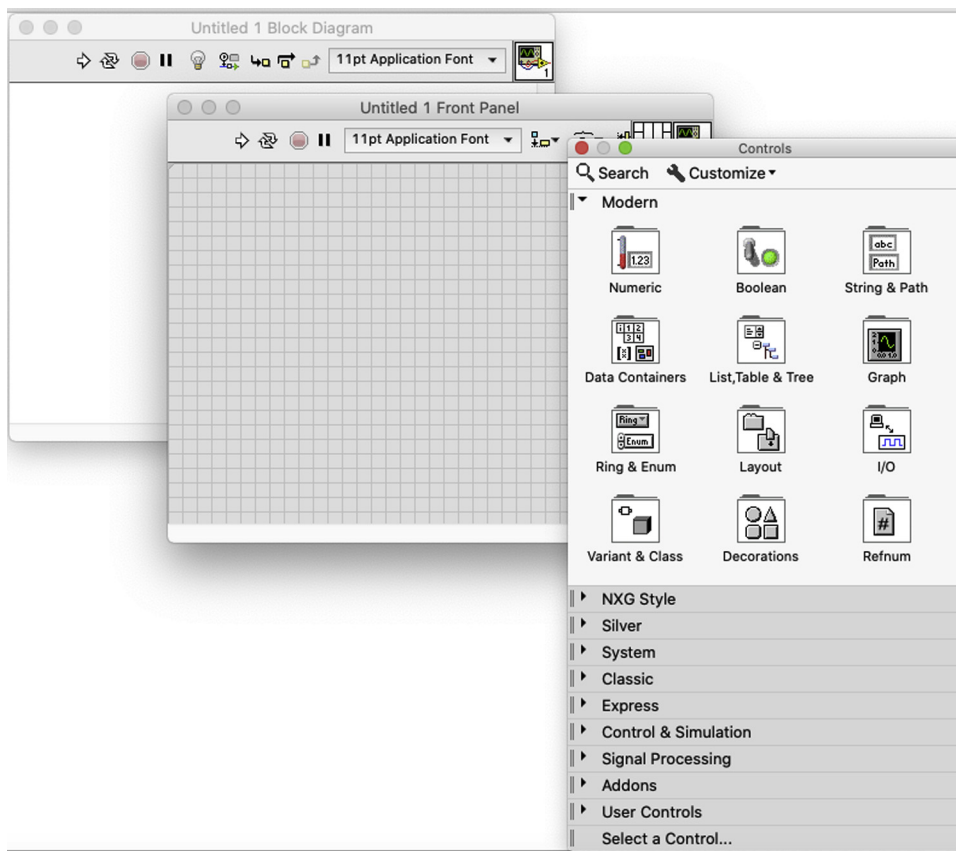


Figure 9.5

Blank VI and controls palette.

indicator in the front panel window is associated with a *terminal* in the block diagram window, also depicted in Fig. 9.6. The wires in the block diagram represent the flow of data within the VI. *Nodes* are additional programming elements that can perform operations on variables, perform input or output functions through the DAQ card, and serve a variety of other functions as well.

The two nodes in the VI in Fig. 9.6 (add and subtract nodes) have two inputs and one output each, as depicted (for instance, in the case of an add node) in Fig. 9.7. Data can be passed to a node through its input terminals (usually on the left) and the results can be accessed through the node's output terminals (usually on the right).

Since LabVIEW diagrams are data flow driven, the sequence in which the various operations in the VI are executed is *not* determined by the *order* of a set of commands. Rather, a block diagram node executes (or is executed by the LabVIEW engine) when data is present at all of its input terminals. As a result, in the case of the block diagram in

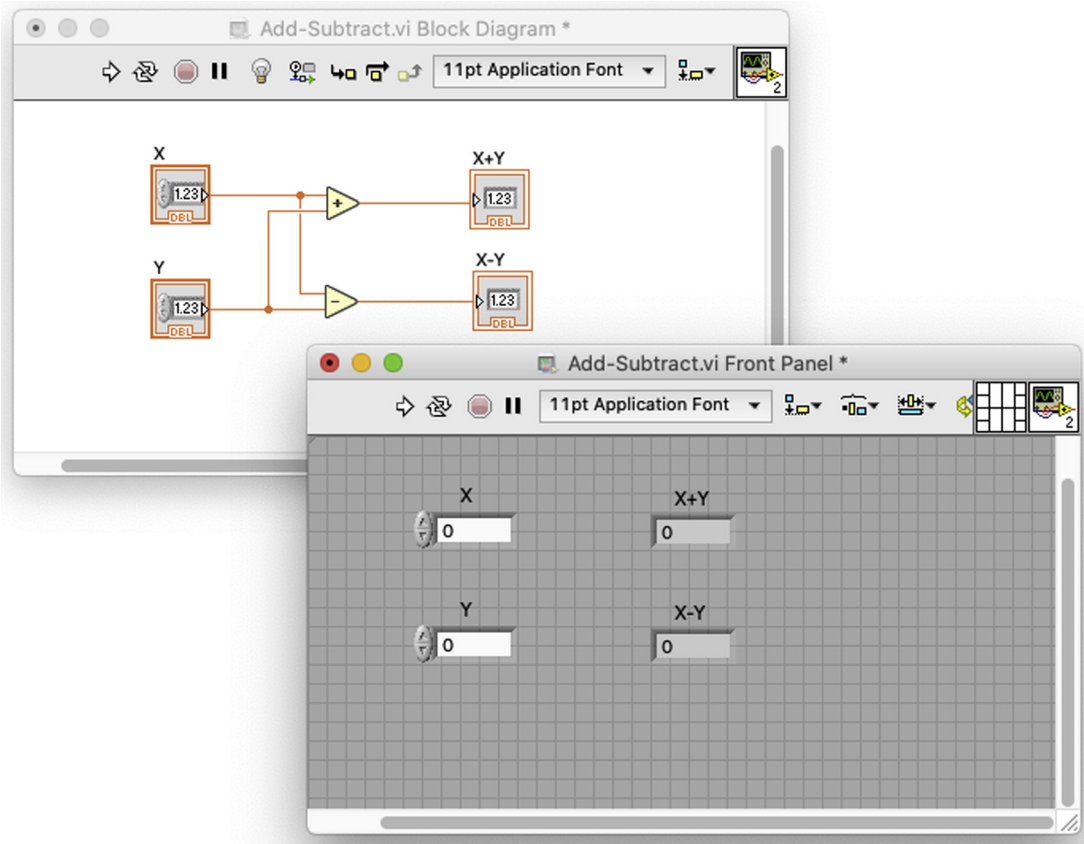


Figure 9.6
Addition and subtraction VI.



Figure 9.7
A LabVIEW add node.

Fig. 9.6, one does not know whether the add node or the subtract node will execute first. This issue has implications in more complex applications but is not particularly important in the present context. On the other hand, one cannot assume an order of execution merely on the basis of the position of the computational nodes (top to bottom or left to right). If a certain execution order is required or desired, one must explicitly build program flow control mechanisms into the VI, which in practice is not always possible nor is it in the spirit in which LabVIEW was originally designed.

9.6 Elements of the tools palette

The mouse pointer can perform a number of different functions in the LabVIEW environment depending on which pointer tool is selected. One can change the mouse pointer tool by selecting the desired tool from the tools palette shown in Fig. 9.8. (If the tools palette does not appear on the screen it can be displayed by selecting *tools* on the *view* menu.) Note, however, that LabVIEW is quite good at using context to select the right tool for you (to move an object to wire objects together, to edit text, etc.) In other words, you may not need to use the tools palette unless you need to force LabVIEW to perform a certain action.

The choices available in the tools palette are as follows:






-  *Automatic Tool Selection*. Automatically selects the tool it assumes you need depending on context. For instance, it can be the *positioning tool*, or the *wiring tool* or



Figure 9.8
The LabVIEW tools palette.

the *text tool* as further noted below. This is in effect the default mode for operation of LabVIEW.

-  *The Operating Tool*. This tool is used operate the front panel controls before or while the VI is running.
-  *The Positioning Tool*. This tool is used to select, move, or resize objects in either the front panel or the block diagram windows. For example, to delete a node in a diagram, one would first select the node with the positioning tool, and then press the delete key.
-  *The Text Tool*. This tool is used to add or change a label. The enter key is used to finalize the task.
-  *The Wiring Tool*. This tool is used to wire objects together in the diagram window. When this tool is selected, you can move the mouse pointer over one of a node's input or output terminals to see the description of that terminal.

One can add controls and indicators to the front panel of a VI by dragging and dropping them from the controls palette, depicted in [Fig. 9.9](#) and which is visible *only* when the front panel window is *active*.

If for some reason the controls palette is not visible, one can access it by clicking anywhere in the front panel window. Once a LabVIEW control or indicator is added to the front panel of a VI, the corresponding node or item is automatically created in the block diagram window. Adding additional nodes to the block diagram requires the use of the functions palette, which is accessible once the block diagram window is visible. One can add arithmetic and logic elements to the block diagram window by dragging and dropping these elements from the functions palette. The functions to add, subtract, etc. can be found in the numeric subpalette of the programming section of the functions palette. One can also use constants from the numeric subpalette and add these into a block diagram, and wire those as inputs to various nodes.

9.7 Logic operations in LabVIEW

In more complex VIs, one may encounter situations where the VI must react differently depending on the condition at hand. For example, in a data acquisition process the VI may need to turn on a warning light when an input voltage exceeds a certain threshold, and therefore it may be necessary to compare the input voltage with a threshold value. The same VI needs to turn on a light on the computer screen or an actual warning light external to the DAQ system, produce an alarm sound, etc. To allow for these types of applications, there are many logic operators available in LabVIEW. These can be found in the comparison subpalette of the programming section of the functions palette. Note that as stated earlier, the functions palette is available when the diagram window is the top (or in focus) window on the screen. To identify the comparison subpalette one can move the

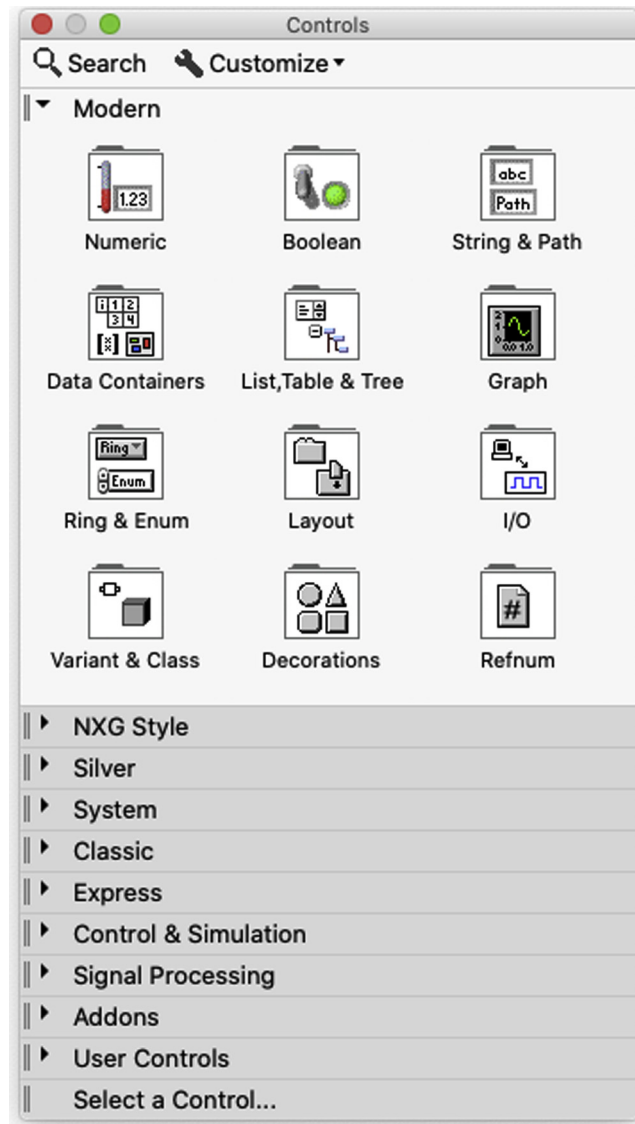


Figure 9.9
The controls palette.

mouse pointer over the programming section of the functions palette to call out the different subpalettes.

Comparison nodes, as for instance depicted in [Figs. 9.10 and 9.11](#), are used to compare two numbers. Their output is either *true* or *false*. This value can be used to make decisions between two numbers as shown in the figure.

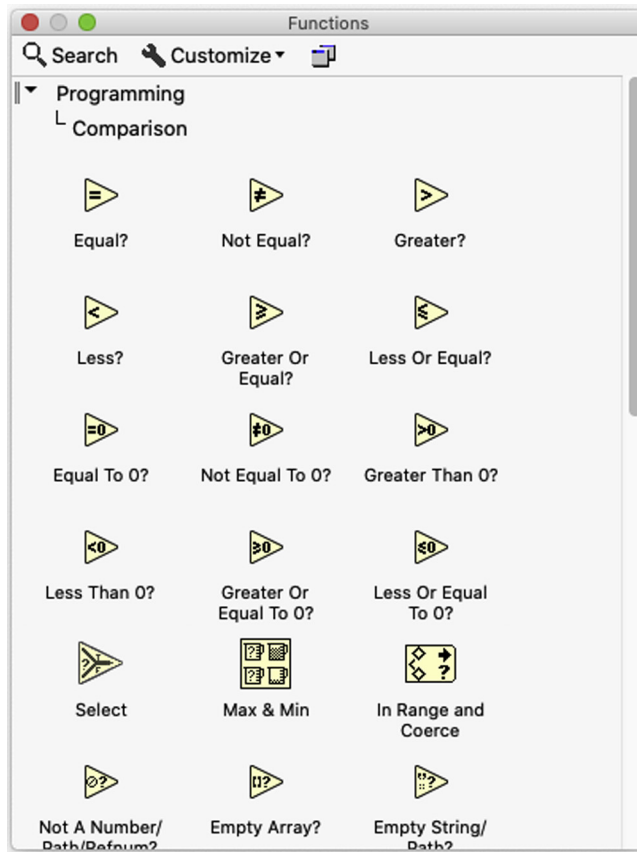


Figure 9.10
The functions palette.



Figure 9.11
Logic/decision example.

Another important node in this respect is the *select* node, which is also available in the same subpalette. This node makes a decision based on the outcome of a *previous* comparison such as depicted in the figure. If the select node's middle input is true, it selects its top input as its output. If its middle input is false, it selects its bottom input as

its output. In this way, one can pair the comparison nodes with a select node to produce an appropriate action for subsequent processing (as for instance to turn on a warning light when a certain threshold is exceeded).

9.8 Loops in LabVIEW

In building more sophisticated VIs it will not be sufficient to simply perform an operation only once. For example, if a LabVIEW VI is being used to compute the factorial of an integer, n , the VI will need to continue to multiply n by $n-1$, $n-2$, and so forth. More pertinently, in data acquisition processes, one needs to periodically acquire samples of data and process these. For this reason, LabVIEW includes several types of *loop* structures. These can be found in the structures subpalette of the programming section in the functions palette. (Note that as with every one of LabVIEW tools, one can use LabVIEW's help feature or its quick help feature to get more information on these constructs.) Here one can find a *while* loop, a *for* loop, a *case* statement, and several other loop structures. Fig. 9.12 depicts a simple program that utilizes a loop. There are several important items to note about using loops. Everything contained inside the loop will be repeated until the *ending condition* of the loop is met. For a *while* loop, this will be some type of logical condition. In the example shown in Fig. 9.12, we have used a *stop* button on the front panel to terminate the loop. The loop will stop when a value of *true* is passed to the stop button in the block diagram at the completion of the loop.

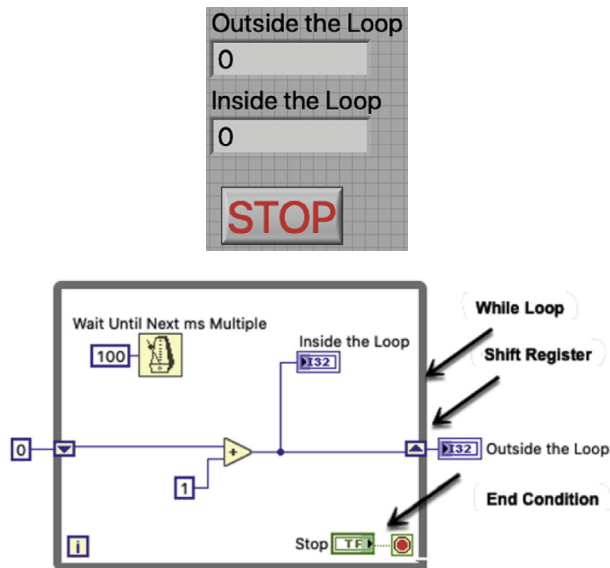


Figure 9.12
Example showing a Loop VI.

In addition, it is often important to be able to pass values from one iteration to the next. In the example above, we needed to take the number from the last iteration and add the number 1 to it. This is accomplished using a *shift register*. To add a shift register, one has to right-click (or control click on a Mac) on the right or left side of the loop and use the menu that opens up to add the shift register. To use the shift register, one must wire the value to be passed to the next iteration to the right side as shown in Fig. 9.12. To use the value from the previous iteration, one draws wire from the left side of the loop box to the terminal of one's choosing. In addition, elements initially wired together can be included in a loop simply by creating one around these elements. The wiring initially in place will be preserved. Finally, note that the metronome in the loop times the loop so that it runs every 100 ms. This element is available from the timing subpalette in the programming section of the functions palette.

9.9 Case structures in LabVIEW

The *case structure* is a programming construct that is useful in emulating a switch, similar to what appears on the front panel of a hard instrument to allow the user to select one of a number of available options. In terms of its appearance in LabVIEW, a case structure works much like a while loop as it is depicted in Fig. 9.13. However, there are significant differences between a case structure and a while loop in that a case structure performs a *separate* operation for each case of the *conditional statement* that drives this structure. The conditional statement takes a value that is chosen by the user at run time from among the set of values for which the given case structure is programmed to execute. This set can be {0,1} as is the case initially when a case structure is added to a VI but can be expanded during the programming stage by right-clicking on the conditional statement and choosing “add case” as necessary. For each case, the VI must include an appropriate execution plan

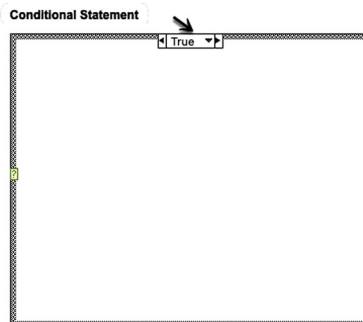


Figure 9.13

A case structure in LabVIEW.

that is placed within the bounding box of the case structure. The conditional statement associated with a case structure is typically driven by a *ring*, which is placed on the front panel of the given VI, and which appears outside the bounding box of the case structure in the block diagram panel of the VI.

This ring is connected to the question mark box on the left side of the case structure in the block diagram. This is illustrated in Fig. 9.14 in conjunction with a four-function calculator implemented in LabVIEW. It is evident in the figure that the *ring*, acting as an operation selector, drives the condition statement while the variables *X* and *Y*, implemented via numeric controls on the front panel, pass their values to the case structure, which embeds the actual mathematical operation for each of the four functions (add, subtract, multiply, and divide) in a dedicated panel. The figure depicts the panel associated with the divide operation. The arrows in the condition statement of the case structure can be used during the programming stage to open each of the cases that the case structure is intended to implement. The ring outside the case structure must have as many elements as there are cases in the respective case structure. Here, it is important to make sure that the order of the selections in the ring and the order of the functions in the case structure correspond to each other. If the first option on the ring is “add”, the first function of the case structure needs to implement the addition function. Exercise 6 deals with this issue at more length. Note that as is the case with other LabVIEW elements, right clicking on a given element allows the user to view a detailed description and examples associated with the given element.

9.10 Data acquisition using LabVIEW

The material in this and subsequent sections assumes access to a laboratory that is equipped with certain basic instruments such as a function generator to produce external signals that can be acquired by a DAQ card and processed by LabVIEW. While it is in principle possible to simulate the generation of data using the computer itself, at the minimum a DAQ card and a connector panel or terminal box (similar to that discussed below) are needed to generate the data and feed that back into the computer. Below we assume that the data is generated by a function generator, however, when feasible. A typical laboratory setup is shown in Fig. 9.15. Aside from the hard instruments shown in the figure, which are also used to perform data acquisition and monitoring tasks, a terminal box is needed to obtain the input from sensors and to allow for a way to produce output voltages from the data acquisition card.

A typical connector panel/terminal box is shown in Fig. 9.16. This is a National Instrument BNC-2120. The pin numbers and the style of the terminal box will vary depending on the

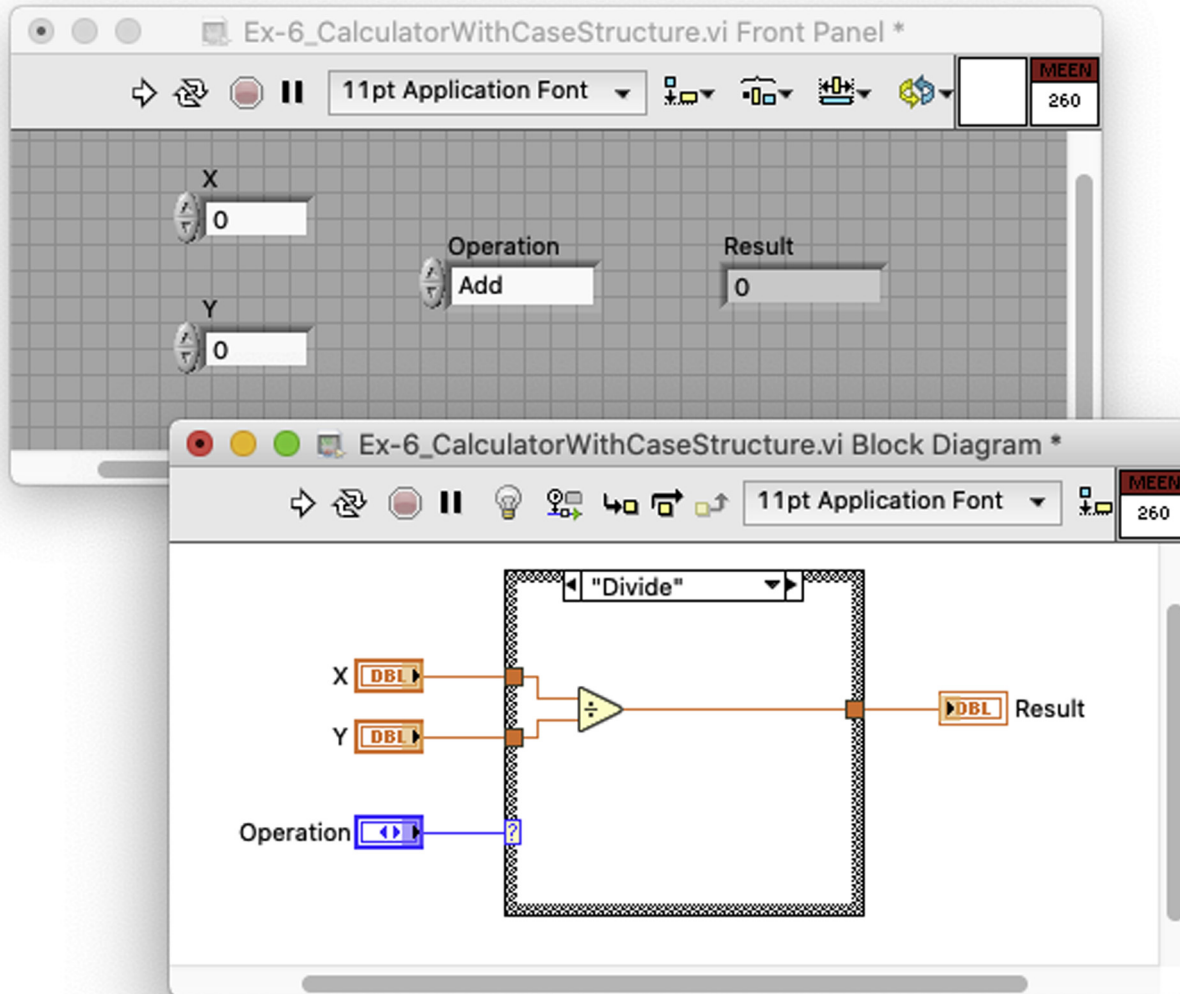


Figure 9.14

A menu ring used in conjunction with a case structure.

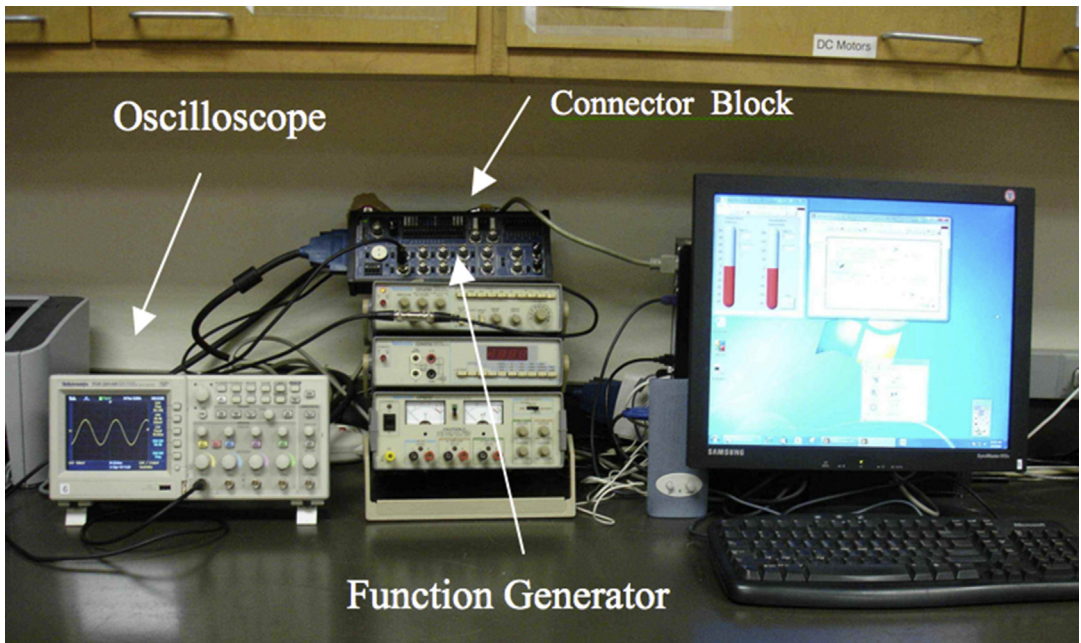


Figure 9.15
Typical setup for LabVIEW-based data acquisition.

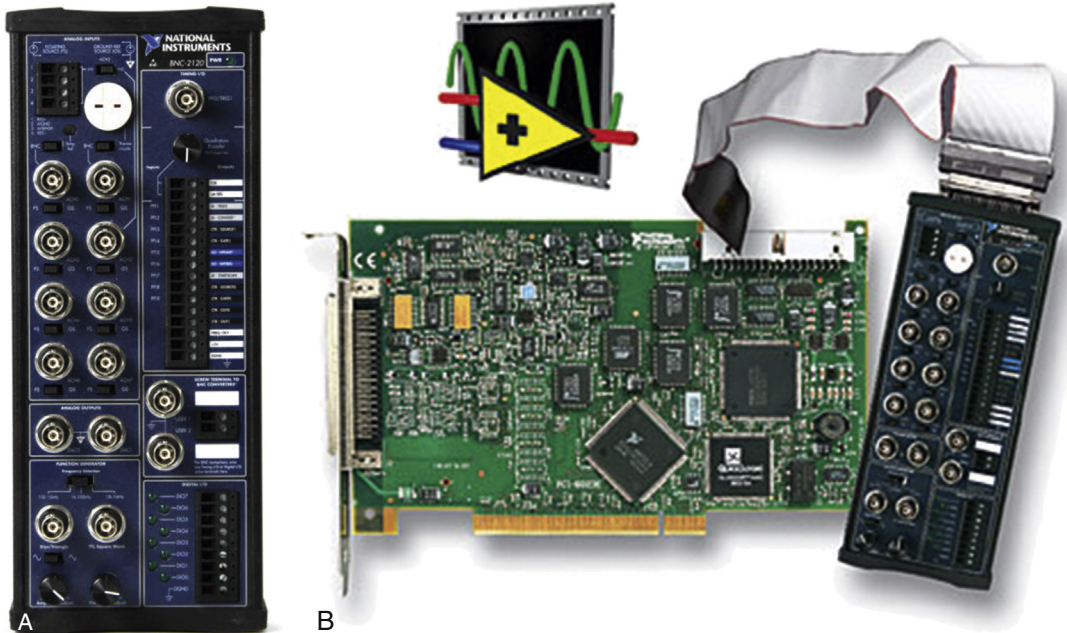


Figure 9.16
NI BNC-2120 connector block on the left and also shown along with a standard DAQ card on the right (note the ribbon cable connecting the DAQ card to the connector block).



Figure 9.17
Typical function generator.

model that you may be using. The experiments described later in this chapter assume a BNC-2120 connector block or terminal panel, but several will use different terminal points that are wired to the input and output channels on the DAQ card, which is typically inside the computer. For the exercise discussed below the data acquisition card's differential mode is used, meaning that it reports the voltage difference across a given input channel (channel 0 in our case).

The card can also operate in an absolute mode where it takes only one input pin and compares this to the ground level. Finally, there is one last piece of equipment that will be needed in the subsequent discussion: function generator. This can be seen in [Fig. 9.15](#), but a larger picture of a slightly different device is shown in [Fig. 9.17](#).

As the name suggests, this piece of equipment is used to generate a sine wave, a triangle wave, or a square wave. The amplitude of the wave can be adjusted by using the amplitude knob of the function generator. The frequency of the wave can also be adjusted using the marked buttons. The main use of this device is to produce an external input (a sinusoidal function, a triangular wave or a square wave) to a custom VI as shown for instance in [Fig. 9.18](#), which converts the voltage produced by the function generator into a temperature reading in Celsius and Fahrenheit units.

The functions relating to communication with the DAQ card are handled by the “DAQ assistant” function depicted in the figure. The board ID and channel settings specify information about the DAQ card that tells the subprogram where to look for the data. For instance, if the function generator is connected via a BNC 2120 ([Fig. 9.16](#)) to Channel 0 of the DAQ card, then the DAQ assistant can be used to set the channel setting to 0 as well. It is important to check this prior to using the VI. The algebraic operations that implement the conversion of the data are straightforward as depicted in the figure.

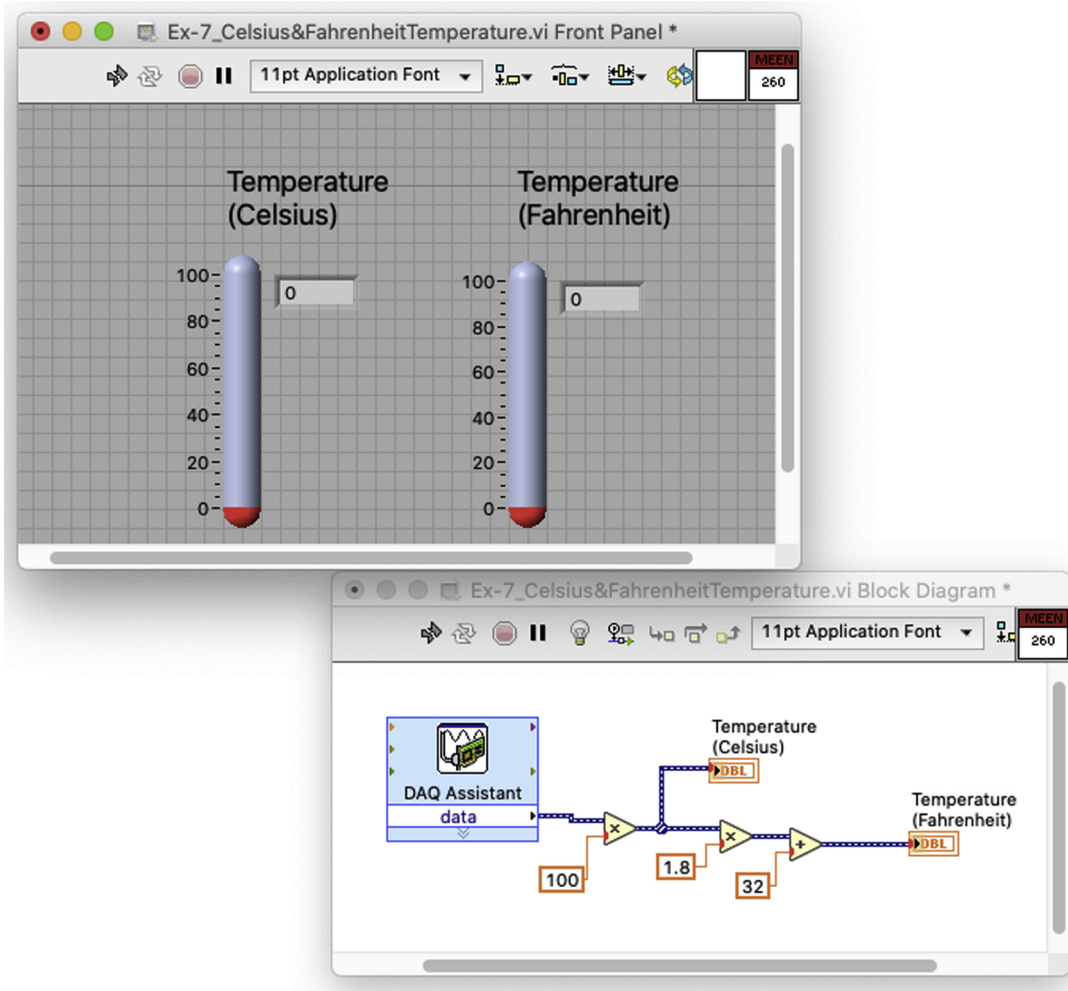


Figure 9.18

Thermometer VI with external input.

9.11 LabVIEW function generation

In this section we will create a VI that emulates a function generator. That is, the VI should produce a periodic signal with a user-selected shape, frequency, and amplitude to an output channel on a National Instruments DAQ card. In addition, the user should be able to select the resolution of the waveform (i.e., the number of data points per period). The VI's front panel should also include a waveform graph that displays one period of the waveform selected as it appears in Fig. 9.19 below.

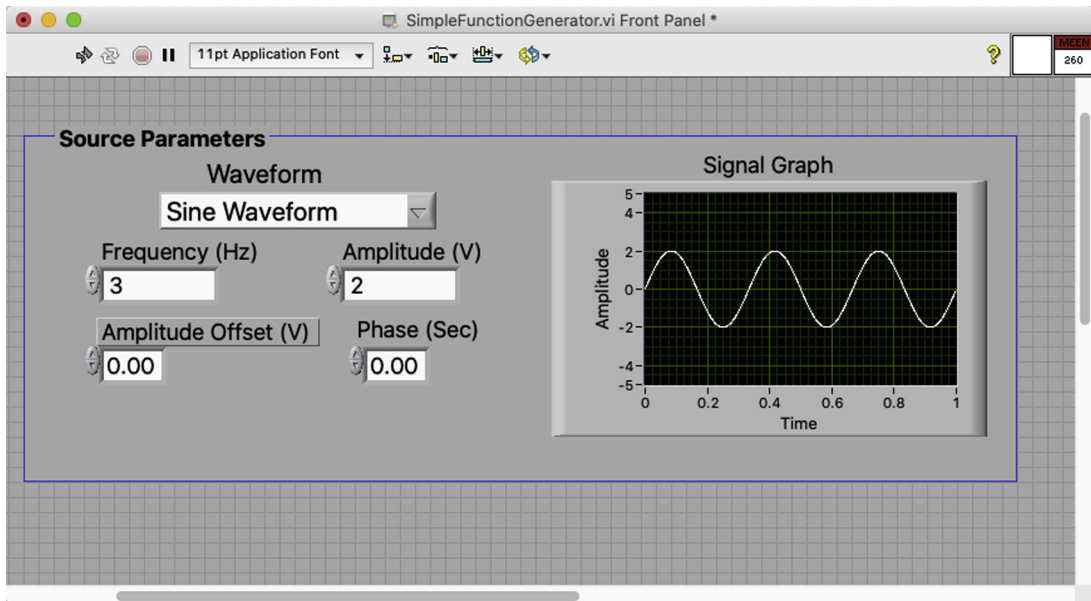


Figure 9.19
Simple function generator front panel.

The VI produces a sine wave, a triangle wave, or a square wave as selected by the user. This requires that a case structure be wrapped around the signal generation block and a ring block to communicate between the front panel and the case structure. This is accomplished in the block diagram for the above function generator as depicted in [Fig. 9.20](#). Note the use of the case structure, the signal graph, and DAQ assistant.

9.12 LabVIEW implementation of digital filters

One can implement a digital filter in LabVIEW as shown in [Fig. 9.21](#). The block diagram of this system appears in [Fig. 9.22](#). The filter parameters are set at 0.5 and 0.5 in the lower left corner of the front panel but can be changed if necessary. The block diagram depicts a sinusoidal signal generator as well as a noise generator on the left side. The in-place operation allows the addition of individual data elements.

The in-place element structure allows simple operations such as addition to occur on the corresponding elements of two dynamic arrays produced by the sinusoidal and noise signal generators. This node can be found in the structures subpalette of the programming section of the functions palette as depicted in [Fig. 9.23](#).

The filter itself is implemented as a *finite impulse response* (FIR) filter node (found in advanced FIR filtering section of the filters subpalette of the signal processing section of

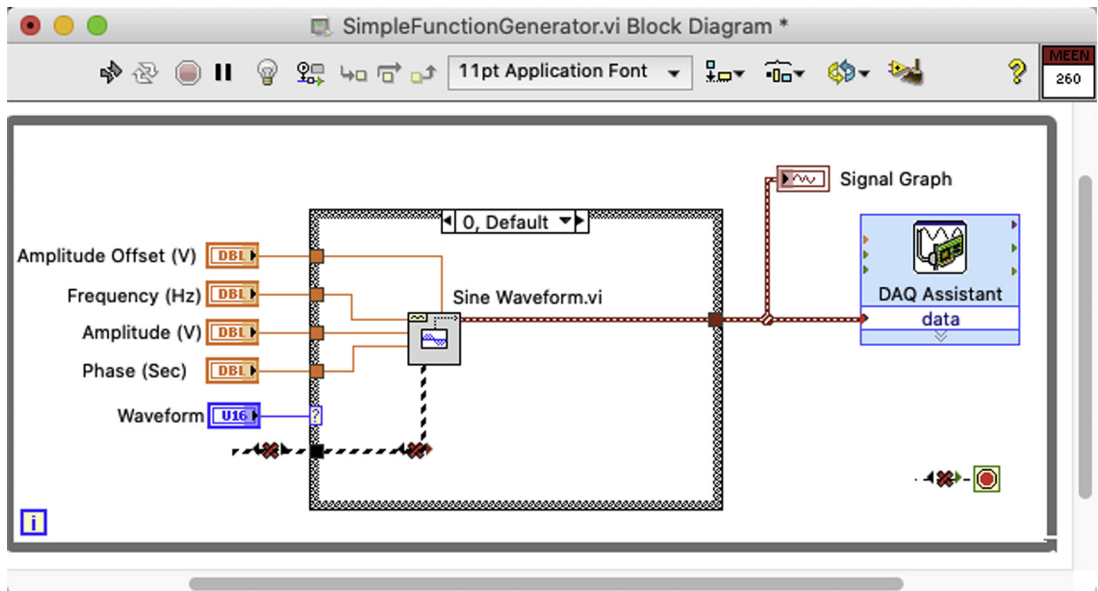


Figure 9.20
Simple function generator block diagram.

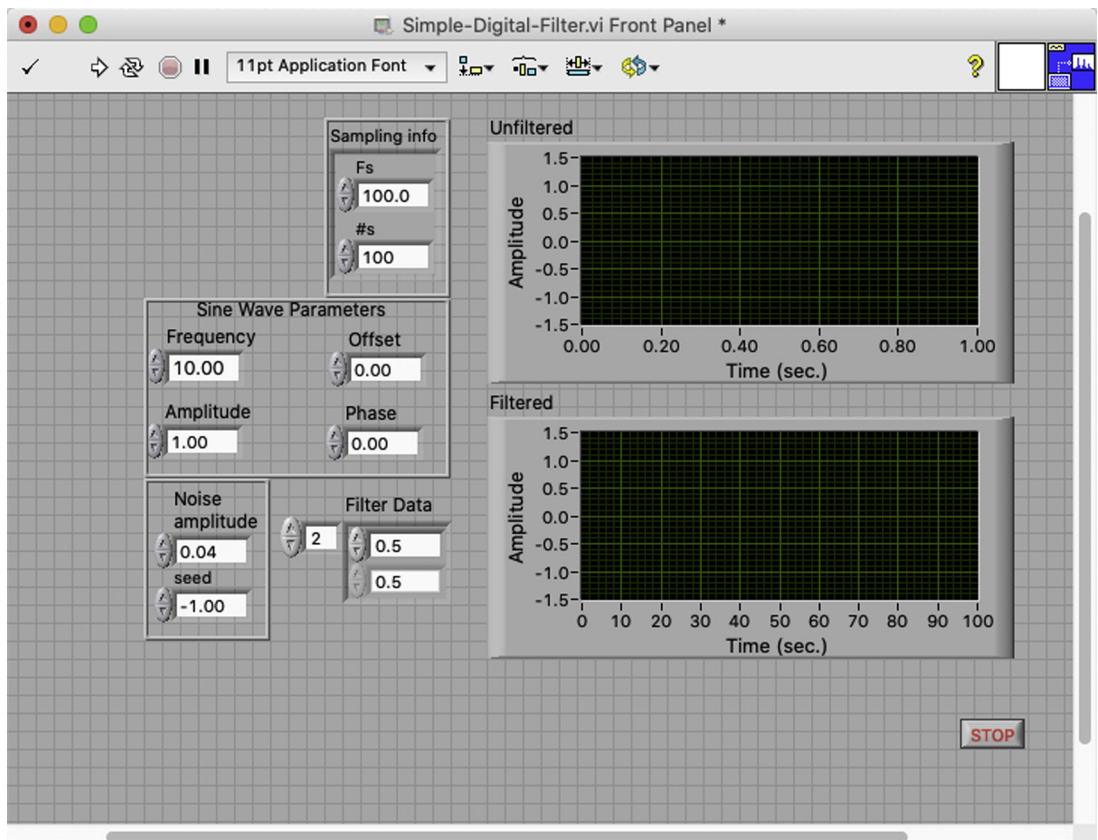


Figure 9.21
Front panel of a simple digital filter.

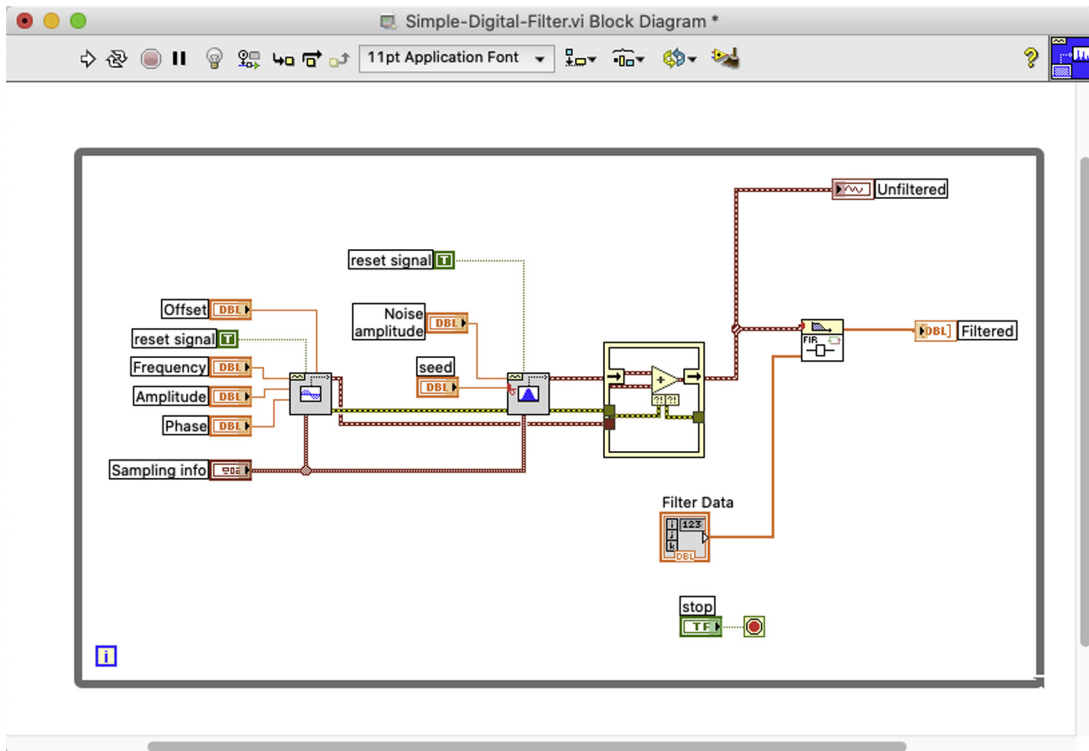


Figure 9.22
Diagram of the simple digital filter.

the functions palette³), which effectively implements a *moving average* (MA) filter type. The array of filter coefficients appears in the front panel of Fig. 9.21. Note that in this case the source signal is generated internally but it is possible to do the same task using an external input signal, as for instance generated by a function generator.

9.13 Higher-order digital filters in LabVIEW

While the simple filter in the previous section works reasonably well, one can build more effective filters using a combination of autoregressive terms and moving average terms, via an ARMA model, also referred to as an infinite impulse response (IIR) filter. The general equation for such a filter is given below:

$$y_k = -a_1y_{k-1} - a_2y_{k-2} - a_3y_{k-3} + \dots + b_0u_k + b_1u_{k-1} + b_2u_{k-2} + b_3u_{k-3} + \dots \quad (9.1)$$

³ The location of these nodes in the respective palette may change depending on the version of LabVIEW. It is, however, possible to search for a specific type of node by name and locate it irrespective of its location.

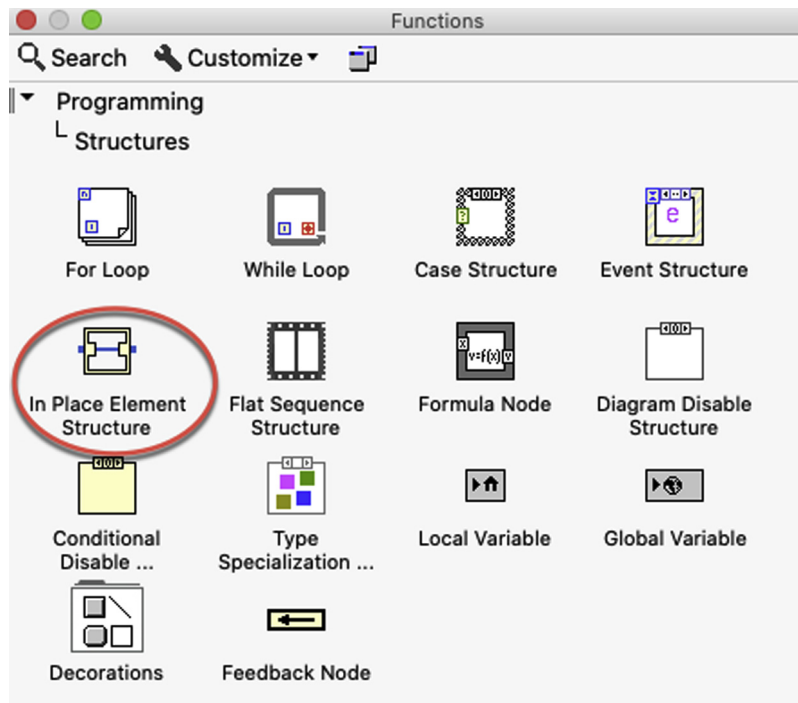


Figure 9.23

Structures subpalette of the functions palette.

Note that the a_i and b_i must be properly chosen for stable and effective performance. This is not a trivial task and requires advanced techniques that extend beyond the scope of the present text. The essential idea is to place the so-called poles and zeros (the roots of the denominator and numerator of the corresponding discrete time transfer function) in reasonable locations in the complex plane. However, there are well-known design strategies that have performed well, including Butterworth (discussed in the previous chapter), Chebyshev, and Bessel that are programmed in LabVIEW. It is also possible to produce the filter coefficients in MATLAB or a similar tool and use a similar technique as in the previous section (albeit using an IIR filter node) to implement the given filter. Implementation using LabVIEW built-in functions is depicted in Fig. 9.24. The filter parameters include the sampling frequency, which in this case is the same the frequency used to generate the signal in the first place. The cutoff frequency is also another required parameter, which is chosen to correspond to the frequency at which the noise components start to dominate the real signal. The block diagram is depicted in Fig. 9.25. The filter node (from the signal processing section of the function palette) requires filter type (set to zero for a low-pass filter) as well as the two parameters mentioned above. Note that the

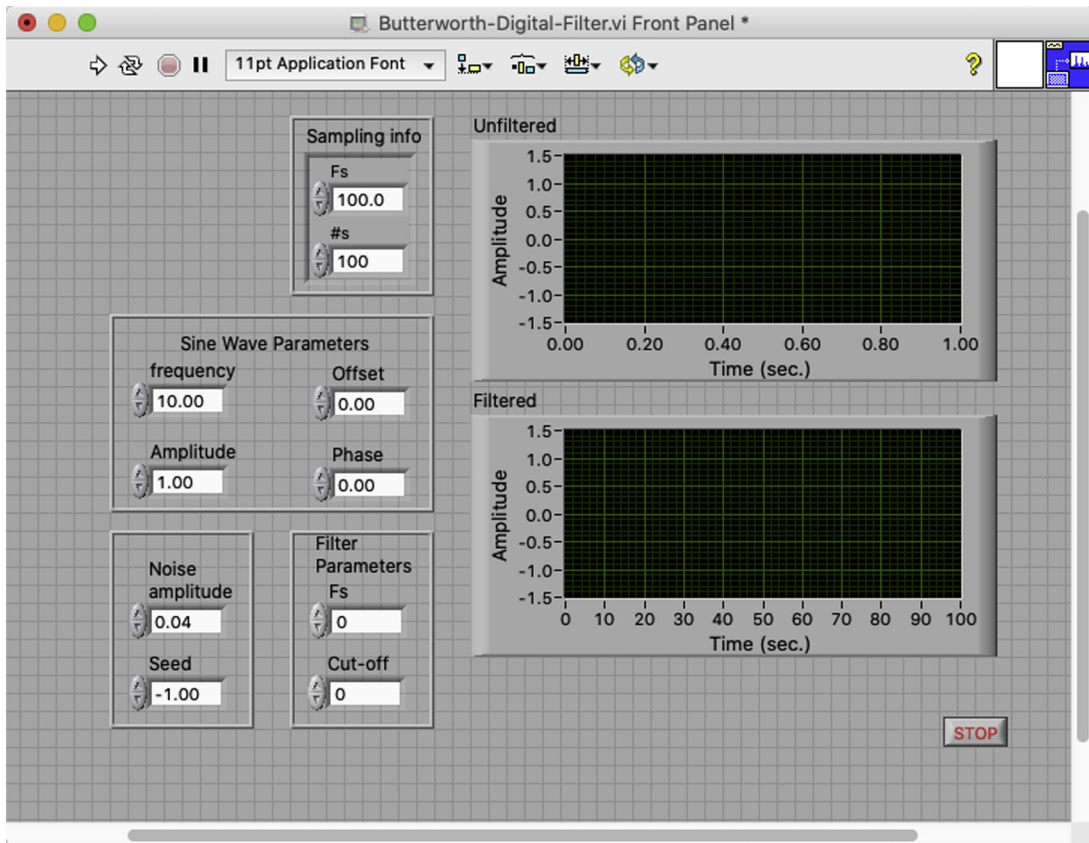


Figure 9.24

Front panel of Butterworth filter design.

unconnected terminal of the filter node (the high-frequency cutoff) is not required for a low-pass or high-pass filter but is required for a band-pass filter.

The response of the filter depicted in the front panel diagram in Fig. 9.24 typifies the low-pass filtering effect of a Butterworth filter, which indeed performs better than the simple filter we designed earlier.

9.14 Summary

This chapter was meant to introduce the reader to the usage of LabVIEW in computer-based data acquisition and signal processing. Basic LabVIEW programming concepts, namely controls, indicators, and simple nodes used to implement algebraic operations as well as program flow control constructs, i.e., while loop and case structure, were introduced. These building blocks were used to implement increasingly more complex

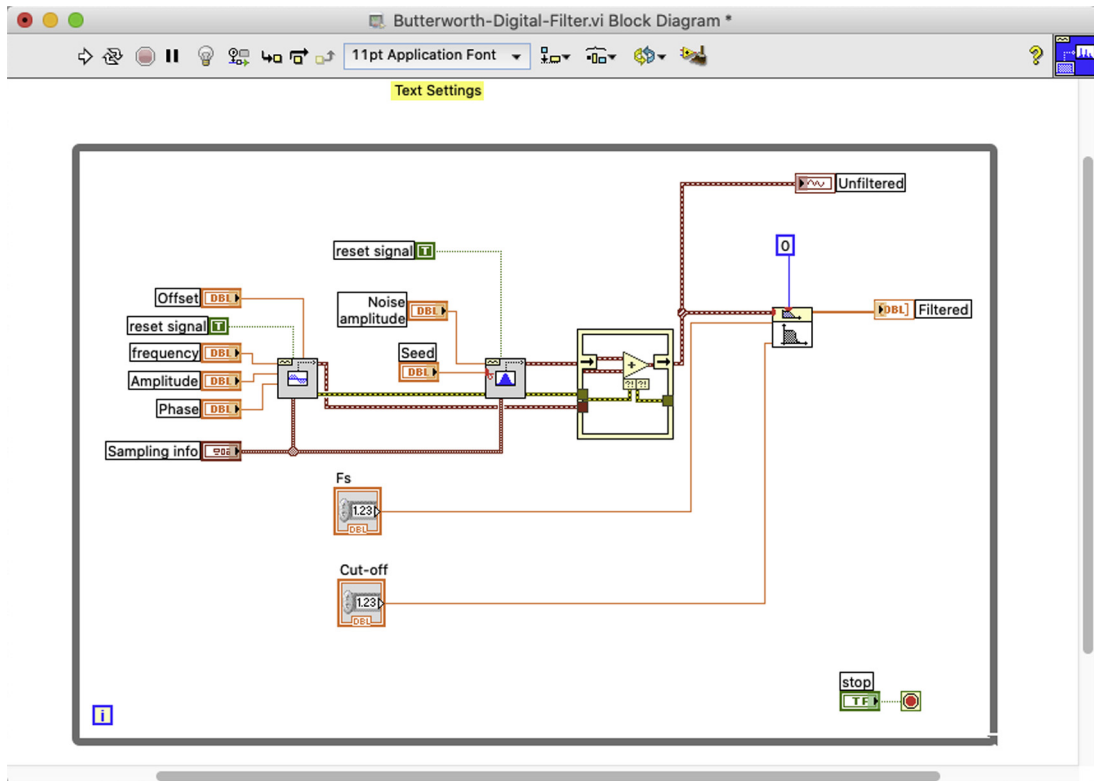


Figure 9.25

Block diagram of Butterworth filter design.

LabVIEW virtual instruments. These VIs can be used as stand-alone LabVIEW programs or expanded to form sophisticated VIs in conjunction with additional LabVIEW elements. Digital filters are also relatively easily built using LabVIEW and can be used to augment LabVIEW data acquisition tools to enable the user to perform meaningful experiments involving mechanical systems. The exercises that follow enable the reader to practice constructing a number of VIs that are of value in laboratory setting.

9.15 Exercises

9.1 In order to demonstrate your understanding of LabVIEW programming, in this exercise you will build the VI depicted previously in Fig. 9.6. We assume that LabVIEW is installed on your computer. Start LabVIEW and select new VI from the file menu. (Note that the startup screen allows you to create new projects but at present we will focus on creating only a new VI.) The blank VI front panel and blank block diagram

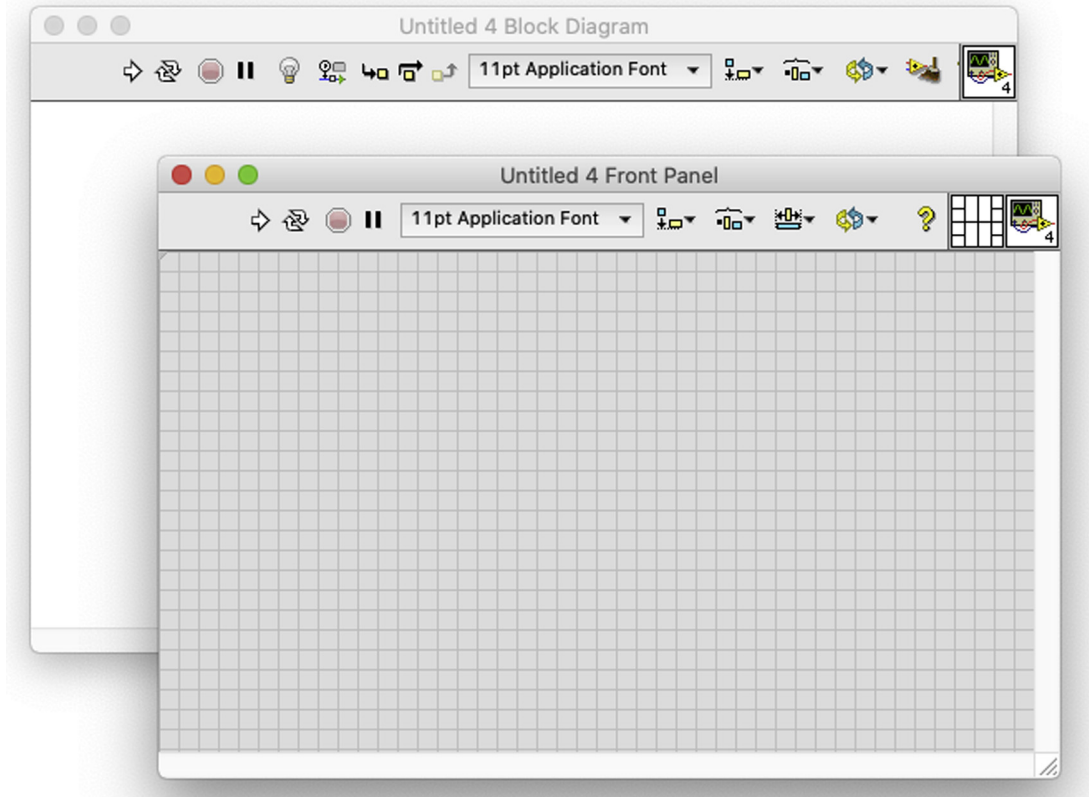




Figure 9.26
Blank VI panels.

window should appear in the background as, for instance, are depicted in [Fig. 9.26](#). If the block diagram window is not visible, choose the show block diagram from the window menu (Ctrl-E in Windows or Cmd-E on a Mac should also do the same thing).

On the front panel diagram, you will need to add the required *numeric controls* for X and Y and *numeric indicators* for $X + Y$ and $X - Y$. These are available in the modern subpalette of the controls palette, which is available by right-clicking in the front panel. You will then use the block diagram window and implement the *add* and *subtract* nodes (available in the numeric section of the programming subpalette of the functions palette, which itself can be viewed by right-clicking in the block diagram window) and connect them to the appropriate controls and indicators. Note that you can change the labels of each entity by using the text tool from the tools palette and further note that changing the label of an indicator or control in the block diagram window changes its label in the front panel as well. Once you have inserted the necessary nodes and wired them together properly, you can run the VI by pressing

the run button  at the upper left corner of the front panel window. The digital indicators should then display the results of the addition and subtraction operations. You can also use the continuous run button , which is next to the run button.

- 9.2 Create a program that will take the slope of a line passing between any two points. In other words, given two points, (X_1, Y_1) and (X_2, Y_2) , find the slope of the line, $Y = mX + b$, which passes through these points. This program should have 4 digital inputs (as numeric controls) that allow the user to select the two coordinates. It should have one numeric indicator to provide the value of the slope of the line between them.
- 9.3 Using ideas similar to those in [Fig. 9.11](#), demonstrate your understanding of the logic operations by adding an additional numeric indicator that will display the *greater* of the two numbers determined from your addition and subtraction operations in Exercise 1. Is the number obtained by the addition operation always the greatest?
- 9.4 Demonstrate your in-depth understanding of *logic* operations by building a VI that takes as input three numbers and outputs the *greatest* and the *smallest* value of the three. This will involve very similar logic operations as the one performed in the earlier VI but requires cascading these operations and generally a more sophisticated use of the LabVIEW logic operations.
- 9.5 In order to drill you in the operation of loops and related structures in LabVIEW, practice by creating the VI depicted in [Fig. 9.12](#). When fully operational, the program should continue to increment by 1 every 0.1 seconds until the stop button is pushed. Verify that this is true and that waiting longer to press the stop button results in a larger number. What is the difference between the *inside* and *outside* loop displays? Why do you think this happens? If you let the program run and failed to turn it off, would your program ever be able to reach *infinity*? Explain your answer.
- 9.6 The specific task that you will perform is to create a calculator that will *add*, *subtract*, *multiply*, or *divide* two numbers using a case structure depicted in [Fig. 9.14](#). This will require that you to use two inputs, an output, and a *ring* on the front panel. You will need a *case structure* in the block diagram window, which initially will have only two cases $\{0,1\}$ but can be extended by right-clicking on the condition statement in the block diagram. The case structure itself can be found in the structures subpalette of the functions palette. The ring should be added to the front panel of the VI and can be found on the modern subpalette of the controls palette. The properties dialog box of the ring, which opens up by right-clicking on the ring, allows you to add cases as you need (in this case, four cases). Make sure the order of the list on the ring corresponds to the same subset in the case list and that you set a *default* case.

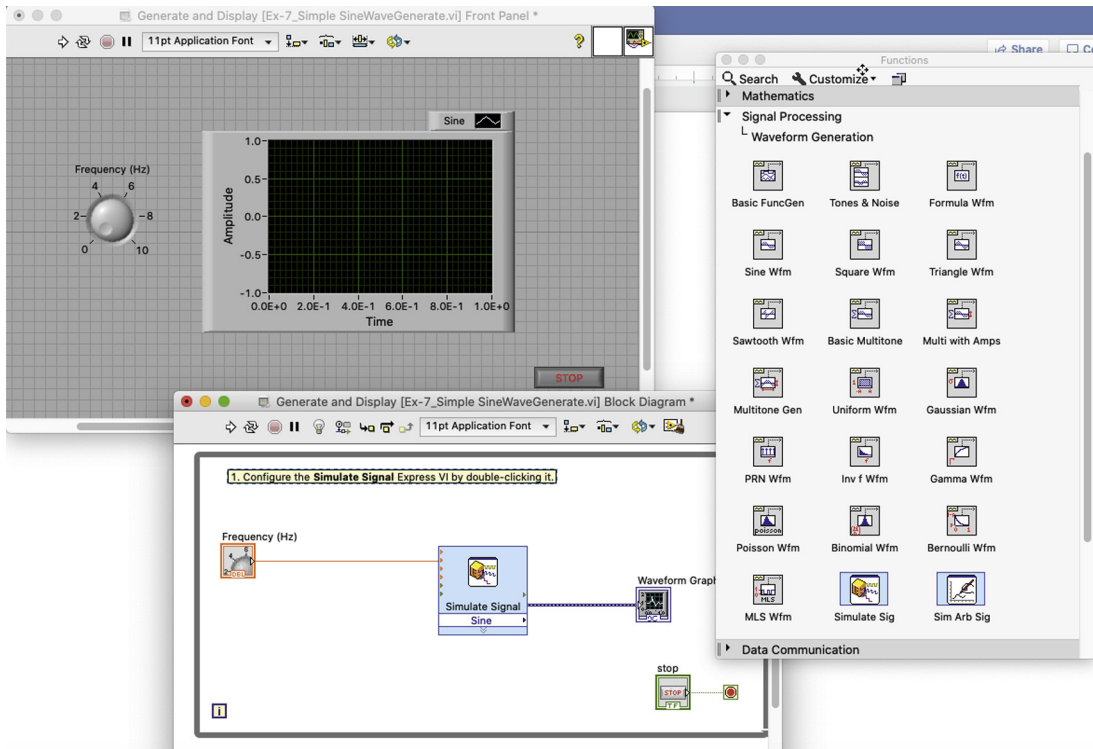




Figure 9.27
Simple sine generator.

- 9.7 In Fig. 9.19 we saw a simple function generator. A more rudimentary one can be developed as shown in Fig. 9.27. Develop this using waveform graph, a knob and a stop button on the front panel. On the block diagram you need a simulate signal. You can search for these using the search button on the tools or functions palette.
- 9.8 Data acquisition is likely the most important function of LabVIEW. To be able to understand completely what takes place during data acquisition, it is important to know what tasks are performed by a VI that acquires data from an external source. The VI in Fig. 9.18 can take a temperature measurement every time the run button is pushed. While this is useful for taking measurements where temperature is constant (for example, the room temperature), it is not useful in tracking temperature changes. (The next exercise addresses this issue via a *loop* structure.) If a low-frequency square wave is used, however, it is possible to verify that the VI functions as intended. Assuming you have access to a physical function generator instrument and that your computer is equipped with a LabVIEW compatible DAQ card, connect the function generator to the computer. Make sure that the function generator is set on a square wave and that the frequency is set to be about 0.1 Hz to

create a signal that changes infrequently. You may wish to view the signal on an oscilloscope to ensure that the function generator is indeed producing the desired waveform. This can typically be done by connecting the output of the function generator via a BNC connector to Channel A (or Channel 1) on a typical two-channel scope. Be sure to set the trigger level of the oscilloscope to external (EXT) and choose Channel A (or Channel 1) as the trigger source. You may have to initially use the ground function of the scope to ensure that the beam is zeroed at the midlevel of the screen and that the vertical and horizontal scales are properly set up. The same BNC connector can be used to connect the function generator to the BNC 2120, which acts as the interface between the DAQ card and the function generator (assuming that this is the device that is enabling you to physically connect a signal source to your computer). The elements of the VI itself are readily obtainable from the controls and functions palettes. For instance the thermometer numeric indicator  is useful in this context as is the DAQ assistant from the measurement I/O subsection of the functions palette. Once the VI is constructed, it is important to ensure that the settings of the DAQ channel are consistent with the actual channel used in the data acquisition process.

- 9.9 In this exercise we need to add a loop to the VI in the previous exercise so that the VI can take readings continuously until a stop button on the front panel of the VI is pushed and the VI is stopped. The loop construct is in the structure subpalette of the programming section of the functions palette. You can also find the stop button in the classic Boolean subpalette of the classic section of the controls palette. Verify that this works and watch how the temperature indicator changes with time. Change the frequency of the function generator and observe the effects on the output. Increase the frequency of the signal using the frequency control knob on the function generator instrument and observe the effect on the reading produced by the thermometer indicator. At what point do you not observe the variation of the external signal?
- 9.10 In this exercise, we will add more functions to the VI from the previous exercise. If we only want to maintain a given temperature (as for instance in a thermostat), we might only care about the highest value of the temperature. Use the *logic* operators, used in the previous exercises, to determine the *highest* temperature among all the measurements (Hint: you will need a shift register). In LabVIEW there are many different options for data types (scalar, dynamic data, waveform, etc.). For this part of the experiment you will need to convert your acquired data from dynamic data type to single scalar type. To do this, right-click in the block diagram, and select express, signal manipulation, from DDT, which appears as  in icon form. The respective dialog box enables you to accomplish the required task.

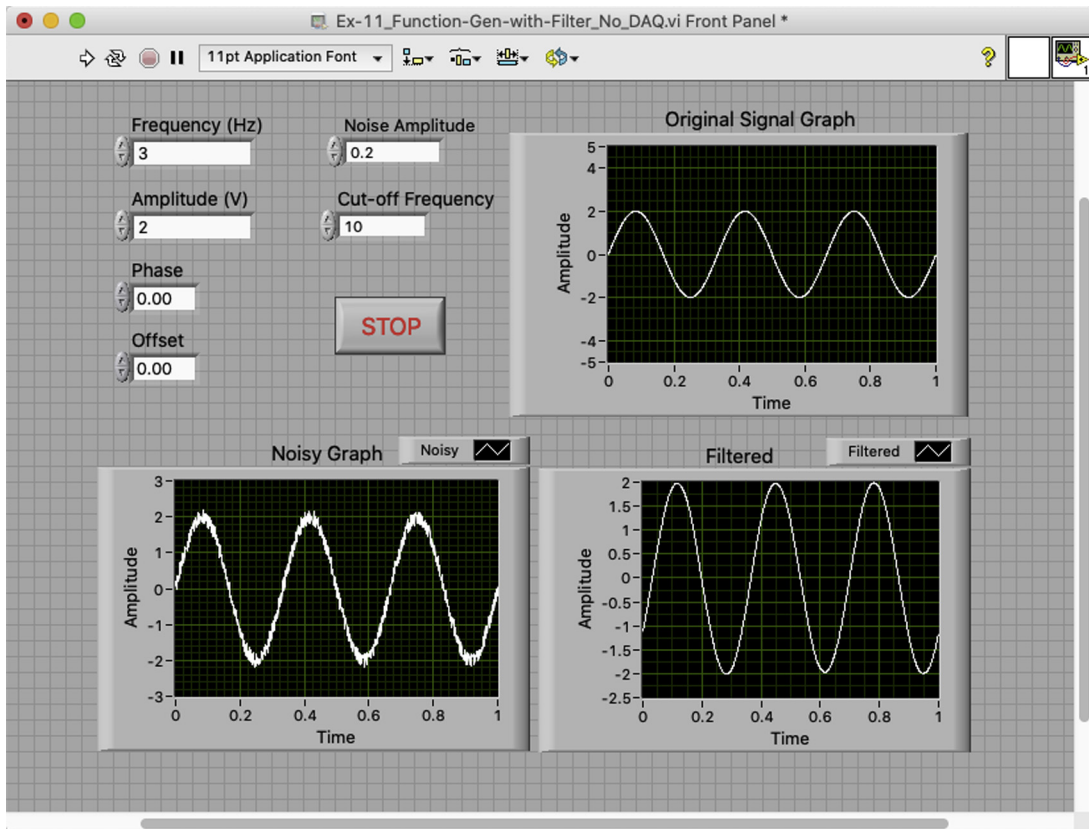


Figure 9.28

Function generator and filter front panel.

- 9.11 This exercise focuses on building a digital filter using LabVIEW built-in filtering functions. Specifically, we will build the VI depicted in [Fig. 9.28](#), which is used to generate a sinusoidal function and after adding noise, filtering the signal, and displaying the filtered signal alongside the original signal. Note that in spite of the scaling factors, the filtered and original signals are of the same range in magnitude. The figure shows the noise amplitude to be about 10% of the signal amplitude, but this can be changed as necessary. The filter cutoff signal can also be changed at will.

Since the design is rather complicated, the block diagram is shown in [Fig. 9.29](#). Note that each of the elements used in the block diagram can be obtained from the subpalettes of the function palette although some require a bit of digging in the various subpalettes, which also helps familiarize you with the various functionalities of LabVIEW.

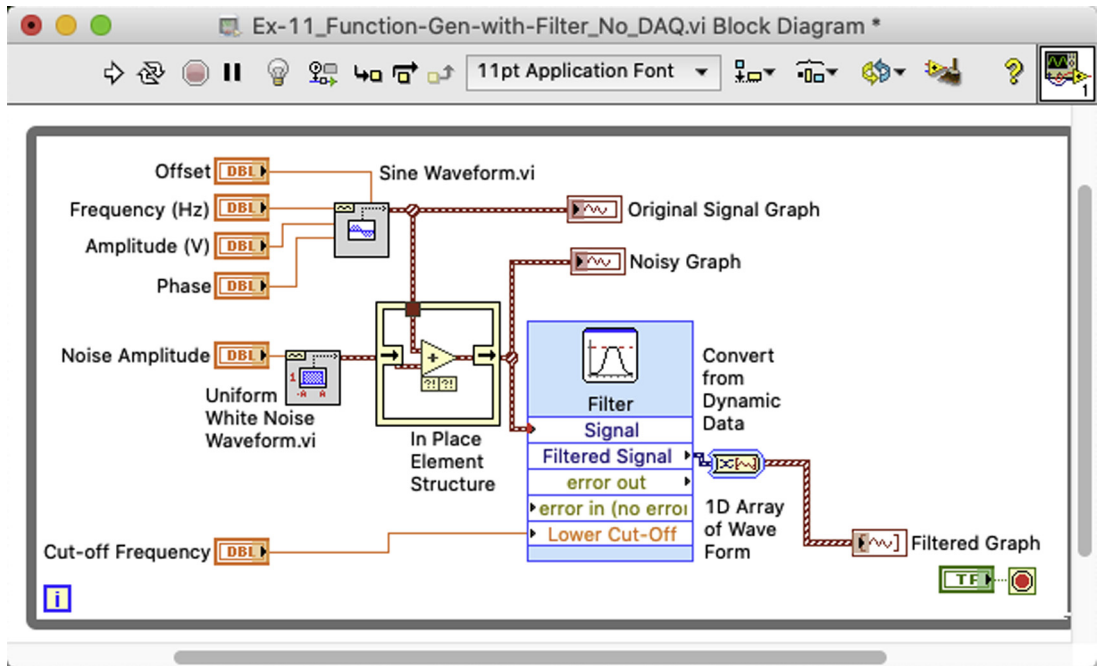


Figure 9.29
Function generator and filter block diagram.

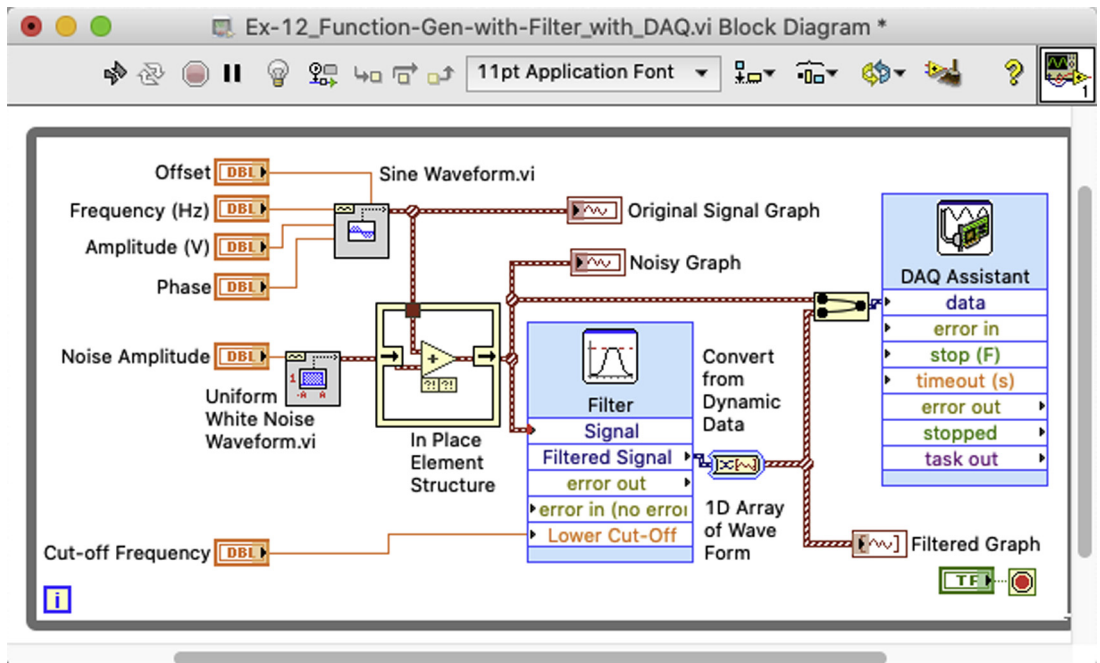


Figure 9.30
Function generator and filter block diagram with DAQ.

- 9.12 The VI in the above exercise can be used to generate external signals that can also be displayed using an oscilloscope. The front panel would look exactly the same as in [Fig. 9.28](#). However, the block diagram would need to be modified as shown in [Figure 9.30](#) to incorporate a DAQ assistant.
- 9.13 Implement the VI in [Figs. 9.21 and 9.22](#). The steps are clear from the block diagram that is shown.
- 9.14 Implement the VI in [Figs. 9.24 and 9.25](#). The steps are clear from the block diagram that is shown.
- 9.15 What are some potential problems of using filters in real systems? Hint: In the earlier exercises, we knew EXACTLY what we were looking for. What if we do not?

Display, recording and presentation of measurement data

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10.1 Introduction

The earlier chapters in this book have been essentially concerned with describing ways of producing high-quality, error-free data at the output of a measurement system. Having obtained the data, the next consideration is how to present it in a form where it can be readily used and analyzed. This chapter therefore starts by covering the techniques available to either display measurement data for immediate use or to record and store it for future use. Following this, standards of good practice for presenting data in either graphical or tabular form are covered, using either paper or a computer monitor screen as the display medium. This leads on to a discussion of mathematical regression techniques for fitting the best lines through data points on a graph. Confidence tests to assess the correctness of the line fitted are also described. Finally, correlation tests are described that determine the degree of association between two sets of data when they are both subject to random fluctuations.

10.2 Display of measurement signals

The traditional ways of displaying measurement signals are to use either an *electrical meter* or an *oscilloscope*. However, newer display options now exist as well, such as electronic output displays or using a computer monitor. All of these apart from oscilloscopes are designed to display the magnitude of measurement signals that are in the form of a varying electrical voltage. If a measurement signal exists in some other form, such as changes in the frequency, phase, or current in a signal, conversion to a varying voltage form has to be carried out first using one of the techniques explained earlier in Chapter 6. However, this is not necessary for oscilloscopes if the measurement signal involves changes in frequency or phase, since an oscilloscope is able to display such frequency and phases changes. It should also be noted that electronic displays and computer monitors are digital devices, and analog signals must be converted to digital form before being inputted to these.

We will start this section by looking at electrical meters. These exist in both digital and analog forms, although the use of the analog form now tends to be restricted to panel meters, where the analog form of the output display means that abnormal conditions of

monitored systems are more readily identified than is the case with the numeric form of output given by digital meters. As well as signal-level voltages (i.e., low voltages typically in the range up to about 5 V in magnitude), many of the meters available can also measure higher-magnitude voltages, and this is indicated where appropriate in the discussion of electrical meters that follows.

Having covered meters, we move on to look at the oscilloscope as a display device. The oscilloscope is particularly useful for interpreting instrument outputs that exist in the form of a varying phase or frequency of an electrical signal. It is a very versatile measuring instrument that is widely used for signal measurement in spite of the measurement accuracy provided being inferior to that of most meters. Although existing in both analog and digital forms, most instruments used professionally are now digital, with analog versions being limited to cheap, low-specification instruments intended for use in educational establishments. Although of little use to professional users, the features of analog instruments are covered in this chapter because students are quite likely to meet these when doing practical work associated with their course. As far as digital oscilloscopes are concerned, the basic type of instrument used is known as a digital storage oscilloscope. More recently, digital phosphor oscilloscopes have been introduced, which have a capability of detecting and recording rapid transients in voltage signals. A third type is the digital sampling oscilloscope, which is able to measure very high-frequency signals. A fourth and final type is the PC-based oscilloscope, which is effectively an add-on unit to a standard PC. All of these different types of oscilloscopes are discussed in [Section 10.3.3](#).

This chapter ends by looking at the newer forms of output display that have emerged in recent years. These new techniques include electronic displays and computer monitor displays.

10.2.1 Digital meters

All types of digital meters are basically modified forms of the *digital voltmeter (DVM)*, irrespective of the quantity that they are designed to measure. Digital meters designed to measure quantities other than voltage are in fact digital voltmeters that contain appropriate electrical circuits to convert current or resistance measurement signals into voltage signals. *Digital multimeters* are also essentially digital voltmeters that contain several conversion circuits, thus allowing the measurement and display of voltage, current, and resistance magnitudes within one instrument.

Digital meters have been developed to satisfy a need for higher measurement accuracies and a faster speed of response to voltage changes than can be achieved with analog instruments. They are technically superior to analog meters in almost every respect.

The binary nature of the output reading from a digital instrument can be readily applied to a display that is in the form of discrete numerals. Where human operators are required to measure and record signal voltage levels, this form of output makes an important contribution to measurement reliability and accuracy, since the problem of analog meter parallax error is eliminated and the possibility of gross error through misreading the meter output is greatly reduced. The availability in many instruments of a direct output in digital form is also very useful in the rapidly expanding range of computer control applications. Quoted inaccuracy values are between $\pm 0.005\%$ (measuring DC voltages) and $\pm 2\%$. Digital meters also have very high input impedance (10 M Ω compared with 1–20 K Ω for analog meters), which avoids the measurement system loading problem (see Chapter 3) that frequently occurs when analog meters are used. Additional advantages of digital meters are their ability to measure signals of frequency up to 1 MHz and the common inclusion of features such as automatic ranging, which prevents overload and reverse polarity connection, etc.

The major part of a digital voltmeter is the circuitry that converts the analog voltage being measured into a digital quantity. As the instrument only measures DC quantities in its basic mode, another necessary component within it is one that performs AC–DC conversion and thereby gives it the capacity to measure AC signals. After conversion, the voltage value is displayed by means of indicating tubes or a set of solid-state light-emitting diodes. Four-, five-, or even six-figure output displays are commonly used, and although the instrument itself may not be inherently more accurate than some analog types, this form of display enables measurements to be recorded with much greater accuracy than that obtainable by reading an analog meter scale.

Digital voltmeters differ mainly in the technique used to effect the analog-to-digital conversion between the measured analog voltage and the output digital reading. As a general rule, the more expensive and complicated conversion methods achieve a faster conversion speed. Some common types of DVM are discussed below.

Voltage-to-time conversion digital voltmeter: This is the simplest form of DVM and is a ramp type of instrument. When an unknown voltage signal is applied to the input terminals of the instrument, a negative-slope ramp waveform is generated internally and compared with the input signal. When the two are equal, a pulse is generated that opens a gate, and at a later point in time a second pulse closes the gate when the negative ramp voltage reaches zero. The length of time between the gate opening and closing is monitored by an electronic counter, which produces a digital display according to the level of the input voltage signal. Its main drawbacks are nonlinearities in the shape of the ramp waveform used and lack of noise rejection, and these problems lead to a typical inaccuracy of $\pm 0.05\%$. It is relatively cheap, however.

Potentiometric digital voltmeter: This uses a servo principle, in which the error between the unknown input voltage level and a reference voltage is applied to a servo-driven potentiometer that adjusts the reference voltage until it balances the unknown voltage. The output reading is produced by a mechanical drum-type digital display driven by the potentiometer. This is also a relatively cheap form of DVM that gives excellent performance for its price.

Dual-slope integration digital voltmeter: This is another relatively simple form of DVM that has better noise-rejection capabilities than many other types and give correspondingly better measurement accuracy (inaccuracy as low as $\pm 0.005\%$). Unfortunately, it is quite expensive. The unknown voltage is applied to an integrator for a fixed time T_1 , following which a reference voltage of opposite sign is applied to the integrator, which discharges down to a zero output in an interval T_2 measured by a counter. The output-time relationship for the integrator is shown in Fig. 10.1, from which the unknown voltage V_i can be calculated geometrically from the triangle as:

$$V_i = V_{ref}(T_1 / T_2) \quad (10.1)$$

Voltage-to-frequency conversion digital voltmeter: In this instrument, the unknown voltage signal is fed via a range switch and an amplifier into a converter circuit whose output is in the form of a train of voltage pulses at a frequency proportional to the magnitude of the input signal. The main advantage of this type of DVM is its ability to reject AC noise.

Digital multimeter: This is an extension of the DVM. It can measure both AC and DC voltages over a number of ranges through inclusion within it of a set of switchable amplifiers and attenuators. It is widely used in circuit test applications as an alternative to the analog multimeter, and includes protection circuits that prevent damage if high voltages are applied to the wrong range.

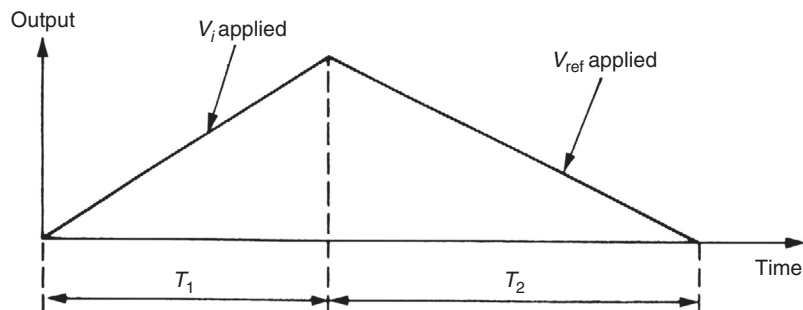


Figure 10.1

Output-time relationship for integrator in a dual-slope digital voltmeter (DVM).

10.2.2 Analog meters

Despite the technical superiority of digital meters, particularly in terms of their greater accuracy and much higher input impedance, analog meters continue to be used in a significant number of applications. Firstly, they are often preferred as indicators in system control panels, where they are known as **panel meters**. This is because deviations of controlled parameters away from the normal expected range are spotted more easily by a pointer moving against a scale in an analog meter rather than by variations in the numeric output display of a digital meter. Analog instruments also tend to suffer less from noise and isolation problems, which favors their use in some applications. In addition, because analog instruments are usually passive instruments that do not need a power supply, this is often very useful in measurement applications where a suitable mains power supply is not readily available. Many examples of analog meters also remain in use for historical reasons. A typical, commercially available analog panel meter is shown in Fig. 10.2.

Analog meters are electromechanical devices that drive a pointer against a scale. They are prone to measurement errors from a number of sources that include inaccurate scale marking during manufacture, bearing friction, bent pointers, and ambient temperature variations. Further human errors are introduced through parallax error (not reading the scale from directly above) and mistakes in interpolating between scale markings. Quoted inaccuracy values are between $\pm 0.1\%$ and $\pm 3\%$. Various types of analog meter are used as discussed below.

Moving-coil meter

A moving-coil meter is a very common form of analog voltmeter because of its sensitivity, accuracy, and linear scale, although it only responds to DC signals. As shown schematically in Fig. 10.3, it consists of a rectangular coil wound round a soft iron core that is suspended in the field of a permanent magnet. The signal being measured is applied



Figure 10.2

Eltime analog panel meter. *Reproduced by kind permission of Eltime Controls.*

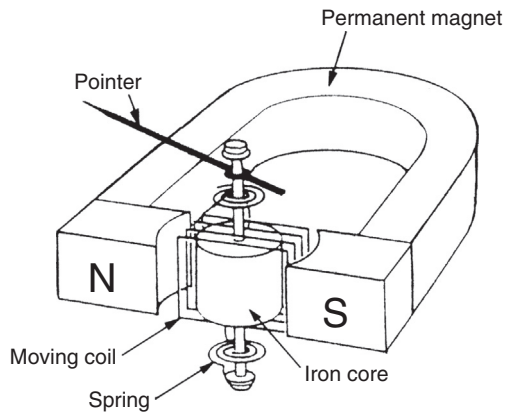


Figure 10.3
Mechanism of moving-coil meter.

to the coil and this produces a radial magnetic field. Interaction between this induced field and the field produced by the permanent magnet causes a torque, which results in rotation of the coil. The amount of rotation of the coil is measured by attaching a pointer to it that moves past a graduated scale. The theoretical torque produced is given by:

$$T = BlhwN \quad (10.2)$$

where B is the flux density of the radial field, I is the current flowing in the coil, h is the height of the coil, w is the width of the coil, and N is the number of turns in the coil. If the iron core is cylindrical and the air gap between the coil and pole faces of the permanent magnet is uniform, then the flux density B is constant, and Eq. (10.2) can be rewritten as:

$$T = KI \quad (10.3)$$

i.e., the torque is proportional to the coil current and the instrument scale is linear.

The torque due to the coil current is opposed by the reaction torque of a torsional spring such that the final rotational displacement of the coil is proportional to the measured current.

As the basic instrument operates at low current levels of one milliamp or so, it is only suitable for measuring voltages up to around 2 volts. If there is a requirement to measure higher voltages, the measuring range of the instrument can be increased by placing a resistance in series with the coil, such that only a known proportion of the applied voltage is measured by the meter. In this situation the added resistance is known as a **shunting resistor**.

While Fig. 10.3 shows the traditional moving-coil instrument with a long U-shaped permanent magnet, many newer instruments employ much shorter magnets made from recently developed magnetic materials such as Alnico and Alcomax. These materials produce a substantially greater flux density, which, besides allowing the magnet to be smaller, has additional advantages in allowing reductions to be made in the size of the coil and in increasing the useable range of deflection of the coil to about 120 degrees. Some versions of the instrument also have either a specially shaped core or specially shaped magnet pole faces to cater for special situations where a nonlinear scale such as a logarithmic one is required.

Moving-iron meter

As well as measuring DC signals, the moving-iron meter can also measure AC signals at frequencies up to 125Hz. It is the cheapest form of meter available and is used in similar numbers to moving-coil meters. The signal to be measured is applied to a stationary coil, and the associated field produced is often amplified by the presence of an iron structure associated with the fixed coil. The moving element in the instrument consists of an iron vane that is suspended within the field of the fixed coil. When the fixed coil is excited, the iron vane turns in a direction that increases the flux through it.

The majority of moving-iron instruments are either of the attraction type or of the repulsion type. A few instruments belong to a third combination type. The attraction type, where the iron vane is drawn into the field of the coil as the current is increased, is shown schematically in Fig. 10.4A. The alternative repulsion type is sketched in Fig. 10.4B. For an excitation current I , the torque produced that causes the vane to turn is given by:

$$T = \frac{I^2 dM}{2d\theta}$$

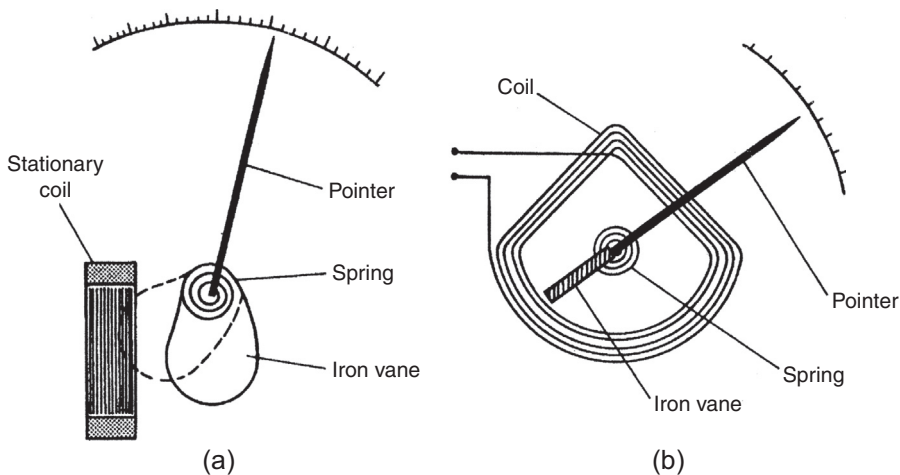


Figure 10.4

Mechanisms of moving-iron meters: (A) attraction-type; (B) repulsion-type.

where M is the mutual inductance and θ is the angular deflection. Rotation is opposed by a spring that produces a backwards torque given by:

$$T_s = K\theta$$

At equilibrium, $T = T_s$, and θ is therefore given by:

$$\theta = \frac{I^2 dM}{2Kd\theta} \quad (10.4)$$

The instrument thus has a square-law response where the deflection is proportional to the square of the signal being measured, i.e., the output reading is a root-mean-squared (**rms**) quantity.

The instrument can typically measure voltages in the range of 0–30 V. However, it can be modified to measure higher voltages by placing a resistance in series with it, as in the case of moving-coil meters. A series resistance is particularly beneficial in AC signal measurements because it compensates for the effect of coil inductance by reducing the total resistance/inductance ratio, and hence measurement accuracy is improved. A switchable series resistance is often provided within the casing of the instrument to facilitate range extension. However, when the voltage measured exceeds about 300 V, it becomes impractical to use a series resistance within the case of the instrument because of heat-dissipation problems, and an external resistance is used instead.

Clamp-on meters

These are used for measuring circuit currents and voltages in a noninvasive manner that avoids having to break the circuit being measured. The meter clamps onto a current-carrying conductor, and the output reading is obtained by transformer action. The principle of operation is illustrated in Fig. 10.5, where it can be seen that the clamp-on jaws of the instrument act as a transformer core and the current-carrying conductor acts as a primary winding. Current induced in the secondary winding is rectified and applied to a moving-coil meter. Although it is a very convenient instrument to use, the clamp-on meter has low sensitivity and the minimum current measurable is usually about one amp.

Analog multimeter

The analog multimeter is now less common than its counterpart, the digital multimeter, but is still widely available. It is a multifunction instrument that can measure current and resistance as well as DC and AC voltage signals. Basically, the instrument consists of a moving-coil analog meter with a switchable bridge rectifier to allow it to measure AC signals, as shown in Fig. 10.6. A set of rotary switches allows the selection of various series and shunt resistors, which make the instrument capable of measuring both voltage and current over a number of ranges. An internal power source is also provided to allow it

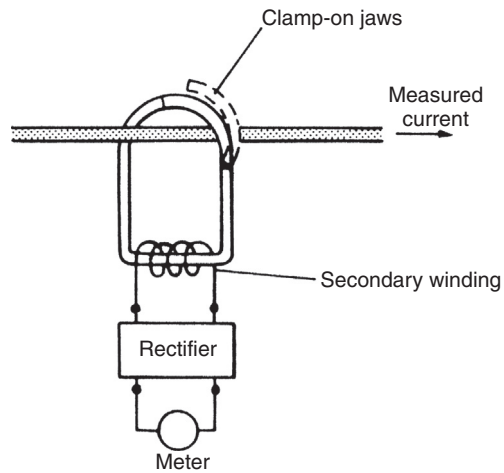


Figure 10.5
Schematic drawing of clamp-on meter.

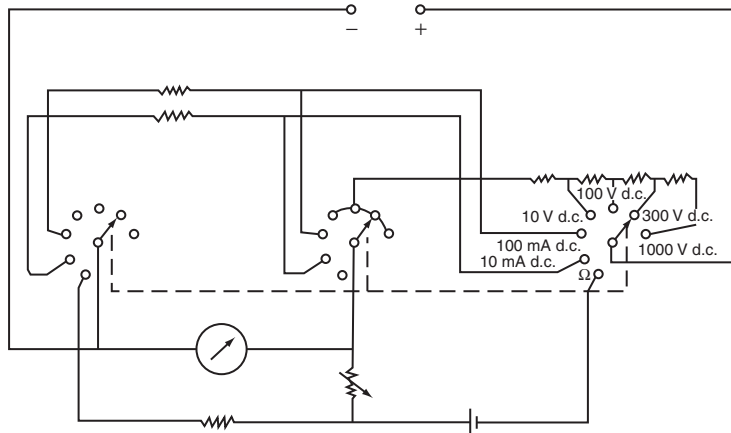
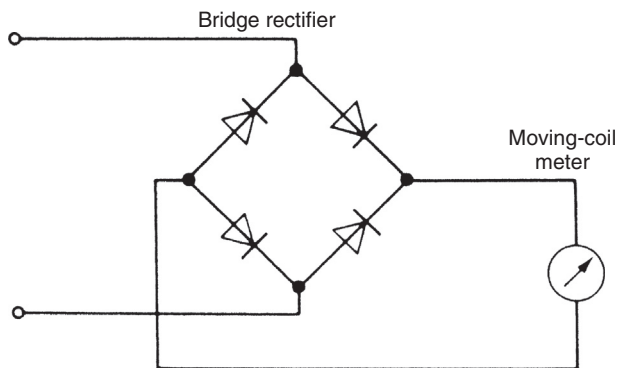


Figure 10.6
Circuitry of analog multimeter.

to measure resistances as well. While this instrument is very useful for giving an indication of voltage levels, the compromises in its design that enable it to measure so many different quantities necessarily mean that its accuracy is not as good as instruments that are purpose-designed to measure just one quantity over a single measuring range.

Measuring high-frequency signals with analog meters

One major limitation in using analog meters for AC voltage measurement is that the maximum frequency measurable directly is low, 2 KHz for the dynamometer voltmeter and only 100 Hz in the case of the moving-iron instrument. A partial solution to this limitation is to rectify the voltage signal and then apply it to a moving-coil meter, as


Figure 10.7

Measurement of high-frequency voltage signals.

shown in Fig. 10.7. This extends the upper measurable frequency limit to 20 KHz. However, the inclusion of the bridge rectifier makes the measurement system particularly sensitive to environmental temperature changes, and nonlinearities significantly affect measurement accuracy for voltages that are small relative to the full-scale value.

Calculation of meter outputs for nonstandard waveforms

The two examples below provide an exercise in calculating the output reading from various types of analog voltmeter. These examples also serve as a useful reminder of the mode of operation of each type of meter and the form that the output takes.

■ Example 10.1

Calculate the reading that would be observed on a moving-coil ammeter when it is measuring the current in the circuit shown in Fig. 10.8.

■ Solution

A moving-coil meter measures mean current.

$$\begin{aligned}
 I_{mean} &= \frac{1}{2\pi} \left(\int_0^\pi \frac{5\omega t}{\pi} d\omega t + \int_\pi^{2\pi} 5 \sin(\omega t) d\omega t \right) = \frac{1}{2\pi} \left(\left[\frac{5(\omega t)^2}{2\pi} \right]_0^\pi + 5[-\cos(\omega t)]_\pi^{2\pi} \right) \\
 &= \frac{1}{2\pi} \left(\frac{5\pi^2}{2\pi} - 0 - 5 - 5 \right) = \frac{1}{2\pi} \left(\frac{5\pi}{2} - 10 \right) = \frac{5}{2\pi} \left(\frac{\pi}{2} - 2 \right) = -0.342 \text{ amps}
 \end{aligned}$$

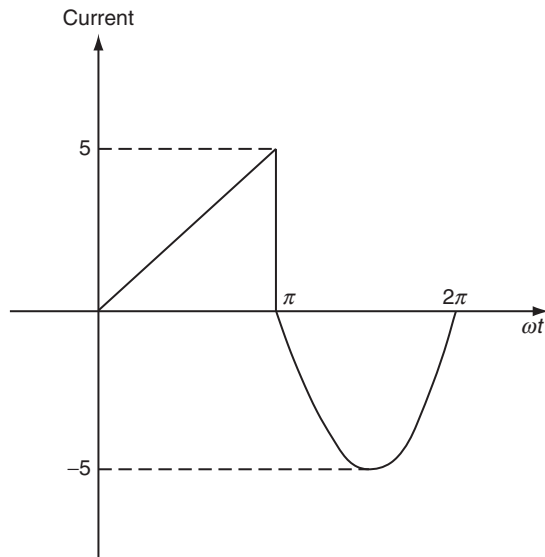


Figure 10.8

Circuit for Examples 10.1 and 10.2.

Example 10.2

Calculate the reading that would be observed on a moving-iron ammeter when it is measuring the current in the circuit shown in Fig. 10.8.

Solution

A moving-iron meter measures rms current.

$$\begin{aligned}
 I_{rms}^2 &= \frac{1}{2\pi} \left(\int_0^{\pi} \frac{25(\omega t)^2}{\pi^2} d\omega t + \int_{\pi}^{2\pi} 25 \sin^2(\omega t) d\omega t \right) \\
 &= \frac{1}{2\pi} \left(\int_0^{\pi} \frac{25(\omega t)^2}{\pi^2} d\omega t + \int_{\pi}^{2\pi} \frac{25(1 - \cos 2\omega t)}{2} d\omega t \right) \\
 &= \frac{25}{2\pi} \left(\left[\frac{(\omega t)^3}{3\pi^2} \right]_0^{\pi} + \left[\frac{\omega t}{2} - \frac{\sin 2\omega t}{4} \right]_{\pi}^{2\pi} \right) = \frac{25}{2\pi} \left(\frac{\pi}{3} + \frac{2\pi}{2} - \frac{\pi}{2} \right) \\
 &= \frac{25}{2\pi} \left(\frac{\pi}{3} + \frac{\pi}{2} \right) = \frac{25}{2} \left(\frac{1}{3} + \frac{1}{2} \right) = 10.416
 \end{aligned}$$

Thus, $I_{rms} = \sqrt{(I_{rms}^2)} = 3.23$ amp

10.2.3 Oscilloscopes

The oscilloscope is probably the most versatile and useful instrument available for signal measurement and display. The example shown in Fig. 10.9 is a digital model that is capable of displaying up to eight signals simultaneously. While oscilloscopes still exist in both analog and digital forms, analog models tend to be found more often in the form of low-specification, low-cost instruments that are produced for educational use in schools, colleges and universities. Most oscilloscopes used for professional work now tend to be digital models. These can be divided into digital storage oscilloscopes, digital phosphor oscilloscopes, and digital sampling oscilloscopes.

The basic function of an oscilloscope is to draw a graph of an electrical signal. In the most common mode of usage, the y -axis (vertical) of the display represents the voltage of a measured signal and the x -axis (horizontal) represents time. Thus, the basic output display is a graph of the variation of the magnitude of the measured voltage with time.

The oscilloscope is able to measure a very wide range of both AC and DC voltage signals, and is used particularly as an item of test equipment for circuit fault-finding. Besides measuring voltage levels, it can also measure other quantities such as the frequency and phase of a signal. It can also indicate the nature and magnitude of noise that may be corrupting the measurement signal. The most expensive models can measure signals at frequencies up to 25 GHz, while the very cheapest models can only measure signals up to 10 MHz. One particularly strong merit of the oscilloscope is its high input



Figure 10.9

Example of a digital oscilloscope. Photograph reproduced by kind permission of Tektronix UK Ltd..

impedance, typically $1\text{ M}\Omega$, which means that the instrument has a negligible loading effect in most measurement situations. As a test instrument, it is often required to measure voltages whose frequency and magnitude are totally unknown. The set of rotary switches that alter its time base so easily, and the circuitry that protects it from damage when high voltages are applied to it on the wrong range, make it ideally suited for such applications. However, it is not a particularly accurate instrument and is best used where only an approximate measurement is required. In the best instruments, inaccuracy can be limited to $\pm 1\%$ of the reading but inaccuracy can approach $\pm 5\%$ in the cheapest instruments.

The most important aspects in the specification of an oscilloscope are its bandwidth, its rise time, and its accuracy. The bandwidth is defined as the range of frequencies over which the oscilloscope amplifier gain is within 3 dB ¹ of its peak value, as illustrated in Fig. 10.10. The -3 dB point is where the gain is 0.707 times its maximum value. In most oscilloscopes, the amplifier is direct-coupled, which means that it amplifies DC voltages by the same factor as low-frequency AC ones. For such instruments, the minimum frequency measurable is zero and the bandwidth can be interpreted as the maximum frequency where the sensitivity (deflection/volt) is within 3 dB of the peak value. In all measurement situations, the oscilloscope chosen for use must be such that the maximum frequency to be measured is well within the bandwidth. The -3 dB specification means that an oscilloscope with a specified inaccuracy of $\pm 2\%$ and bandwidth of 100 MHz will have an inaccuracy of $\pm 5\%$ when measuring 30 MHz signals, and this inaccuracy will

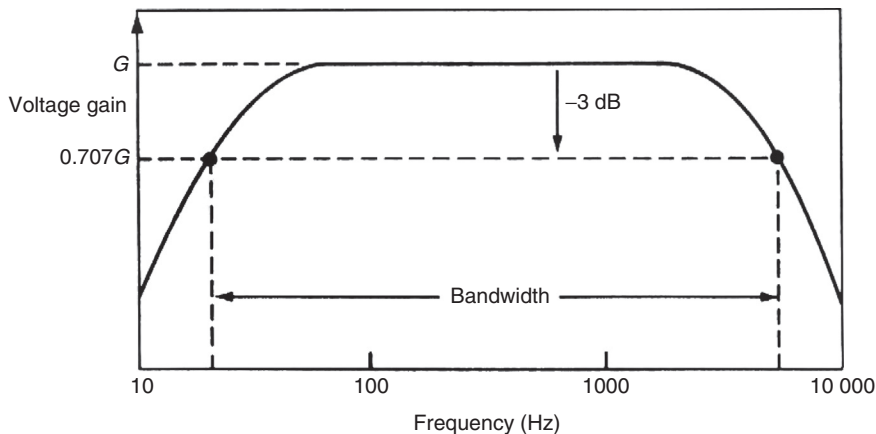


Figure 10.10
Bandwidth.

¹ The decibel, commonly written dB, is used to express the ratio between two quantities. For two voltage levels V_1 and V_2 , the difference between the two levels is expressed in decibels as $20\log_{10}(V_1/V_2)$. It follows from this that $20\log_{10}(0.7071) = -3\text{ dB}$.

increase still further at higher frequencies. Thus, when applied to signal-amplitude measurement, the oscilloscope is only useable at frequencies up to about 0.3 times its specified bandwidth.

The rise time is the transit time between the 10% and 90% levels of the response when a step input is applied to the oscilloscope. Oscilloscopes are normally designed such that:

$$\text{Bandwidth} \times \text{Rise time} = 0.35$$

Thus, for a bandwidth of 100 MHz, rise time = $0.35/100,000,000 = 3.5$ ns.

All oscilloscopes are relatively complicated instruments that are constructed from a number of subsystems, and it is necessary to consider each of these in turn in order to understand how the complete instrument functions.

Channel: One channel describes the basic subsystem in the instrument for measuring one voltage input signal. This subsystem is usually duplicated one or more times within the instrument to provide a capability of displaying two or more signals at the same time on the screen. An oscilloscope with two channels can therefore display two separate signals simultaneously.

Single-ended input: This type of input only has one input terminal plus a ground terminal per oscilloscope channel and, consequently, only allows signal voltages to be measured relative to ground. It is normally only used in simple oscilloscopes.

Differential input: This type of input is provided on more expensive oscilloscopes. Two input terminals plus a ground terminal are provided for each channel, which allows the potentials at two nongrounded points in a circuit to be compared. This type of input can also be used in single-ended mode to measure a signal relative to ground by using just one of the input terminals plus ground.

Input coupling: Coupling describes the method used to connect each measured electrical signal into the oscilloscope. It can be switched between DC, AC, and GROUND.

DC coupling displays all of the input signal.

AC coupling filters out the DC component (bias) in the input signal so that the waveform displayed is centered about 0 volts.

GROUND is only used temporarily. It disconnects the input signal to allow the 0 volts position to be displayed on the screen (as a horizontal line).

Vertical sensitivity control: This consists of a series of attenuators and preamplifiers at the input to the oscilloscope. These condition the measured signal to the optimum magnitude for input to the main amplifier, thus enabling the instrument to measure a very wide range of different signal magnitudes. Selection of the appropriate input

amplifier/attenuator is made by setting a *volts/div* control associated with each oscilloscope channel. This defines the magnitude of the input signal that will cause a deflection of one division on the screen.

Horizontal sensitivity control (timebase): This sets the rate at which the waveform is drawn across the screen and is often known as the timebase. It defines the time represented by one horizontal division on the display screen.

Display position control: This allows the position at which a signal is displayed on the screen to be controlled in two ways. The horizontal position is adjusted by a *horizontal position* knob on the oscilloscope front panel, and similarly a *vertical position* knob controls the vertical position. These controls adjust the position of the display by biasing the measured signal with DC voltage levels. There is a separate vertical position control for each channel in the oscilloscope so that the trace belonging to each input can be easily identified. Identification is further facilitated in most digital oscilloscopes by displaying each signal in a different color.

Trigger system: The trigger system allows a repeating signal waveform (for example, a sine wave) to appear stationary on the oscilloscope screen, thus facilitating analysis of the signal. It achieves this by synchronizing the horizontal sweep of the display to the same point on the input signal waveform. The various trigger modes available vary from one oscilloscope to the next, and the reader requiring further information is advised to consult the instruction manual for whatever instrument is currently being used.

Analog oscilloscope (Cathode ray oscilloscope)

Analog oscilloscopes were originally called cathode ray oscilloscopes because a fundamental component within them is a cathode ray tube. In recent times, digital oscilloscopes have largely replaced analog versions in professional use. A particular attribute of analog oscilloscopes is that they are less expensive than digital ones. This means that they still find some usage in schools, colleges, and universities. However, the low cost of basic analog models is their only real merit, since the inclusion of a cathode ray tube makes them very fragile, and the technical performance of digital equivalents is greatly superior.

Digital storage oscilloscopes

Digital storage oscilloscopes are the most basic form of digital oscilloscopes, but even these usually have the ability to perform extensive waveform processing and provide permanent storage of measured signals. When first created, a digital storage oscilloscope consisted of a conventional analog cathode ray oscilloscope with the added facility that the measured analog signal could be converted to digital format and stored in computer

memory within the instrument. This stored data could then be reconverted to analog form at the frequency necessary to refresh the analog display on the screen, producing a nonfading display of the signal on the screen.

While examples of such early digital oscilloscopes might still be found in some workplaces, modern digital storage oscilloscopes no longer use cathode ray tubes and are entirely digital in construction and operation. The block diagram in Fig. 10.11 shows the typical components used in the digital storage oscilloscope. The first component (as in an analog oscilloscope) is an amplifier/attenuator unit that allows adjustment of the magnitude of the input voltage signal to an appropriate level. This is followed by an analog-to-digital converter that samples the input signal at discrete points in time. The sampled signal values are stored in the acquisition memory component before passing into a microprocessor. This carries out signal-processing functions, manages the front panel control settings, and prepares the output display. Following this, the output signal is stored in a display memory module before being output to the display itself. This consists of either a monochrome or multicolor liquid crystal display (see Section 10.3.4). The signal displayed is actually a sequence of individual dots rather than a continuous line as displayed by an analog oscilloscope. However, as the density of dots increases, the display becomes closer and closer to a continuous line. The density of the dots is entirely dependent upon the sampling rate at which the analog signal is digitized and the rate at which the memory contents are read to reconstruct the original signal. As the speed of sampling and signal processing is a function of instrument cost, more expensive instruments give better performance in terms of dot density and the accuracy with which the analog signal is recorded and represented. Nevertheless, the cost of computing power is now sufficiently low to mean that all but the very cheapest instruments now have a display that looks very much like a continuous trace.

Besides their ability to display the magnitude of voltage signals and other parameters such as signal phase and frequency, most digital oscilloscopes can also carry out analysis

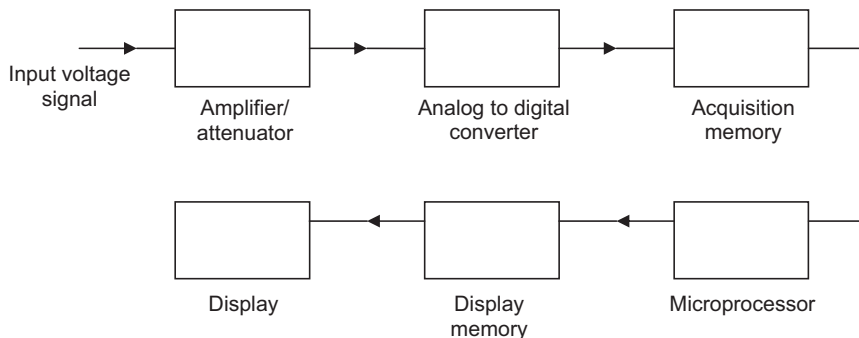


Figure 10.11
Components of a digital storage oscilloscope.

of the measured waveform and compute signal parameters such as maximum and minimum signal levels, peak-peak values, mean values, rms values, rise time, and fall time. These additional functions are controlled by extra knobs and push buttons on the front panel. They are also ideally suited to capturing transient signals when set to single-sweep mode. This avoids the problem of the very careful synchronization that is necessary to capture such signals on an analog oscilloscope. In addition, digital oscilloscopes often have facilities to output analog signals to devices like chart recorders and output digital signals in a form that is compatible with standard interfaces like IEEE488 and RS232.

The principal limitation of a digital storage oscilloscope is that the only signal information captured is the status of the signal at each sampling instant. Thereafter, no new signal information is captured during the time that the previous sample is being processed. This means that any signal changes occurring between sampling instants, such as fast transients, are not detected. This problem is overcome in the digital phosphor oscilloscope.

Digital phosphor oscilloscope

This newer type of oscilloscope, first introduced in 1998, uses a parallel-processing architecture rather than the serial-processing architecture found in digital storage oscilloscopes. The components of the instrument are shown schematically in Fig. 10.12. The amplifier/attenuator and analog-to-digital converter are the same as in a digital storage oscilloscope. However, the signal-processing mechanism is substantially different. The output from the analog-to-digital converter passes into a digital phosphor memory unit, which is in fact entirely electronic and not composed of chemical phosphor as its name might imply. Thereafter, data follows two parallel paths. Firstly, a microprocessor processes the data acquired at each sampling instant according to the settings on the

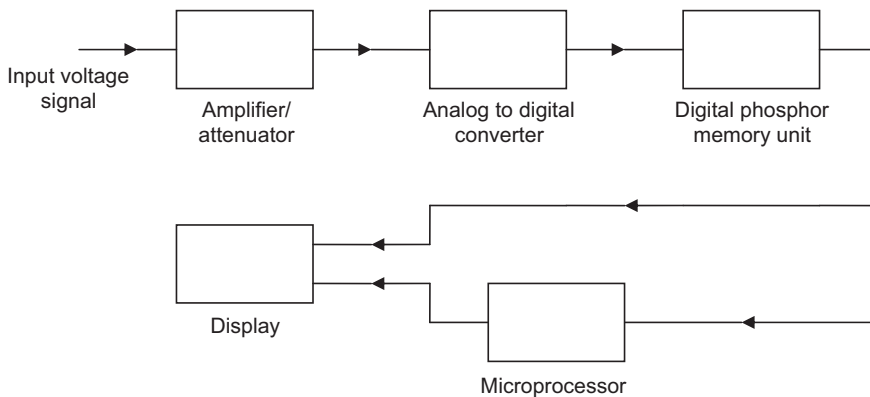


Figure 10.12
Components of a digital phosphor oscilloscope.

control panel and sends the processed signal to the instrument display unit. In addition to this, a snapshot of the input signal is sent directly to the display unit at a rate of 30 images per second. This enhanced processing capability enables the instrument to have a higher waveform capture rate and to detect very fast signal transients that are missed by digital storage oscilloscopes.

Digital sampling oscilloscope

The digital sampling oscilloscope has a bandwidth of up to 25 GHz, which is about 10 times better than that achieved by other types of oscilloscope. This increased bandwidth is achieved by reversing the positions of the analog-to-digital converter and the amplifier, as shown in the block diagram in Fig. 10.13. This reversal means that the sampled signal applied to the amplifier has a much lower frequency than the original signal, allowing the use of a low-bandwidth amplifier. However, the fact that the input signal is applied directly to the analog-to-digital converter without any scaling means that the instrument can only be used to measure signals whose peak magnitude is within a relatively small range of typically 1 V peak-peak. By contrast, both digital storage and digital phosphor oscilloscopes can typically deal with inputs up to 500 V.

PC-based oscilloscope

A PC-based oscilloscope consists of a hardware unit that connects to a standard PC via either a USB or a parallel port. The hardware unit provides the signal scaling, analog-to-digital conversion, and buffer memory functions found in a conventional oscilloscope. More expensive PC-based oscilloscopes also provide some high-speed digital signal-processing functions within the hardware unit. The host PC itself provides the control interface and display facilities.

The primary advantage of a PC-based oscilloscope over other types is one of cost, the cost savings being achieved because the use of the PC obviates the need for the display unit

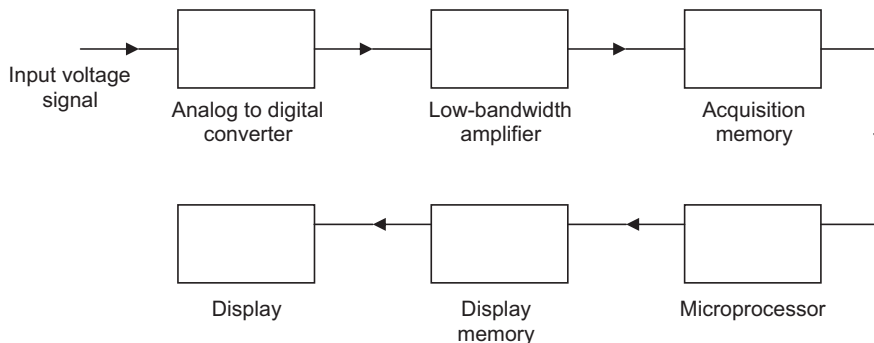


Figure 10.13
Components of a digital sampling oscilloscope.

and the front control panel found in other forms of oscilloscopes. The larger size of a PC display compared with a conventional oscilloscope often makes the output display easier to read. A further advantage is one of portability, since a laptop plus add-on hardware unit is usually smaller and lighter than a conventional oscilloscope. PC-based oscilloscopes also facilitate the transfer of output data into standard PC software such as spreadsheets and word processors.

Although pc-based oscilloscopes have a number of advantages over conventional oscilloscopes, they also have disadvantages. Firstly, electromagnetic noise originating in PC circuits requires the hardware unit to be well shielded in order to avoid corruption of the measured signal. Secondly, signal sampling rates can be limited by the mode of connection of the hardware unit into the PC.

10.2.4 Electronic output displays

Electronic displays enable a parameter value to be read immediately, thus allowing for any necessary response to be made immediately. The main requirement for displays is that they should be clear and unambiguous. Two common types of character format used in displays, seven-segment and 7×5 dot matrix, are shown in Fig. 10.14. Both types of display have the advantage of being able to display alphabetic as well as numeric information, although the seven-segment format can only display a limited 9-letter subset of the full 26-letter alphabet. This allows added meaning to be given to the number displayed by including a word or letter code. It also allows a single display unit to send information about several parameter values, cycling through each in turn and including alphabetic information to indicate the nature of the variable currently displayed.

Electronic output units usually consist of a number of side-by-side cells, where each cell displays one character. Generally, these accept either serial or parallel digital input signals, and the input format can be either binary-coded decimal (BCD) or ASCII. Technologies used for the individual elements in the display are either light-emitting diodes (LEDs) or liquid-crystal elements.

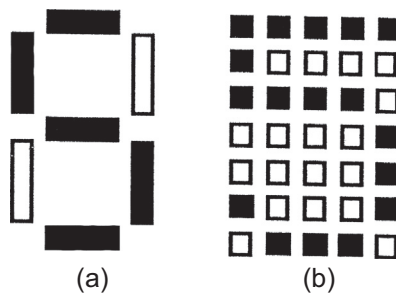


Figure 10.14

Character formats used in electronic displays: (A) seven-segment; (B) 7×5 dot matrix.

10.2.5 Computer monitor displays

Now that computers are part of the furniture in most homes, the ability of computers to display information is widely understood and appreciated. Computers are now both cheap and highly reliable, and they provide an excellent mechanism for both displaying and storing information. As well as alphanumeric displays of industrial plant variable and status data, for which the plant operator can vary the size of font used to display the information at will, it is also relatively easy to display other information such as plant layout diagrams, process flow layouts, etc. This allows not only the value of parameters that go outside control limits to be displayed but also their location on a schematic map of the plant. Graphical displays of the behavior of a measured variable are also possible. However, this poses a difficulty when there is a requirement to display the variable's behavior over a long period of time since the length of the time axis is constrained by the size of the monitor's screen. To overcome this, the display resolution has to decrease as the time period of the display increases.

Touch screens have the ability to display the same sort of information as a conventional computer monitor, but they also provide a command-input facility in which the operator simply has to touch the screen at points where images of keys or boxes are displayed. A full "qwerty" keyboard is often provided as part of the display. The sensing elements behind the screen are protected by the glass and continue to function even if the glass gets scratched. Touch screens are usually totally sealed, and thus provide intrinsically safe operation in hazardous environments.

10.3 Recording of measurement data

As well as displaying the current values of measured parameters, there is often a need to make continuous recordings of measurements for later analysis. Such records are particularly useful when faults develop in systems, as analysis of the changes in measured parameters in the time before the fault is discovered can often quickly indicate the reason for the fault. The options for recording data include chart recorders, digital oscilloscopes, digital data recorders, and hard copy devices such as inkjet and laser printers. The various types of recorders used are discussed below.

10.3.1 Chart recorders

Chart recorders have particular advantages in providing a noncorruptible record that has the merit of instant "viewability." This means that all but paperless forms of chart recorders satisfy regulations set for many industries that require variables to be monitored and recorded continuously with hard copy output. ISO9000 quality assurance procedures and ISO14000 environmental protection systems set similar requirements, and special

regulations in the defense industry go even further by requiring hard copy output to be kept for 10 years. Hence, while many people have been predicting the demise of chart recorders, the reality of the situation is that they are likely to be needed in many industries for many years to come.

Originally, all chart recorders were electromechanical in operation and worked on the same principle as a moving-coil meter (see analog meters in [Section 10.2.2](#)) except that the moving coil to which the measured signal was applied carried a pen as shown in [Fig. 10.15](#) rather than carrying a pointer moving against a scale as it would do in a meter. The pen drew an ink trace on a strip of ruled chart paper that was moved past the pen at constant speed by an electrical motor. The resultant trace on the chart paper showed the variations with time in the magnitude of the measured signal. Even early recorders commonly had two or more pens of different colors so that several measured parameters could be recorded simultaneously.

The first improvement to this basic recording arrangement was to replace the galvanometric mechanism with a servo system, as shown in [Fig. 10.16](#), in which the pen is driven by a servomotor, and a sensor on the pen feeds back a signal proportional to pen position. In this form, the instrument is known as a *potentiometric recorder*. The servo system reduces the typical inaccuracy of the recorded signal to $\pm 0.1\%$, compared to $\pm 2\%$ in a galvanometric mechanism recorder. Typically, the measurement resolution is around 0.2% of the full scale reading. Originally, the servo motor was a standard DC motor, but brushless servomotors are now invariably used to avoid the commutator problems that occur with DC motors. The position signal is measured by a potentiometer in cheaper

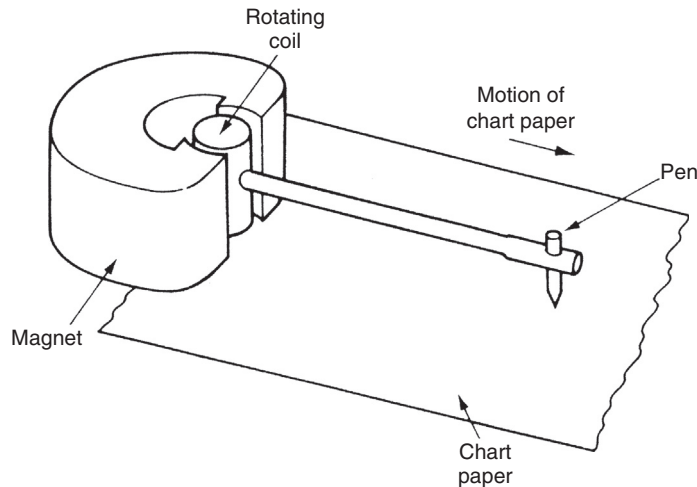


Figure 10.15

Original form of galvanometric chart recorder.

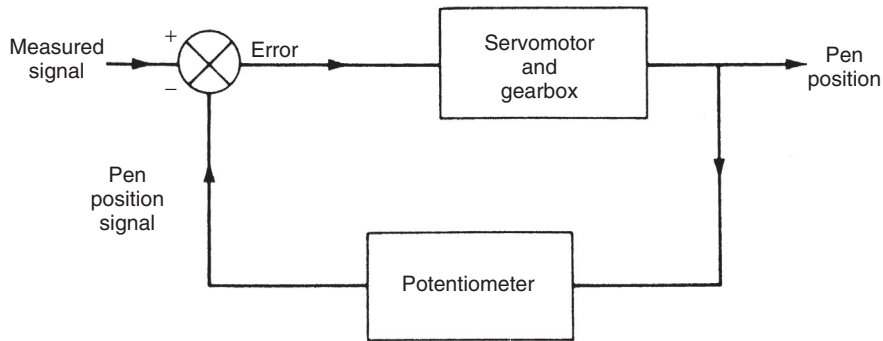


Figure 10.16

Servo system of potentiometric chart recorder.

models, but more expensive models achieve better performance and reliability by using a noncontacting ultrasonic sensor to provide feedback on pen position. The difference between the pen position and the measured signal is applied as an error signal that drives the motor. One consequence of this electromechanical balancing mechanism is that the instrument has a slow response time in the range of 0.2–2.0 seconds, which means that electromechanical potentiometric recorders are only suitable for measuring DC and slowly time-varying signals.

All potentiometric chart recorders currently available contain a microprocessor controller, where the functions vary according to the particular chart recorder. Common functions are selection of range and chart speed, and also specification of alarm modes and levels to detect when measured variables go outside acceptable limits. Basic recorders can record up to three different signals using three different colored pens. However, multipoint recorders can have 24 or more inputs and plot 6 or more different colored traces simultaneously. Another common device is the circular chart recorder, in which the chart paper is circular in shape and is rotated rather than moving translationally. Finally, paperless forms of recorders exist where the output display is generated entirely electronically. These various forms are discussed in more detail below.

Pen strip chart recorder

The pen strip chart recorder refers to the basic form of electromechanical potentiometric chart recorder mentioned earlier. It is also called a **hybrid chart recorder** by some manufacturers. The word *hybrid* was originally used to differentiate chart recorders that had a microprocessor controller from those that did not. However, since all chart recorders now contain a microprocessor, the term *hybrid* has become superfluous.

Strip chart recorders typically have up to four inputs and four pens in different colors, allowing up to four different signals to be recorded. A typical commercially available



Figure 10.17

Honeywell DPR100 strip chart recorder. *Reproduced by kind permission of Honeywell International, Inc.*

model is shown in Fig. 10.17. Chart paper comes in either roll or fan-fold form. The drive mechanism can be adjusted to move the chart paper at different speeds. The fastest speed is typically 6000 mm/h and the slowest is typically 1 mm/h.

As well as recording signals as a continuous trace, many models also allow for the printing of alphanumeric data on the chart to record date, time, and other process information. Some models also have a digital numeric display to provide information on the current values of recorded variables.

Multipoint strip chart recorder

The multipoint strip chart recorder is a modification of the pen strip chart recorder that uses a dot matrix print head striking against an ink ribbon instead of pens. A typical model might allow up to 24 different signal inputs to be recorded simultaneously, using a six-color ink ribbon. Certain models of such recorders also have the same enhancements as pen strip chart recorders in terms of printing alphanumeric information on the chart and providing a digital numeric output display.

Circular chart recorder

The circular chart recorder consists of a servo-driven pen assembly that records the measured signal on a rotating circular paper chart, as shown in Fig. 10.18. The rotational speed of the chart can be typically adjusted between one revolution in 1 hour to one revolution in 31 days. Recorded charts are replaced and stored after each revolution, which means replacement intervals that vary between hourly and monthly according to the chart

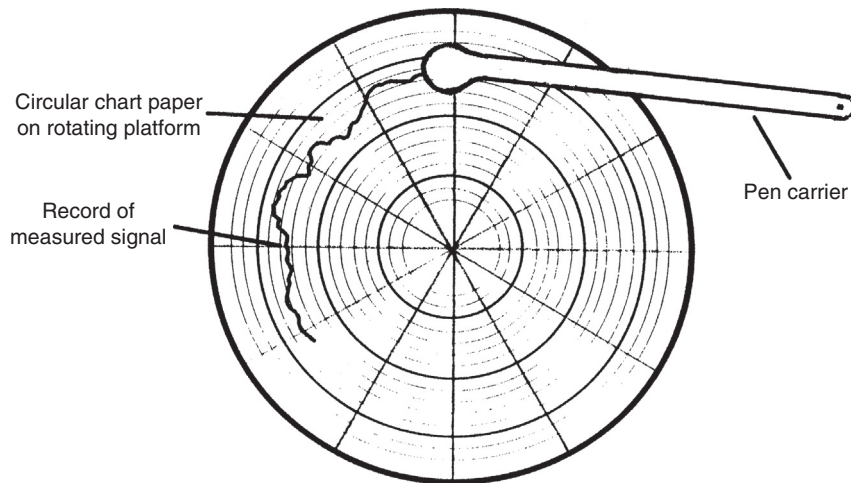


Figure 10.18
Circular chart recorder.

speed. The major advantage of the circular chart recorder over other forms is compactness. Some models have up to four different-colored pen assemblies, allowing up to four different parameters to be recorded simultaneously.

Paperless chart recorder

A paperless chart recorder, sometimes alternatively called a *virtual chart recorder* or a *digital chart recorder*, displays the time history of measured signals electronically, using a color-matrix liquid crystal display. This avoids the chore of periodically replacing chart paper and ink cartridges associated with other forms of chart recorder. Reliability is also enhanced compared with electromechanical recorders. As well as displaying the most recent time history of measured signals on its screen, the instrument also stores a much larger past history. This stored data can be recalled in batches and redisplayed on the screen as required. The only downside compared with other forms of chart recorder is this limitation of only displaying one screenful of information at a time. Of course, conventional recorders allow the whole past history of signals to be viewed at the same time on the hard copy, paper recordings. Otherwise, specifications are very similar to other forms of chart recorder, with vertical motion of the screen display varying between 1 and 6000 mm/h, typical inaccuracy less than $\pm 0.1\%$ and capability of recording multiple signals simultaneously in different colors.

Videographic recorder

The videographic recorder provides exactly the same facilities as a paperless chart recorder but has additional display modes such as bar graphs (histograms) and digital numbers. However, it should be noted that the distinction is becoming blurred between the various

forms of paperless recorder described earlier and videographic recorders as manufacturers enhance the facilities of their instruments. For historical reasons, many manufacturers retain the names that they have traditionally used for their recording instruments but there is now much overlap between their respective capabilities as the functions provided are extended.

10.3.2 Ink-jet and laser printers

Standard computer output devices in the form of ink-jet and laser printers are now widely used as an alternative means of storing measurement system output in paper form. Since a computer is a routine part of many data-acquisition and processing operations, it often makes sense to output the data in a suitable form to a computer printer rather than a chart recorder. This saves the cost of a separate recorder and is facilitated by the ready availability of software that can output measurement data in a graphical format.

10.3.3 Other recording instruments

Many of the display devices mentioned earlier in this chapter also have facilities for storing measurement data digitally. These include data logging acquisition devices and digital storage oscilloscopes. This data can then be converted into hard copy form as required by transferring it to either a chart recorder or a computer and printer.

10.3.4 Digital data recorders

Digital data recorders, also known as *data loggers*, have already been introduced in Chapter 8 in the context of data acquisition. They provide a further alternative way of recording measurement data in digital format. Data so recorded can then be transferred at a future time either to a computer for further analysis, to any of the forms of measurement display devices discussed in [Section 10.2](#), or to one of the hard copy output devices described earlier in this section.

Features contained within a data recorder/data logger obviously vary according to the particular manufacturer/model under discussion. However, most recorders have facilities to handle measurements in the form of both analog and digital signals. Common analog input signals allowed include DC voltages, DC currents, AC voltages, and AC currents. Digital inputs can usually be either in the form of data from digital measuring instruments or discrete data representing events such as switch closures or relay operations. Some models also provide alarm facilities to alert operators to abnormal conditions during data recording operations.

Many data recorders provide special input facilities that are optimized for particular kinds of measurement sensor such as accelerometers, thermocouples, thermistors, resistance

thermometers (RTDs), strain gauges (including strain gauge bridges), linear variable differential transformers, and rotational differential transformers. Some instruments also have special facilities for dealing with inputs from less-common devices like encoders, counters, timers, tachometers, and clocks. A few recorders also incorporate integral sensors when they are designed to measure a particular type of physical variable.

The quality of the data recorded by a digital recorder is a function of the cost of the instrument. Paying more usually means getting more memory to provide a greater data storage capacity, greater resolution in the analog-to-digital converter to give better recording accuracy, and faster data processing to allow a greater data sampling frequency.

10.4 Presentation of data

The two formats available for presenting data on paper are tabular and graphical ones. The relative merits of these are compared below. In some circumstances, it is clearly best to use only one or other of these two alternatives alone. However, in many data collection exercises, part of the measurements and calculations are expressed in tabular form and part graphically, so making best use of the merits of each technique. Very similar arguments apply to the relative merits of graphical and tabular presentations if a computer screen is used for the presentation instead of paper.

10.4.1 Tabular data presentation

A tabular presentation allows data values to be recorded in a precise way that exactly maintains the accuracy to which the data values were measured. In other words, the data values are written down exactly as measured. Besides recording the raw data values as measured, tables often also contain further values calculated from the raw data. An example of a tabular data presentation is given in [Table 10.1](#). This records the results of an experiment to determine the strain induced in a bar of material that is subjected to a range of stresses. Data were obtained by applying a sequence of forces to the end of the bar and using an extensometer to measure the change in length. Values of the stress and strain in the bar are calculated from these measurements and are also included in the table. The final row, which is of crucial importance in any tabular presentation, is the estimate of possible error in each calculated result.

A table of measurements and calculations should conform to several rules as illustrated in [Table 10.1](#):

- (i) The table should have a title that explains what data are being presented within the table.
- (ii) Each column of figures in the table should refer to the measurements or calculations associated with one quantity only.

Table 10.1: Sample tabular presentation of data.

Table of measured applied forces and extensometer readings and calculations of stress and strain				
	Force applied (kN)	Extensometer reading (Divisions)	Stress (N/m ²)	Strain
	0	0	0	0
	2	4.0	15.5	19.8×10^{-5}
	4	5.8	31.0	28.6×10^{-5}
	6	7.4	46.5	36.6×10^{-5}
	8	9.0	62.0	44.4×10^{-5}
	10	10.6	77.5	52.4×10^{-5}
	12	12.2	93.0	60.2×10^{-5}
	14	13.7	108.5	67.6×10^{-5}
Possible error in measurements (%)	±0.2	±0.2	±1.5	$\pm 1.0 \times 10^{-5}$

- (iii) Each column of figures should be headed by a title that identifies the data values contained in the column.
- (iv) The units in which quantities in each column are measured should be stated at the top of the column.
- (v) All headings and columns should be separated by bold horizontal (and sometimes vertical) lines.
- (vi) The errors associated with each data value quoted in the table should be given. The form shown in Table 10.1 is a suitable way to do this when the error level is the same for all data values in a particular column. However, if error levels vary, then it is preferable to write the error boundaries alongside each entry in the table.

10.4.2 Graphical presentation of data

Presentation of data in graphical form involves some compromise in the accuracy to which the data are recorded, as the exact values of measurements are lost. However, graphical presentation has important advantages over tabular presentation.

- Graphs provide a pictorial representation of results that is more readily comprehended than a set of tabular results.
- Graphs are particularly useful for expressing the quantitative significance of results and showing whether a linear relationship exists between two variables. Fig. 10.19 shows a graph drawn from the stress and strain values given in Table 10.1. Construction of the graph involves first of all marking the points corresponding to the stress and strain values. The next step is to draw a line through these data points that best represents the relationship between the two variables. This line will normally be either a straight one or a smooth curve. The data points will not usually lie exactly on this line but instead will lie on either

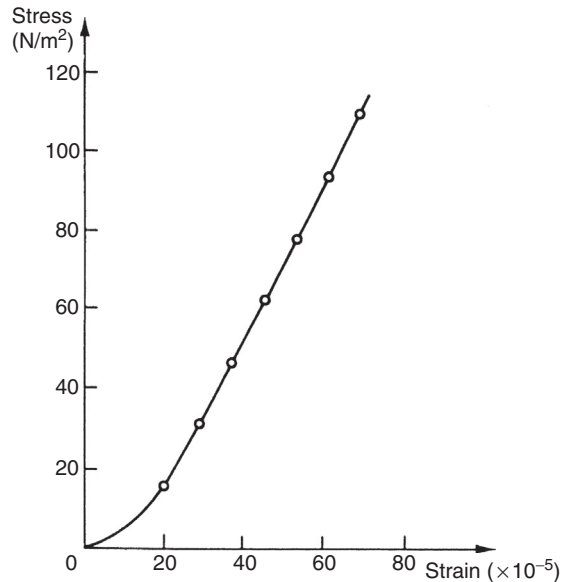


Figure 10.19

Sample graphical presentation of data: graph of stress against strain.

side of it. The magnitude of the excursions of the data points from the line drawn will depend on the magnitude of the random measurement errors associated with the data.

- Graphs can sometimes show up a data point that is clearly outside the straight line or curve that seems to fit the rest of the data points. Such a data point is probably due either to a human mistake in reading an instrument or else to a momentary malfunction in the measuring instrument itself. If the graph shows such a data point where a human mistake or instrument malfunction is suspected, the proper course of action is to repeat that particular measurement and then discard the original data point if the mistake or malfunction is confirmed.

Like tables, the proper representation of data in graphical form has to conform to certain rules:

- The graph should have a title or caption that explains what data are being presented in the graph
- Both axes of the graph should be labeled to express clearly what variable is associated with each axis and to define the units in which the variables are expressed.
- The number of points marked along each axis should be kept reasonably small—about five divisions is often a suitable number
- No attempt should be made to draw the graph outside the boundaries corresponding to the maximum and minimum data values measured, i.e., in Fig. 10.19, the graph stops at a point corresponding to the highest-measured stress value of 108.5.

Fitting curves to data points on a graph

The procedure of drawing a straight line or smooth curve as appropriate that passes close to all data points on a graph, rather than joining the data points by a jagged line that passes through each data point, is justified on account of the random errors that are known to affect measurements. Any line between the data points is mathematically acceptable as a graphical representation of the data if the maximum deviation of any data point from the line is within the boundaries of the identified level of possible measurement errors.

However, within the range of possible lines that could be drawn, only one will be the optimum one. This optimum line is where the sum of negative errors in data points on one side of the line is balanced by the sum of positive errors in data points on the other side of the line. The nature of the data points is often such that a perfectly acceptable approximation to the optimum can be obtained by drawing a straight line through the data points by eye. In other cases, however, it is necessary to fit a line mathematically, using regression techniques.

Regression techniques

Regression techniques consist of finding a mathematical relationship between measurements of two variables y and x , such that the value of variable y can be predicted from a measurement of the other variable x . However, regression techniques should not be regarded as a magic formula that can fit a good relationship to measurement data in all circumstances, as the characteristics of the data must satisfy certain conditions. In determining the suitability of measurement data for the application of regression techniques, it is recommended practice to draw an approximate graph of the measured data points, as this is often the best means of detecting aspects of the data that make it unsuitable for regression analysis. Drawing a graph of the data will indicate, for example, whether there are any data points that appear to be erroneous. This may indicate that human mistakes or instrument malfunctions have affected the erroneous data points, and it is assumed that any such data points will be checked for correctness.

Regression techniques cannot be successfully applied if the deviation of any particular data point from the line to be fitted is greater than the maximum possible error that is calculated for the measured variable (i.e., the predicted sum of all systematic and random errors). The nature of some measurement data sets is such that this criterion cannot be satisfied, and any attempt to apply regression techniques is doomed to failure. In that event, the only valid course of action is to express the measurements in tabular form. This can then be used as an x - y lookup table, from which values of the variable y corresponding to particular values of x can be read off. In many cases, this problem of large errors in some data points only becomes apparent during the process of attempting to fit a relationship by regression.

A further check that must be made before attempting to fit a line or curve to measurements of two variables x and y is to examine the data and look for any evidence that both variables are subject to random errors. It is a clear condition for the validity of regression techniques that only one of the measured variables is subject to random errors, with no error in the other variable. If random errors do exist in both measured variables, regression techniques cannot be applied and recourse must be made instead to correlation analysis (covered later in this chapter). A simple example of a situation where both variables in a measurement data set are subject to random errors are measurements of human height and weight, and no attempt should be made to fit a relationship between them by regression.

Having determined that the technique is valid, the regression procedure is simplest if a straight-line relationship exists between the variables, which allows a relationship of the form $y = a + bx$ to be estimated by linear least squares regression. Unfortunately, in many cases, a straight-line relationship between the points does not exist, which is readily shown by plotting the raw data points on a graph. However, knowledge of physical laws governing the data can often suggest a suitable alternative form of relationship between the two sets of variable measurements, such as a quadratic relationship or a higher-order polynomial relationship. Also, in some cases, the measured variables can be transformed into a form where a linear relationship exists. For example, suppose that two variables y and x are related according to: $y = ax^c$. A linear relationship from this can be derived, using a logarithmic transformation, as: $\log(y) = \log(a) + c\log(x)$. Thus, if a graph is constructed of $\log(y)$ plotted against $\log(x)$, the parameters of a straight-line relationship can be estimated by linear least squares regression.

In some cases, the background theory describing the relationship between two sets of measured variables is not known. The way to proceed in this case is to fit two relationships to the data, one using linear least squares regression and one using quadratic (second order) least squares regression. A confidence test is then applied (see later section headed *confidence tests in curve fitting*) to compare the two relationships fitted. If this shows that the linear relationship is best, the curve-fitting process ends. However, if the confidence test shows that a quadratic relationship is better than a straight-line one, it is necessary to fit a third line to the data using a third-order relationship. A confidence test is then applied again, this time comparing the quadratic relationship to the third-order relationship. This continues until the next higher relationship tried fits the data less well than the previous relationship.

All quadratic and higher-order relationships relating one variable y to another variable x can be represented by a power series of the form:

$$y = a_0 + a_1x + a_2x^2 + \cdots + a_px^p$$

Estimation of the parameters $a_0 \cdots a_p$ is very difficult if p has a large value. Fortunately, a relationship where p only has a small value can be fitted to most data sets. Quadratic least squares regression is used to estimate parameters where p has a value of 2, and for larger values of p , polynomial least squares regression is used for parameter estimation. Where the appropriate form of relationship between variables in measurement data sets is not obvious either from visual inspection or from consideration of physical laws, a method that is effectively a trial-and-error one has to be applied. This consists of estimating the parameters of successively higher-order relationships between y and x until a curve is found that fits the data sufficiently closely. What level of closeness is acceptable is considered in the later section on confidence tests.

Linear least squares regression

Linear least squares regression estimates the optimal values for the constant coefficients a and b of a linear relationship $y = a + bx$ for a set of n measurements $y_1 \cdots y_n, x_1 \cdots x_n$, such that the relationship gives the best fit to the measurement data. Linear regression is applicable whenever the relationship between two sets of measurement variables is either known to be linear or is suspected to be so. For example, the deflection of a spring is linearly proportional to the force applied according to the known background theory for a spring (as long as the magnitude of the force applied is not sufficient to cause the elastic limit for the spring to be exceeded).

Typical data for a set of output measurements $y_1 \cdots y_n$ for a corresponding set of inputs $x_1 \cdots x_n$ subject to random errors is shown in the graph in Fig. 10.20. In the absence of errors, all data points would fall exactly on a straight line. However, because of measurement errors, the actual data points fall above and below the straight line expected according to the theoretical relationship between the data points. The deviation of each point (x_i, y_i) from the line can be expressed as d_i , where $d_i = y_i - (a + bx_i)$.

The best-fit line is obtained when the sum of the squared deviations, S , is a minimum,

$$\text{i.e. when } S = \sum_{i=1}^n (d_i^2) = \sum_{i=1}^n (y_i - a - bx_i)^2 \text{ is a minimum.}$$

The minimum can be found by setting the partial derivatives $\partial S/\partial a$ and $\partial S/\partial b$ to zero and solving the resulting two simultaneous (normal) equations:

$$\partial S/\partial a = \sum 2(y_i - a - bx_i)(-1) = 0 \quad (10.5)$$

$$\partial S/\partial b = \sum 2(y_i - a - bx_i)(-x_i) = 0 \quad (10.6)$$

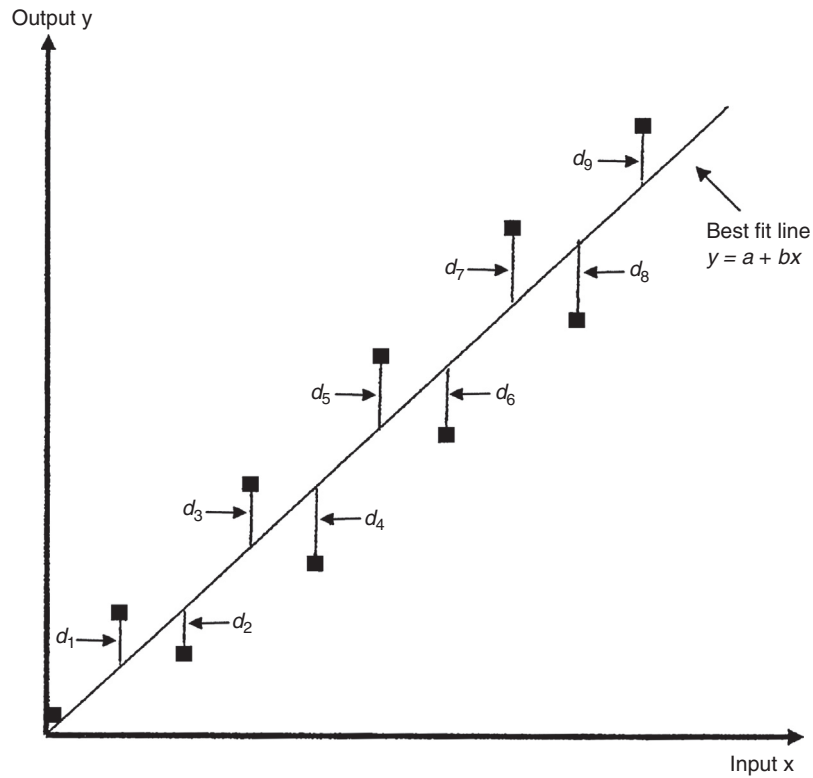


Figure 10.20
Deviations of data points away from best-fit straight line.

The values of the coefficients a and b at the minimum point can be represented by \hat{a} and \hat{b} , which are known as the least squares estimates of a and b . These can be calculated as follows:

From (Eq. 10.5),

$$\sum y_i = \sum \hat{a} + \hat{b} \sum x_i = n\hat{a} + \hat{b} \sum x_i$$

and thus,

$$\hat{a} = \frac{\sum y_i - \hat{b} \sum x_i}{n} \quad (10.7)$$

From (Eq. 10.6),

$$\sum (x_i y_i) = \hat{a} \sum x_i + \hat{b} \sum x_i^2 \quad (10.8)$$

Now substitute for \hat{a} in Eq. (10.8) using (Eq. 10.7):

$$\sum (x_i y_i) = \frac{(\sum y_i - \hat{b} \sum x_i)}{n} \sum x_i + \hat{b} \sum x_i^2$$

Collecting terms in \hat{b} ,

$$\hat{b} \left[\sum x_i^2 - \frac{(\sum x_i)^2}{n} \right] = \sum (x_i y_i) - \frac{\sum x_i \sum y_i}{n}$$

Rearranging gives:

$$\hat{b} \left[\sum x_i^2 - n \left\{ \left(\frac{\sum x_i}{n} \right)^2 \right\} \right] = \sum (x_i y_i) - n \left(\frac{\sum x_i}{n} \right) \left(\frac{\sum y_i}{n} \right).$$

This can be expressed as:

$$\hat{b} \left[\sum x_i^2 - n x_m^2 \right] = \sum (x_i y_i) - n x_m y_m,$$

where x_m and y_m are the mean values of x and y .

Thus:

$$\hat{b} = \frac{\sum (x_i y_i) - n x_m y_m}{\sum x_i^2 - n x_m^2} \quad (10.9)$$

And, from (Eq. 10.7):

$$\hat{a} = \frac{\sum y_i - \hat{b} \sum x_i}{n} = y_m - \hat{b} x_m \quad (10.10)$$

■ Example 10.3

In an experiment to determine the characteristics of a displacement sensor with a voltage output, the following output voltage values were recorded when a set of standard displacements was measured:

Displacement (cm)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Voltage (V)	2.1	4.3	6.2	8.5	10.7	12.6	14.5	16.3	18.3	21.2

Fit a straight line to this set of data using least squares regression and estimate the output voltage when a displacement of 4.5 cm is measured.



■ Solution

Let y represent the output voltage and x represent the displacement. Then a suitable straight line is given by $y = a + bx$. We can now proceed to calculate estimates for the coefficients a and b using Eqs. (10.9) and (10.10) above. The first step is to calculate the mean values of x and y . These are found to be $x_m = 5.5$ and $y_m = 11.47$. Next, we need to tabulate $x_i y_i$ and x_i^2 for each pair of data values:

x_i	y_i	$x_i y_i$	x_i^2
1.0	2.1	2.1	1
2.0	4.3	8.6	4
3.0	6.2	18.6	9
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
10.0	21.2	212.0	100

Now calculate the values needed from this table: $n = 10$; $\sum (x_i y_i) = 801.0$; $\sum (x_i^2) = 385$ and enter these values into Eqs. (10.9) and (10.10).

$$\hat{b} = \frac{801.0 - (10 \times 5.5 \times 11.47)}{385 - (10 \times 5.5^2)} = 2.067; \quad \hat{a} = 11.47 - (2.067 \times 5.5) = 0.1033;$$

$$\text{i.e. } y = 0.1033 + 2.067x$$

Hence, for $x = 4.5$, $y = 0.1033 + (2.067 \times 4.5) = 9.40\text{V}$. Note that in this solution, we have only specified the answer to an accuracy of three figures, which is the same accuracy as the measurements. Any greater number of figures in the answer would be meaningless.

Least squares regression is often appropriate for situations where a straight-line relationship is not immediately obvious, for example, where $y \propto x^2$ or $y \propto \exp(x)$.

■ Example 10.4

From theoretical considerations, it is known that the voltage (V) across a charged capacitor decays with time (t) according to the relationship: $V = K \exp(-t/\tau)$. Estimate values for K and τ if the following values of V and t are measured.

V	8.67	6.55	4.53	3.29	2.56	1.95	1.43	1.04	0.76
t	0	1	2	3	4	5	6	7	8

■ Solution

If $V = K \exp(-T/\tau)$ then, $\log_e(V) = \log_e(K) - t/\tau$. Now let $y = \log_e(V)$, $a = \log(K)$, $b = -1/\tau$ and $x = t$. Hence, $y = a + bx$, which is the equation of a straight line whose coefficients can be estimated by applying (Eqs. 10.9 and 10.10). Therefore, proceed in the same way as Example 10.3 and tabulate the values required:

V	$\log_e(V) (y_i)$	$t (x_i)$	$(x_i y_i)$	(x_i^2)
8.67	2.16	0	0	0
6.55	1.88	1	1.88	1
4.53	1.51	2	3.02	4
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
0.76	-0.27	8	-2.16	64

Now calculate the values needed from this table: $n = 9$; $\sum (x_i y_i) = 15.86$; $\sum (x_i^2) = 204$; $x_m = 4.0$; $y_m = 0.9422$, and enter these values into Eqs. (10.9) and (10.10).

$$\hat{b} = \frac{15.86 - (9 \times 4.0 \times 0.9422)}{204 - (9 \times 4.0^2)} = -0.301; \quad \hat{a} = 0.9422 + (0.301 \times 4.0) = 2.15$$

$K = \exp(a) = \exp(2.15) = 8.58$; $\tau = -1/b = -1/(-0.301) = 3.32$.

Quadratic least squares regression

Quadratic least squares regression is used to estimate the parameters of a relationship $y = a + bx + cx^2$ between two sets of measurements $y_1 \cdots y_n$, $x_1 \cdots x_n$. It is applied when theory suggests that this is the appropriate form of relationship for the data.

The deviation of each point (x_i, y_i) from the line can be expressed as d_i , where $d_i = y_i - (a + bx_i + cx_i^2)$.

The best-fit line is obtained when the sum of the squared deviations, S , is a minimum,

$$\text{i.e., when } S = \sum_{i=1}^n (d_i^2) = \sum_{i=1}^n (y_i - a - bx_i - cx_i^2)^2 \text{ is a minimum.}$$

The minimum can be found by setting the partial derivatives $\partial S/\partial a$, $\partial S/\partial b$, and $\partial S/\partial c$ to zero and solving the resulting simultaneous equations, as for the linear least squares regression case above. Standard computer packages to estimate the parameters a , b , and c

by numerical methods are widely available (for example, LabVIEW and MATLAB) and therefore a detailed solution is not presented here.

Polynomial least squares regression

Polynomial least squares regression is used to estimate the parameters of the p th-order relationship $y = a_0 + a_1x + a_2x^2 + \dots + a_px^p$ between two sets of measurements $y_1 \dots y_n, x_1 \dots x_n$.

The deviation of each point (x_i, y_i) from the line can be expressed as d_i , where:

$$d_i = y_i - (a_0 + a_1x_i + a_2x_i^2 + \dots + a_px_i^p)$$

The best-fit line is obtained when the sum of the squared deviations given by $S = \sum_{i=1}^n (d_i^2)$ is a minimum.

The minimum can be found as before by setting the p partial derivatives $\partial S/\partial a_0 \dots \partial S/\partial a_p$ to zero and solving the resulting simultaneous equations. Again, as for the quadratic least squares regression case, standard computer programs to estimate the parameters $a_0 \dots a_p$ by numerical methods are widely available (for example, LabVIEW and MATLAB) and therefore a detailed solution is not presented here.

Confidence tests in curve fitting by least squares regression

Once data has been collected and a mathematical relationship that fits the data points has been determined by regression, the level of confidence that the mathematical relationship fitted is correct must be expressed in some way. The first check that must be made is whether the fundamental requirement for the validity of regression techniques is satisfied, i.e., whether the deviations of data points from the fitted line are all less than the maximum error level predicted for the measured variable. If this condition is violated by any data point that a line or curve has been fitted to, then use of the fitted relationship is unsafe and recourse must be made to tabular data presentation, as described earlier.

The second check concerns whether or not random errors affect both measured variables. If attempts are made to fit relationships by regression to data where both measured variables contain random errors, any relationship fitted will only be approximate and it is likely that one or more data points will have a deviation from the fitted line or curve that is greater than the maximum error level predicted for the measured variable. This will show up when the appropriate checks are made.

Having carried out the above checks to show that there are no aspects of the data that suggest that regression analysis is not appropriate, the next step is to apply least squares regression to estimate the parameters of the chosen relationship (linear, quadratic, etc.).

After this, some form of follow-up procedure is clearly required to assess how well the estimated relationship fits the data points. A simple curve-fitting confidence test is to calculate the sum of squared deviations S for the chosen y/x relationship and compare it with the value of S calculated for the next higher-order regression curve that could be fitted to the data. Thus, if a straight-line relationship is chosen, the value of S calculated should be of a similar magnitude or less than that obtained by fitting a quadratic relationship. If the value of S were substantially lower for a quadratic relationship, this would indicate that a quadratic relationship was a better fit to the data than a straight-line one and further tests would be needed to examine whether a cubic or higher-order relationship was a better fit still.

The stages of this simple confidence test can be summarized as follows:

1. Calculate $S = \sum (d_i)^2$ for the relationship fitted (for example, a linear relationship)
2. Calculate $S' = \sum (d_i)^2$ for the next higher-order relationship, for example, a quadratic relationship

If S' is significantly less than S , then the first relationship fitted (in this example, the linear relationship) is incorrect.

3. Repeat steps (1) and (2) until S' is greater than or equal to S .

Curve-fitting programs within packages such as LabVIEW and MATLAB are able to assist this procedure by calculating the value of S for each order of regression fit tried.

Other more sophisticated confidence tests exist such as the *F-ratio test*. However, these are outside the scope of this book.

Correlation tests

Where both variables in a measurement data set are subject to random fluctuations, correlation analysis is applied to determine the degree of association between the variables. For example, in the case already quoted of a data set containing measurements of human height and weight, we certainly expect some relationship between the variables of height and weight because a tall person is heavier *on average* than a short person. Correlation tests determine the strength of the relationship (or interdependence) between the measured variables, which is expressed in the form of a correlation coefficient.

For two sets of measurements $y_1 \cdots y_n, x_1 \cdots x_n$ with means x_m and y_m , the correlation coefficient Φ is given by:

$$\Phi = \frac{\sum (x_i - x_m)(y_i - y_m)}{\sqrt{\left[\sum (x_i - x_m)^2 \right] \left[\sum (y_i - y_m)^2 \right]}} \quad (10.11)$$

The value of $|\Phi|$ always lies between 0 and 1, with 0 representing the case where the variables are completely independent of one another and 1 being the case where they are totally related to one another. Because of the presence of measurement errors in almost all situations, it is very unlikely that value of $|\Phi|$ will ever equal 1, which would indicate that the data fitted a straight line exactly. Hence, how close to 1 does $|\Phi|$ have to be for there to be reasonable confidence that the data fits a straight-line relationship? The usual criterion applied is to regard any value given by $\pm 0.9 \leq |\Phi| \leq 1.0$ as indicating that a straight line exists between the data.

For $0 < |\Phi| < 1$, linear least squares regression can be applied to find a relationship between the variables, which allows x to be predicted from a measurement of y , and y to be predicted from a measurement of x . This involves finding two separate regression lines of the form:

$$y = a + bx \quad \text{and} \quad x = c + dy$$

These two lines are not normally coincident as shown in Fig. 10.21. Both lines pass through the centroid of the data points but their slopes are different.

As $|\Phi| \rightarrow 1$, the lines tend to coincidence, representing the case where the two variables are totally dependent upon one another.

As $|\Phi| \rightarrow 0$, the lines tend to orthogonal ones parallel to the x - and y -axes. In this case, the two sets of variables are totally independent. The best estimate of x given any measurement of y is x_m and the best estimate of y given any measurement of x is y_m .

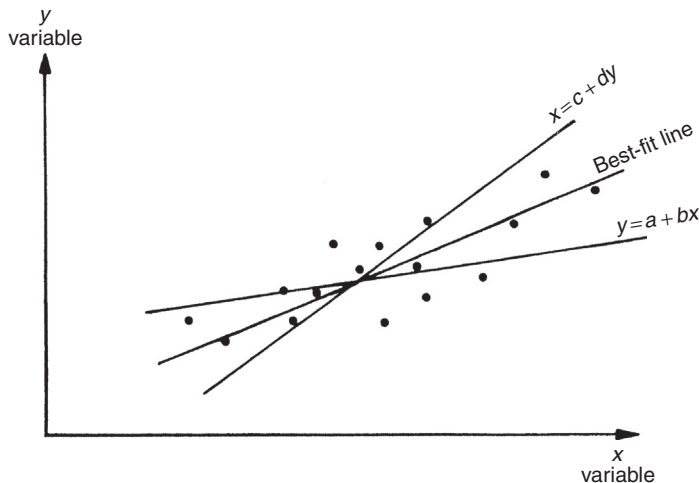


Figure 10.21

Relationship between two variables with random fluctuations.

For the general case, the best fit to the data is the line that bisects the angle between the lines on Fig. 10.21.

Example 10.5

The height and weight of 33 students is measured and recorded as follows:

y (weight in kg)	70.3	75.2	88.1	75.0	72.8	80.7	81.3	76.6	73.8	85.2	81.2
x (height in meters)	1.69	1.68	1.82	1.69	1.81	1.76	1.84	1.74	1.70	1.81	1.75
y (weight in kg)	77.3	77.9	75.3	84.7	68.5	70.7	74.4	73.0	69.4	78.8	83.5
x (height in meters)	1.80	1.71	1.74	1.77	1.72	1.67	1.75	1.66	1.75	1.78	1.80
y (weight in kg)	75.5	64.2	77.2	76.1	90.6	66.6	79.5	64.1	65.8	69.4	73.5
x (height in meters)	1.76	1.68	1.75	1.79	1.85	1.70	1.83	1.76	1.64	1.82	1.73

Use linear least squares regression to find the optimal values for a , b , c , d in relationships between the variables of the form $y = a + bx$ and $x = c + dy$. Draw both of these relationships on a graph. Using these two plotted relationships as a guide, draw a single line that represents the best straight-line fit to the height/weight data given and calculate the gradient of this line. Also, calculate the correlation coefficient Φ .

Solution

Eqs. (10.9) and (10.10) provide expressions for estimating the optimal values of a and b for a relationship $y = a + bx$ between the data.

$$\hat{b} = \frac{\sum(x_i y_i) - n x_m y_m}{\sum x_i^2 - n x_m^2} \quad \text{and} \quad \hat{a} = \frac{\sum y_i - \hat{b} \sum x_i}{n} = y_m - \hat{b} x_m$$

Similar expressions for estimating the optimal values of c and d for a relationship $x = c + dy$ between the data can be found by an identical procedure of minimizing the sum of squared deviations as used earlier for calculating a and b (however, these expressions are found most simply by just interchanging x and y in Eqs. 10.9 and 10.10):

$$\hat{d} = \frac{\sum(x_i y_i) - n x_m y_m}{\sum y_i^2 - n y_m^2} \quad \text{and} \quad \hat{c} = \frac{\sum x_i - \hat{d} \sum y_i}{n} = x_m - \hat{d} y_m$$

It would be extremely tedious to calculate a , b , c , and d by hand for 33 data pairs, and so it is usual to write a computer program or use a spreadsheet package to evaluate the expressions for a , b , c , and d . The author used Microsoft Excel and obtained the following values:

$$\hat{a} = -53.23; \hat{b} = 73.64; \hat{c} = 1.3433; \hat{d} = 0.005377$$

Thus, the best-fit lines are

$$y = -53.23 + 73.64x \quad (10.12)$$

and

$$x = 1.3433 + 0.005377y \quad (10.13)$$

Data points for these two best-fit lines can be calculated as follows:

Using (Eq. 10.12), for $x = 1.67$, $y = 69.8$ and for $x = 1.85$, $y = 83.0$.

Using (Eq. 10.13), for $y = 64$, $x = 1.687$ and for $y = 92$, $x = 1.838$.

These two relationships have been plotted in Fig. 10.22.

A line that bisects these two relationships can then be estimated by eye as shown by the dashed line in Fig. 10.22. This represents the best straight-line fit to the height/weight data given. The gradient of this line can be calculated by selecting two data points on it:

At $y = 67.0$, $x = 1.680$; at $y = 86.0$, $x = 1.834$.

Hence, the gradient of the line is $y/x = (86.0 - 67.0)/(1.834 - 1.680) = 123.4 \text{ kg/m}$.

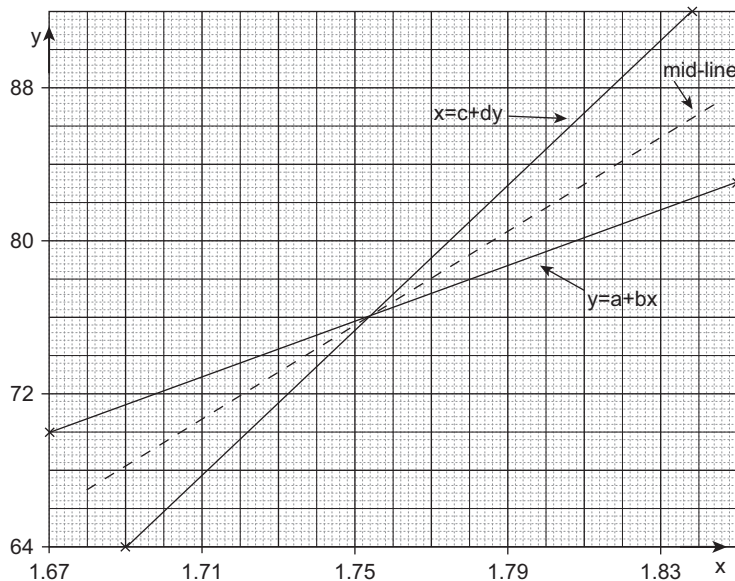


Figure 10.22

Figure showing solution of worked Example 10.5.

The equation for calculating Φ is given in Eq. (10.11). Using a computer program or a spreadsheet package to evaluate this gives $\Phi = 0.629$. ■

The noncoincidence of the lines for $y = a + bx$ and $x = c + dy$ calculated in Example 10.5 is explained by the value of 0.629 calculated for Φ . We have previously commented that a value of at least 0.9 is expected for Φ if a linear relationship exists between the data.

10.5 Summary

This chapter has covered what is essentially the final stage in a measurement system, that of using the data collected. Such use might mean immediate use by displaying the data on a display device. Alternatively, the requirement may be to store the data for future use and analysis.

The chapter started with a discussion of the techniques available to display measurement data for immediate use. Within this discussion, we looked at the merits and usage of various types of electrical meters and also of oscilloscopes. We then went on to look at two newer ways of displaying measurement data, using an electronic display device and using a computer monitor.

Following this, we looked at the various means available to record and store measurement data for future use. We learned that the options available for recording data are numerous and include chart recorders, digital oscilloscopes, digital data recorders, and hard copy devices such as ink-jet and laser printers. We gave consideration to each of these and indicated some of the circumstances in which each alternative recording device might be used.

The chapter then ended with a discussion about standards of good practice in presenting data. This included coverage of both graphical and tabular forms of presentation, using either paper or a computer monitor screen as the display medium. We then went on to consider the best way of fitting lines through data points on a graph. This led us to a discussion of mathematical regression techniques and the associated confidence tests that are necessary to assess the correctness of the line fitted using regression. Finally, correlation tests were described that determine the degree of association between two sets of data when they are both subject to random fluctuations.

10.6 Problems

- 10.1 Summarize the advantages of digital meters over their analog counterparts.
- 10.2 Explain the four main alternative mechanisms used for effecting analog-to-digital conversion in a digital voltmeter.
- 10.3 What sort of applications are analog meters still commonly found in?
- 10.4 (a) Explain the mode of operation of a moving-coil meter.
 (b) Calculate the reading that would be observed on a moving-coil ammeter when the current for the waveform shown in Fig. 10.23 is being measured. (The waveform between $\omega t = \pi$ and $\omega t = 2\pi$ is the positive half of a sine wave with a peak current of +20 amps.)
- 10.5 (a) Explain the mode of operation of a moving iron meter.
 (b) Calculate the reading that would be observed on a moving iron ammeter when the current for the waveform shown in Fig. 10.23 is being measured. (The waveform between $\omega t = \pi$ and $\omega t = 2\pi$ is the positive half of a sine wave with a peak current of +20 amps.)
- 10.6 Calculate the reading that would be observed on (a) a moving-coil ammeter and (b) a moving iron ammeter when the current for the waveform shown in Fig. 10.24 is being measured. (The waveform between $\omega t = 0$ and $\omega t = \pi$ is the positive half of a sine wave with a peak current of +10 amps.)
- 10.7 How does an oscilloscope work?
- 10.8 What are the main differences between analog and digital oscilloscopes?
- 10.9 Explain the following terms: (a) bandwidth, (b) rise time.
 In designing oscilloscopes, what relationship is sort between bandwidth and rise time?
- 10.10 Explain the following terms in relation to an oscilloscope: (a) channel, (b) single-ended input, (c) differential input, and (d) input coupling.

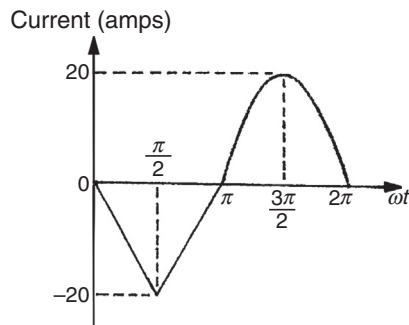


Figure 10.23

Waveform for Problems 10.4 and 10.5.

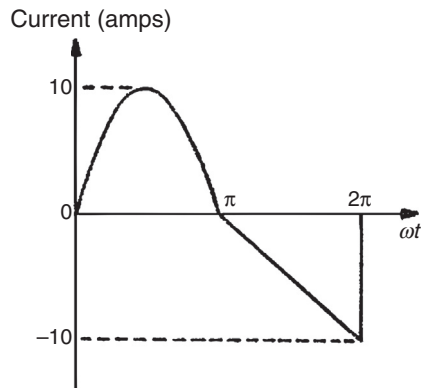


Figure 10.24

Waveform for Problem 10.6.

- 10.11 Explain the following terms in relation to an oscilloscope: (a) time base, (b) vertical sensitivity, (c) display position control, and (d) trigger system.
- 10.12 Sketch a block diagram showing the main components in a digital storage oscilloscope and explain the mode of operation of the instrument.
- 10.13 Draw a block diagram showing the main components in a digital phosphor oscilloscope. What advantages does a digital phosphor oscilloscope have over a digital storage one?
- 10.14 Illustrate the main components in a digital sampling oscilloscope by sketching a block diagram of them. What performance advantages does a digital sampling oscilloscope have over a digital storage one?
- 10.15 What is a PC-based oscilloscope? Discuss their advantages and disadvantages compared with digital oscilloscopes.
- 10.16 What are the main differences between a digital storage oscilloscope, a digital phosphor oscilloscope, and a digital sampling oscilloscope? How do these differences affect their performance and typical usage?
- 10.17 What is the main use of electronic displays in measurement systems? Discuss the features of electronic displays.
- 10.18 What is the main use of computer monitor displays in measurement systems? Discuss the features of computer monitor displays.
- 10.19 Discuss the range of instruments and techniques available for recording measurement signals, mentioning particularly the frequency response characteristics of each instrument or technique and the upper frequency limit for signals in each case.
- 10.20 Discuss the features of the main types of chart recorder that are available for recording measurement signals.
- 10.21 What is a digital data recorder and how does it work?

- 10.22 Discuss the relative merits of tabular and graphical methods of recording measurement data.
- 10.23 What would you regard as good practice in recording measurement data in graphical form?
- 10.24 What would you regard as good practice in recording measurement data in tabular form?
- 10.25 Explain the technique of linear least squares regression for finding a relationship between two sets of measurement data.
- 10.26 (a) Show that the optimal coefficients \hat{a} and \hat{b} for a linear relationship $y = a + bx$ between a set of n measurements $y_1 \cdots y_n$ and $x_1 \cdots x_n$ are given by:

$$\hat{b} = \frac{\sum(x_i y_i) - nx_m y_m}{\sum x_i^2 - nx_m^2} \quad \text{and} \quad \hat{a} = y_m - \hat{b}x_m$$

where x_m and y_m are the mean values of x and y .

- (b) The following set of measurements is obtained for values of an output variable y and an input variable x that are believed to be related by a linear expression of the form: $y = a + bx$.

i	1	2	3	4	5	6	7	8	9	10
x_i	40	45	50	55	60	65	70	75	80	85
y_i	13.8	22.7	33.6	42.9	53.5	63.3	75.0	82.8	94.1	103.4

Apply linear least squares regression to find the values of a and b that produce the best-fit straight line.

- 10.27 What checks have to be made before using regression techniques to fit a relationship to measurement data? In what circumstances is it not possible to fit a valid relationship using regression techniques?
- 10.28 (a) What is linear least squares regression and in what situations is it useful?
- (b) If a set of n measurements $y_1 \cdots y_n$ and $x_1 \cdots x_n$ are believed to be related by a linear relationship given by $y = a + bx$, show that the optimal coefficients \hat{a} and \hat{b} between are given by:

$$\hat{b} = \frac{\sum(x_i y_i) - nx_m y_m}{\sum x_i^2 - nx_m^2} \quad \text{and} \quad \hat{a} = y_m - \hat{b}x_m$$

where x_m and y_m are the mean values of x and y .

- (c) An experiment is carried out to determine the characteristics of a spring. A set of weights are added to the spring and the corresponding deflections are measured. The following set of measurements is obtained for the values of the

output deflection y and the input weight x . It is known from the background theory for a spring that the deflection y is related to the weight x according to the linear expression: $y = a + bx$.

i	1	2	3	4	5	6	7	8	9	10	11	12
x_i (gram)	5	10	15	20	25	30	35	40	45	50	55	60
y_i (mm)	11.4	21.9	32.3	44.8	54.2	66.7	75.7	89.1	98.5	111.0	120.6	133.4

Apply linear least squares regression to find the values of a and b that produce the best-fit straight line.

- 10.29 Explain the techniques of (a) quadratic least squares regression and (b) polynomial least squares regression. How would you determine whether either quadratic or polynomial least squares regression provides a better fit to a set of measurement data than linear least squares regression?
- 10.30 During calibration of a platinum resistance thermometer, the following temperature and resistance values were measured:

Resistance (Ω)	212.8	218.6	225.3	233.6	240.8	246.6
Temperature ($^{\circ}\text{C}$)	300	320	340	360	380	400

The temperature measurements were made using a primary reference standard instrument for which the measurement errors can be assumed to be zero. The resistance measurements were subject to random errors but it can be assumed that there are no systematic errors in them.

- Determine the sensitivity of measurement in $\Omega/^{\circ}\text{C}$ in as accurate a manner as possible.
 - Write down the temperature range that this sensitivity value is valid for.
 - Explain the steps that you would take to test the validity of the type of mathematical relationship that you have used for the data.
- 10.31 Theoretical considerations show that quantities x and y are related in a linear fashion such that: $y = ax + b$. Show that the best estimate of the constants a and b are given by:

$$\hat{a} = \frac{\sum (x_i - x_m)(y_i - y_m)}{\sum (x_i - x_m)^2}; \quad \hat{b} = y_m - \hat{a}x_m$$

Explain carefully the meaning of all the terms in the above two equations.

- 10.32 The characteristics of a chromel-constantan thermocouple are known to be approxi-

mately linear over the temperature range of 300°C–800°C. The output e.m.f. was measured practically at a range of temperatures and the following table of results obtained. Using least squares regression, calculate the coefficients a and b for the relationship $T = a + bE$ that best describes the temperature–e.m.f. characteristic.

Temp(°C)	300	325	350	375	400	425	450	475	500	525	550
e.m.f. (mV)	21.0	23.2	25.0	26.9	28.6	31.3	32.8	35.0	37.2	38.5	40.7
Temp (°C)	575	600	625	650	675	700	725	750	775	800	
e.m.f. (mV)	43.0	45.2	47.6	49.5	51.1	53.0	55.5	57.2	59.0	61.0	

- 10.33 Measurements of the current (I) flowing through a resistor and the corresponding voltage drop (V) are shown below:

I (amps)	1	2	3	4	5
V (volts)	10.8	20.4	30.7	40.5	50.0

The instruments used to measure voltage and current were accurate in all respects except that they each had a zero error that the observer failed to take account of or to correct at the time of measurement. Determine the value of the resistor from the data measured.

- 10.34 A measured quantity y is known from theoretical considerations to depend on a variable x according to the relationship: $y = a + bx^2$. For the following set of measurements of x and y , use linear least squares regression to determine the estimates of the parameters a and b that fit the data best.

x	0	1	2	3	4	5
y	0.9	9.2	33.4	72.5	130.1	200.8

- 10.35 The mean-time-to-failure (MTTF) of an integrated circuit is known to obey a law of the following form: $\text{MTTF} = C \exp T_0/T$, where T is the operating temperature and C and T_0 are constants.

The following values of MTTF at various temperatures were obtained from accelerated-life tests.

MTTF (hours)	54	105	206	411	941	2145
Temperature (°K)	600	580	560	540	520	500

- (a) Estimate the values of C and T_0 . Hint: $\log_e(\text{MTTF}) = \log_e(C) + T_0/T$. This equation is now a straight-line relationship between $\log(\text{MTTF})$ and $1/T$, where $\log(C)$ and T_0 are constants.
- (b) For a MTTF of 10 years, calculate the maximum allowable temperature.

- 10.36 The height and weight of 15 men are measured and recorded as follows, where y is the weight in kg and x is the height in meters of each man:

y	70.3	76.1	73.5	88.1	72.8	66.5	75.5	80.7	88.7	76.6	79.7	75.3	68.5	69.4	78.8
x	1.69	1.79	1.73	1.82	1.81	1.68	1.76	1.76	1.84	1.74	1.71	1.74	1.72	1.75	1.78

Use linear least squares regression to find the optimal values for a , b , c , d in relationships between the variables of the form $y = a + bx$ and $x = c + dy$. Draw both of these relationships on a graph. Using these two plotted relationships as a guide, draw a single line that represents the best straight-line fit to the height/weight data given and calculate the gradient of this line. Also, calculate the correlation coefficient Φ .

- 10.37 The height and weight of 15 male students are measured and recorded as follows, where y is the weight in pounds and x is the height in inches of each student:

y	68.0	69.5	68.5	72.5	69.0	70.5	67.5	70.0	68.0	71.5	66.0	70.0	71.5	66.5	69.5
x	151	166	172	198	153	168	180	174	162	181	147	178	161	155	176

Use linear least squares regression to find the optimal values for a , b , c , d in relationships between the variables of the form $y = a + bx$ and $x = c + dy$. Draw both of these relationships on a graph. Using these two plotted relationships as a guide, draw a single line that represents the best straight-line fit to the height/weight data given and calculate the gradient of this line. Also, calculate the correlation coefficient Φ .

- 10.38 What are confidence tests in relation to curve fitting by regression techniques? Explain how they work.

Intelligent sensors

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11.1 Introduction

We find references to devices with names such as “intelligent sensors,” “intelligent instruments,” “smart sensors,” and “smart transmitters” whenever we open a technical magazine or browse through an instrument manufacturer’s catalog. This reflects the fact that intelligent devices have achieved widespread use in measurement applications. The term “intelligent” is used to denote any measurement device that uses computational power to enhance its measurement performance.

Digital computers have been used in conjunction with measurement systems for many years in the typical control system scenario in which a computer uses data to process variables supplied by a measurement system to compute a control signal that is then applied to an actuator to modify some aspect of the controlled process. In this case, the computer was not actually part of the measurement system but merely worked with it by taking data from the system.

As the cost of computers fell and their power increased, it became common practice to use the computer assigned to a process control function to enhance the quality of measurements by performing various signal processing operations digitally that were previously carried out by analogue electronic circuits. However, in these early applications of digital signal processing, the computer remained a distinctly separate component within the measurement system.

We have moved one stage further to the point where the computer that performs digital signal processing to enhance measurement quality is incorporated into the measurement device. Such devices that incorporate digital signal processing are given the generic name *intelligent devices*. Individual intelligent devices attract various names such as *intelligent instrument*, *intelligent sensor*, *smart sensor*, and *smart transmitter*. There are no hard distinctions between the functions of any of these, and which term is used to refer to an intelligent device is largely due to the preference adopted by different manufacturers for one name or another. Similar variation exists in the name used to describe the computational power within the intelligent device; terms such as “microcomputer” and “microprocessor” are common.

The subject of this chapter is therefore intelligent devices. However, to start, we will look more generally at some basic principles of digital computation, because this will enable us to understand better how intelligent devices function and what potential difficulties exist in their application and operation.

11.2 Principles of digital computation

11.2.1 Elements of a computer

The primary function of a digital computer is the manipulation of data. Three elements essential to the fulfilment of this task are the central processing unit, memory, and the input–output (I/O) interface, as shown in Fig. 11.1. These elements are collectively known as the computer hardware, and each element exists physically as one or more integrated circuit chips mounted onto a printed circuit board. When the central processing unit (CPU) consists of a single microprocessor, it is usual to regard the system as a microcomputer. The distinctions among the terms “microcomputer,” “minicomputer,” and “mainframe computer” are arbitrary and are made according to relative computer power. However, this classification has become meaningless; modern “microcomputers” are more powerful than mainframe computers of only a few years ago.

The CPU part of a computer can be regarded as the brain of system. A relatively small CPU is commonly called a *microprocessor*. The CPU determines what computational operations are carried out and the sequence in which the operations are executed. During such an operation, the CPU uses one or more special storage locations within itself, known as *registers*. Another part of the CPU is the *arithmetic and logic unit*, which is where all arithmetic operations are evaluated. The CPU operates according to a sequential list of required operations defined by a computer program, known as the computer software. This program is held in the second of the three system components, the computer memory.

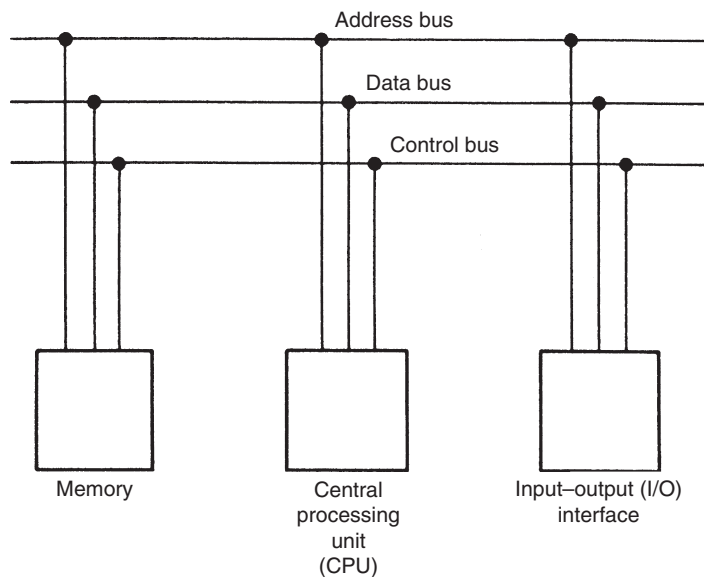


Figure 11.1
Elements of a microcomputer.

The *computer memory* also serves several other functions besides this role of holding the computer program. One is to provide temporary storage locations that the CPU uses to store variables during execution of the computer program. A further common use of memory is to store data tables for scaling and variable conversion purposes during program execution.

Memory can be visualized as a consecutive sequence of boxes in which various items are stored, as shown in [Fig. 11.2](#) for a typical memory size of 65,536 storage units. If this storage mechanism is to be useful, it is essential for a means to be provided to give a unique label to each storage box. This is achieved by labeling the first box as 0, the next one as 1, and so on for the rest of the storage locations. These numbers are known as the *memory addresses*. Although these can be labeled by decimal numbers, it is more usual to use hexadecimal notation.

Two main types of computer memory exist, and there are important differences between them: *random access memory* (RAM) and *read-only memory* (ROM). The CPU can both read from and write to the former, but it can only read from the latter. The importance of ROM becomes apparent when considering the behavior of each kind of memory when the

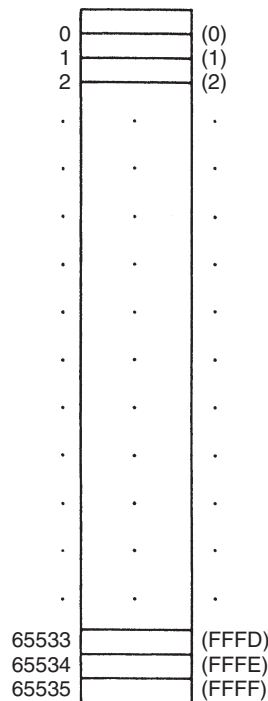


Figure 11.2

Schematic representation of computer memory (numbers in parentheses are memory addresses in hexadecimal notation).

power supply is turned off. At power-off time, RAM loses its contents but ROM maintains them, and this is the value of ROM. Intelligent devices normally use ROM to store the program and data tables and just have a small amount of RAM that is used by the CPU for temporary variable storage during program execution.

The third essential element of a computer system is the *I/O interface*, which allows the computer to communicate with the outside world by reading in data values and outputting results after the appropriate computation has been executed. In the case of a microcomputer performing a signal processing function within an intelligent device, this means reading in the values obtained from one or more sensors and outputting a processed value for presentation at the instrument output. All such external peripherals are identified by a unique number, as for memory addresses.

Communication among these three computer elements is provided by three electronic highways known as the *data bus*, *address bus*, and *control bus*. At each data transfer operation executed by the CPU, two items of information must be conveyed along the electronic highway: the item of data being transferred and the address where it is being sent. Although both of these items of information could be conveyed along a single bus, it is more usual to use two buses that are called the data bus and the address bus. The timing of data transfer operations is important, particularly when transfers take place to peripherals such as disk drives and keyboards, in which the CPU often has to wait until the peripheral is free before it can initialize a data transfer. This timing information is carried by a third highway known as the control bus.

The current trend made possible by advances in very—large scale integration technology is to incorporate all three functions of the CPU, memory, and I/O within a single chip (known as a computer on a chip or *microcomputer*). The term “microprocessor” is often used to describe such an integrated unit, but this is strictly incorrect because the device contains more than just processing power.

11.2.2 Computer operation

As mentioned, the fundamental role of a computer is the manipulation of data. Numbers are used both in quantifying items of data and in the form of codes that define the computational operations to be executed. All numbers that are used for these two purposes must be stored within the computer memory and also transported along the communication buses.

Programming and program execution

In most modes of use, including as part of intelligent devices, computers are involved in manipulating data. This requires data values to be input, processed, and output according to

a sequence of operations defined by the computer program. However, in practice, programming the microprocessor within an intelligent device is not normally the province of the instrument user; indeed, there is rarely a provision for the user to create or modify operating programs even if he or she wished to do so. There are several reasons for this. First, the signal processing needed within an intelligent device is usually well-defined, and therefore it is more efficient for a manufacturer to produce this rather than to have each individual user produce near-identical programs separately. Second, better program integrity and instrument operation is achieved if a standard program produced by the instrument manufacturer is used. Finally, use of a standard program allows it to be burned into ROM, thereby protecting it from failure of the instrument power supply. This also facilitates software maintenance and updates, by the mechanism of the manufacturer providing a new ROM that simply plugs into the slot previously occupied by the old ROM.

However, although it is not normally a task undertaken by the user, some appreciation of microprocessor programming for an intelligent device is useful background knowledge. To illustrate techniques involved in programming, consider a simple program that reads in a value from a sensor, adds a prestored value to it to compensate for a bias in the sensor measurement, and outputs a corrected reading to a display device.

Let us assume that the addresses of the sensor and output display device are 00C0 and 00C1, respectively (hexadecimal numbering notation) and that the required scaling value has been stored in memory address 0100. The instructions below make use of CPU registers A and B:

```
IN A,C0
IN B,100
ADD A,B
OUT C1,A
```

This list of four instructions constitutes the computer program necessary to execute the required task. The CPU normally executes the instructions one at a time, starting at the top of the list and working downward (although jump and branch instructions can change this order). The first instruction (IN A,C0) reads in a value from the sensor at address C0 and places the value in CPU register A (often called the accumulator). The mechanics of the execution of this instruction consist of the CPU putting the required address C0 on the address bus and then putting a command on the control bus that causes the contents of the target address (C0) to be copied onto the data bus and subsequently transferred into the A register. The next instruction (IN B,100) reads in a value from address 100 (the prestored biasing value) and stores it in register B. The following instruction (ADD A,B) adds together the contents of registers A and B and stores the result in register A. Register A now contains the measurement read from the sensor but corrected for bias. The final instruction (OUT C1,A) transfers the contents of register A to the output device on address C1.

11.2.3 Computer input–output interface

The I/O interface connects the computer to the outside world and is therefore an essential part of the computer system. When the CPU puts the address of a peripheral onto the address bus, the I/O interface decodes the address and identifies the unique computer peripheral with which a data transfer operation is to be executed. The interface also has to interpret the command on the control bus so that the timing of the data transfer is correct. One further important function of the I/O interface is to provide a physical electronic highway for the flow of data between the computer data bus and the external peripheral. In many computer applications, including their use within intelligent devices, the external peripheral requires signals to be in analogue form. Therefore, the I/O interface must provide for conversion between these analogue signals and the digital signals required by a digital computer. This is satisfied by analogue-to-digital and digital-to-analogue conversion elements within the I/O interface.

The rest of this section presents some elementary concepts of interfacing in simple terms.

Address decoding

To simplify explanation of the address decoding process, an address bus that is 16 bits wide will be used. (However, 32- and 64-bit address fields are more commonly found.) A 16-bit address field allows 65,536 separate addresses to be accessed in the range 0000–FFFF (in hexadecimal representation). Special commands on some computers are reserved for accessing the bottom end 256 of these addresses in the range 0000–00FF, and if these commands are used, only 8 bits are needed to specify the required address. For the purpose of explaining address-decoding techniques, the subsequent scheme shows how the lower 8 bits of the 16-bit address line are decoded to identify the unique address referenced by one of these special commands. Decoding of all 16 address lines follows a similar procedure but requires a substantially greater number of integrated circuit chips.

Address decoding is performed by a suitable combination of logic gates. Fig. 11.3 shows a simple hardware scheme for decoding 8 address lines. This consists of 256 eight-input NOT-AND (NAND) gates, each of which uniquely decodes one of 256 addresses. A NAND gate is a logic element that gives only a logic level 1 output when all inputs are zero and gives a logic level 0 output for any other combination of inputs. The inputs to the NAND gates are connected onto the lower 8 lines of the address bus and the computer peripherals are connected to the output of the particular gates that decode their unique addresses. There are two pins for each input to the NAND gates that respectively invert and do not invert the input signal. By connecting the 8 address lines appropriately to these two alternative pins at each input, the gate is made to decode a unique address. Consider, for instance, the pin connections shown in Fig. 11.4. This NAND gate decodes address C5

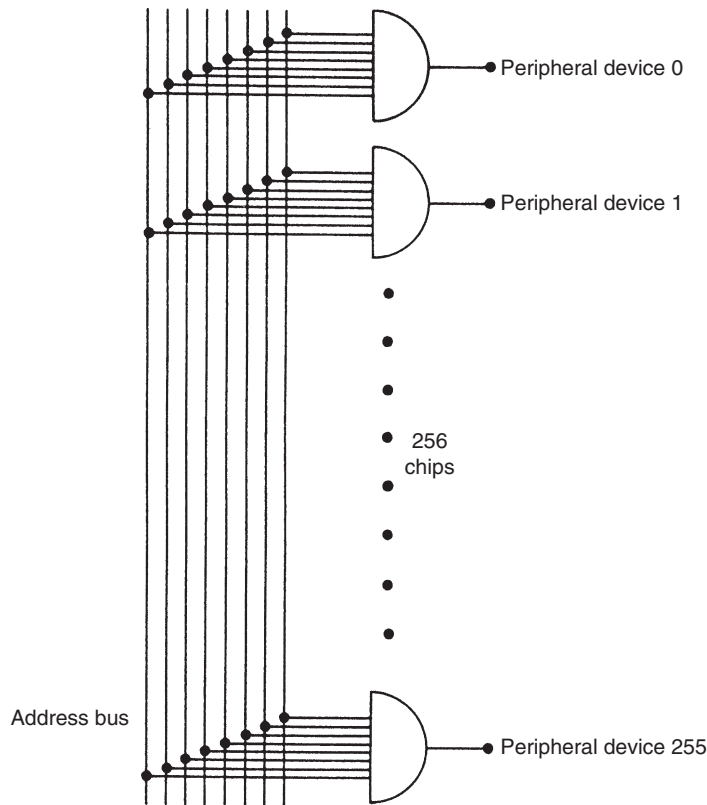


Figure 11.3

Simple hardware scheme for decoding eight address lines.

(hexadecimal), which is 11000101 in binary. Because of the way in which the input pins to the chip are connected, the NAND gate will see all zeros at its input when 11000101 is on the lower 8 bits of the address bus and therefore will have an output of 1. Any other binary number on the address bus will cause this NAND gate to have a zero output.

Data transfer control

The transfer of data between the computer and peripherals is managed by control and status signals carried on the control bus that determine the exact sequencing and timing of I/O operations. Such management is necessary because of the different operating speeds of the computer and its peripherals and because of the multitasking operation of many computers. This means that at any particular instant when a data transfer operation is requested, either the computer or the peripheral may not be ready to take part in the transfer. Typical control and status lines, and their meanings when set at a logic level of 1, are:

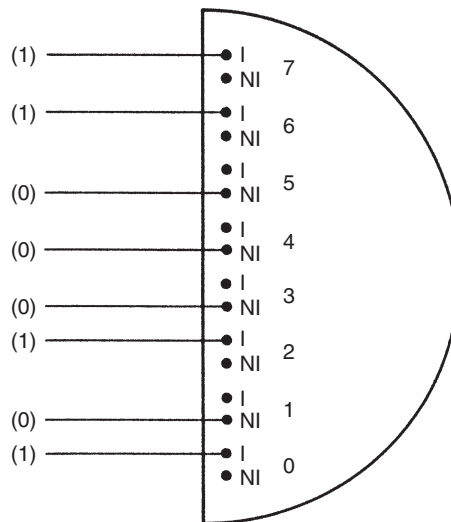


Figure 11.4

Pin connections to NOT-AND (NAND) gate to decode address C5.

- | | |
|----------|---|
| • BUSY | Peripheral device busy |
| • READY | Peripheral device ready for data transfer |
| • ENABLE | CPU ready for data transfer |
| • ERROR | Malfunction on peripheral device |

Similar control signals are set up by both the computer and peripherals, but different conventions are often used to define the status of each device. Differing conventions occur particularly when the computer and peripherals come from different manufacturers. They might mean, for instance, that the computer interprets a logic level of 1 as defining a device to be busy but the peripheral device uses logic level 0 to define “device busy” on the appropriate control line. Therefore, translation of the control lines between the computer and peripherals is required, which is achieved by a further series of logic gates within the I/O interface.

11.2.4 Practical considerations in adding computers to measurement systems

This discussion has presented some necessary elements in an I/O interface in a relatively simplistic manner that is sufficient to give the reader the flavor of what is involved in an interface. Much fine detail has been omitted, and the amount of work involved in the practical design of a real interface should not be underestimated. One significant omission so far is a discussion of the scaling that is generally required within the analogue–digital interface of a computer. The raw analogue input and output signals are generally either too large or too small for compatibility with the operating voltage levels of a digital computer,

and they have to be scaled upward or downward. This is normally achieved by operational amplifiers and/or potentiometers. The main features of an operational amplifier are its high gain (typically $\times 1,000,000$) and its large bandwidth (typically 1 MHz or better). However, when one is used at very high frequencies, the bandwidth becomes significant. The quality of an amplifier is often measured by a criterion called the gain-bandwidth product, which is the product of its gain and bandwidth. Other important attributes of the operational amplifier, particularly when used in a computer I/O interface or within intelligent devices, are its distortion level, overload recovery capacity, and offset level. Special instrumentation amplifiers that are particularly good in these attributes have been developed for instrumentation applications, as described in Chapter 8.

Suitable care must always be taken when introducing a computer into a measurement system, to avoid creating sources of measurement noise. This applies particularly where one computer is used to process the output of several transducers and is connected to them by signal wires. Under such circumstances, the connections and connecting wires can create noise through electrochemical potentials, thermoelectric potentials, offset voltages introduced by common mode impedances, and alternating current noise at power, audio, and radio frequencies. Recognition of all of these possible noise sources allows them to be eliminated in most cases by employing good practice when designing and constructing the measurement system.

11.3 Intelligent devices

The term “intelligent device” is used to describe a package containing either a complete measurement system or a component within a measurement system, which incorporates a digital processor. Processing of the output of measurement sensors to correct for errors inherent in the measurement process brings large improvements in measurement accuracy. Such intelligent devices are known by various names such as *intelligent instrument*, *intelligent sensor*, *smart sensor*, and *smart transmitter*. *Smart microelectromechanical systems devices* have become available (these are described in Chapter 13 and in later chapters covering the measurement of specific physical variables). *Smart nanoelectromechanical system devices* are also the subject of research for use in applications such as hearing aids, in which a nanoscale microphone is integrated with signal conditioning and a digital interface. There is no formal definition for any of these names, and there is considerable overlap between the characteristics of particular devices and the name given to them. The name used for any particular device depends largely on the whims and style of device manufacturers. The subsequent discussion lays out the historical development of intelligent devices and summarizes a general understanding of characteristics possessed by the various forms of intelligent devices. Details of their application to measure particular physical variables will be covered in more detail in various chapters in Part B of this book.

11.3.1 Intelligent instruments

The first intelligent instrument appeared many years ago, although high prices when such devices first became available meant that their use within measurement systems initially grew slowly. However, since those early days, there has been a dramatic reduction in the price of all intelligent devices, and the cost differential between intelligent and conventional devices is now small. Indeed, an intelligent device is sometimes cheaper than its nonintelligent equivalent because of the greater sales volume for the intelligent version. Thus, intelligent devices are routinely bought in many cases instead of nonintelligent versions.

The processor within an intelligent instrument allows it to apply preprogrammed signal processing and data manipulation algorithms to measurements. This prewritten software is often known by the name *embedded software*. A main function performed by the first intelligent instruments that were available was compensation for environmental disturbances to measurements that cause systematic errors. Thus, apart from a primary sensor to measure the variable of interest, intelligent instruments usually have one or more secondary sensors to monitor the value of environmental disturbances. These extra measurements allow the output reading to be corrected for the effects of environmentally induced errors, subject to the following preconditions being satisfied:

- (a) The physical mechanism by which a measurement sensor is affected by ambient condition changes must be fully understood and all physical quantities that affect the output must be identified.
- (b) The effect of each ambient variable on the output characteristic of the primary sensor must be quantified.
- (c) Suitable secondary sensors for monitoring the value of all relevant environmental variables must be available that will be operate satisfactorily under the prevailing environmental conditions.

Condition (a) means that the thermal expansion and contraction of all elements within a sensor must be considered to evaluate how it will respond to ambient temperature changes. Similarly, the sensor response, if any, to changes in ambient pressure, humidity, gravitational force, or power supply level (active instruments) must be examined.

Quantification of the effect of each ambient variable on the characteristics of the measurement sensor is then necessary, as stated in condition (b). Analytic quantification of ambient condition changes from purely theoretical consideration of the construction of a sensor is usually extremely complex and so is normally avoided. Instead, the effect is quantified empirically in laboratory tests. In such tests, the output characteristic of the sensor is observed as the ambient environmental conditions are changed in a controlled manner.

One early application of intelligent instruments was for volume flow rate measurement, in which the flow rate is inferred by measuring the differential pressure across an orifice

plate placed in a fluid-carrying pipe (see Chapter 15). The flow rate is proportional to the square root of the difference in pressure across the orifice plate. For a given flow rate, this relationship is affected by both the temperature and the mean pressure in the pipe, and changes in the ambient value of either of these cause measurement errors. A typical intelligent flowmeter therefore contains three sensors, a primary one measuring pressure difference across the orifice plate and secondary ones measuring absolute pressure and temperature. The instrument is programmed to correct the output of the primary differential-pressure sensor according to the values measured by the secondary sensors, using appropriate physical laws that quantify the effect of ambient temperature and pressure changes on the fundamental relationship between flow and differential pressure. Such intelligent flow-measuring instruments achieve typical inaccuracy levels of $\pm 0.1\%$ or better, compared with $\pm 0.5\%$ for their nonintelligent equivalents.

Although automatic compensation for environmental disturbances is an important attribute of intelligent instruments, many versions of such devices perform additional functions, and this was so even in the early days of their development. For example, the orifice-plate flowmeter just discussed usually converts the square root relationship between flow and signal output into a linear one, which makes the output much easier to interpret. Other examples of the sort of functions performed by intelligent instruments are:

- correction for the loading effect of measurement on the measured system
- signal damping with selectable time constants
- switchable ranges (using several primary sensors within the instrument that each measure over a different range)
- switchable output units (e.g., display in imperial units or SI Units)
- linearization of the output
- self-diagnosis of faults
- remote adjustment and control of instrument parameters from up to 1500 m away

These features will be discussed in greater detail under the later headings of Smart Sensors and Smart Transmitters.

Over the intervening years since their first introduction, the size of intelligent instruments has gradually reduced and the functions performed have steadily increased. One particular development has been the inclusion of a microprocessor within the sensor itself in devices usually known as *smart sensors*. As further size reduction and device integration has taken place, such smart sensors have been incorporated into packages with other sensors and signal processing circuits, and so forth. Although such a package conforms to the definition of an intelligent instrument given previously, most manufacturers tend to call the package a *smart transmitter* rather than an intelligent instrument, although the latter term has continued in use in some cases.

11.3.2 Smart sensors

The name *smart sensor* is most commonly used to describe any sensor with local processing power that enables it to react to local conditions without having to refer back to a central controller. Smart sensors are usually at least twice as accurate as nonsmart devices; they have reduced maintenance costs and require less wiring to the site where they are used. In addition, long-term stability is improved, reducing the required calibration frequency.

The functions possessed by smart sensors vary widely, but they consist of at least some of the following:

- Remote calibration capability
- Self-diagnosis of faults
- Automatic calculation of measurement accuracy and compensation for random errors
- Adjustment for measurement nonlinearities to produce a linear output
- Compensation for the loading effect of the measuring process on the measured system

Calibration capability

Self-calibration is simple in some cases. Sensors with an electrical output can use a known reference voltage level to carry out self-calibration. Also, load-cell types of sensor, which are used in weighing systems, can adjust the output reading to zero when there is no applied mass. In the case of other sensors, two methods of self-calibration are possible: use of a lookup table and an interpolation technique. Unfortunately, a *lookup table* requires a large memory capacity to store correction points. Also, a large amount of data have to be gathered from the sensor during calibration. In consequence, the interpolation calibration technique is preferable. This uses an interpolation method to calculate the correction required to any particular measurement and requires only a small matrix of calibration points.

Self-diagnosis of faults

Smart sensors perform self-diagnosis by monitoring internal signals for evidence of faults. Although it is difficult to achieve a sensor that can carry out self-diagnosis of all possible faults that might arise, it is often possible to make simple checks that detect many of the more common faults. One example of self-diagnosis in a sensor is measuring the sheath capacitance and resistance in insulated thermocouples to detect breakdown of the insulation. Usually, a specific code is generated to indicate each type of possible fault (e.g., a failing of insulation in a device).

One difficulty that often arises in self-diagnosis is in differentiating between normal measurement deviations and sensor faults. Some smart sensors overcome this by storing

multiple measured values around a set point and then calculating minimum and maximum expected values for the measured quantity.

Uncertainty techniques can be applied to measure the impact of a sensor fault on measurement quality. This makes it possible under certain circumstances to continue to use a sensor after it has developed a fault.

Automatic calculation of measurement accuracy and compensation for random errors

Many smart sensors can calculate measurement accuracy online by computing the mean over a number of measurements and analyzing all factors affecting accuracy. This averaging process also serves to reduce the magnitude of random measurement errors greatly.

Adjustment for measurement nonlinearities

In the case of sensors that have a nonlinear relationship between the measured quantity and the sensor output, digital processing can convert the output to a linear form, providing that the nature of the nonlinearity is known so that an equation describing it can be programmed into the sensor.

11.3.3 Smart transmitters

In concept, a smart transmitter is almost identical to other intelligent devices described earlier. Although the name “smart transmitter” is sometimes used interchangeably with “smart sensor,” it is perhaps more commonly used to describe an intelligent device that has greater functionality than just the computer-assisted sensing of a variable that a smart sensor conventionally does, particularly with respect to output functions and the ability to compensate for environmental disturbances. In some instances, smart transmitters are known alternatively as *intelligent transmitters*. The term *multivariable transmitter* is also sometimes used, particularly for a device such as a smart flow-measuring instrument. This latter device measures absolute pressure, differential pressure, and process temperature, and computes both the mass flow rate and volume flow rate of the measured fluid.

Many smart transmitters currently available still have an analogue output because of the continuing popularity and investment in 4- to 20-mA current transmission systems. Most devices available have a digital output, but many users convert this to analogue form to maintain compatibility with existing instrumentation systems.

Comparison of performance with other forms of transmitter

The capabilities of smart transmitters can perhaps be emphasized best by comparing them with the attributes of analogue transmitters and with devices known as *programmable transmitters*. The latter have computational power but do not have bidirectional

communication ability, meaning that they are not truly intelligent. The respective attributes of these devices are:

- (a) Analogue transmitters:
 - Require one transmitter for every sensor type and every sensor range
 - Require additional transmitters to correct for environmental changes
 - Require frequent calibration.
- (b) Programmable transmitters:
 - Include a microprocessor but do not have bidirectional communication (hence, they are not truly intelligent)
 - Require field calibration
- (c) Smart transmitters:
 - Include a microprocessor and have bidirectional communication
 - Include secondary sensors that can measure and thus compensate for environmental disturbances
 - Usually incorporate signal conditioning and analogue to digital conversion
 - Often incorporate multiple sensors covering different measurement ranges and allow automatic selection of required range. The range can be readily altered if incorrectly estimated initially.
 - Have a self-calibration capability that allows removal of zero-drift and sensitivity-drift errors.
 - Have a self-diagnostic capability that allows them to report problems or requirements for maintenance.
 - Can adjust for nonlinearities to produce a linear output

Summary of advantages of smart transmitters

The main disadvantage that could be cited for using a smart transmitter instead of a nonsmart one is that it is usually a little larger and heavier than its nonsmart equivalent, but this is not a problem in most applications. There is also normally a greater associated purchase cost. However, these potential disadvantages are minor under most circumstances and are greatly outweighed by the advantages that smart transmitters have:

- Improved accuracy and repeatability
- Automatic calculation of measurement accuracy and compensation for random errors
- Compensation for the loading effect of the measuring process on the measured system
- Improvement in long-term stability and reduction in the required recalibration frequency
- Provision for adjustment for measurement of nonlinearities to produce a linear output
- Reduced maintenance costs
- Self-diagnosis of faults
- Large-range coverage, allowing interoperability and giving increased flexibility

- Remote adjustment of output range, on command from a portable keyboard or from a PC. This saves on technician time compared with carrying out adjustment manually.
- Reduction in number of spare instruments required, because one spare transmitter can be configured to cover any range and so replace any faulty transmitter.
- Possibility of including redundant sensors, which can be used to replace failed sensors and so improve device reliability.
- Allowing remote recalibration or re-ranging by sending a digital signal to them
- Ability to store last calibration date and indicate when next calibration is required
- Single penetration into the measured process rather than the multiple penetration required by discrete devices, making installation easier and cheaper
- Ability to store data so that plant and instrument performance can be analyzed. For example, data relating to the effects of environmental variations can be stored and used to correct output measurements over a large range

Self-calibration

The common use of multiple primary and secondary sensors to measure environmental parameters means that the self-calibration procedure for smart transmitters is more complicated than that for simpler smart sensors. Although the general approach to self-calibration remains similar to that explained earlier for smart sensors, the calibration procedure has to be repeated for each primary and secondary sensor within the transmitter. Recommended practice is to use the simplest calibration procedures available for each sensor in the transmitter. However, care has to be taken to ensure that any interaction between measured variables is considered. This often means that lookup tables in a smart transmitter need to have a particularly large memory requirement to consider the cross-sensitivity of other parameters (e.g., temperature), because a matrix of correction values has to be stored. This means that interpolation calibration is preferable to lookup table calibration compared with when calibrating smart sensors.

Self-diagnosis and fault detection

Fault diagnosis in the sensors within a smart transmitter is often difficult because it is not easy to distinguish between measurement deviation due to a sensor fault and deviation due to a plant fault. The best theoretical approach to this difficulty is to apply mathematical modeling techniques to each sensor and the plant in which it is working, with the aim of detecting inconsistencies in data from the sensor. However, there are few industrial applications of this approach to fault detection in practice, first, because of the cost of implementation, and second, because of the difficulty of obtaining plant models that are robust to plant disturbances. Thus, it is usually necessary to resort to having multiple sensors and using a scheme such as two-out-of-three voting (see Chapter 12 for further details).

11.4 Communication with intelligent devices

The inclusion of computer processing power in intelligent instruments and intelligent actuators creates the possibility of building an instrumentation system in which several intelligent devices collaborate, transmit information to one another, and execute process control functions. Such an arrangement is often known as a *distributed control system*. Additional computer processors can be added to the system as necessary to provide necessary computational power when the computation of complex control algorithms is required. Such an instrumentation system is far more fault-tolerant and reliable than older control schemes in which data from several discrete instruments are carried to a centralized computer controller via long instrumentation cables. This improved reliability occurs because the presence of computer processors in every unit injects a degree of redundancy into the system. Therefore, measurement and control action can continue, albeit in a degraded form, if one unit fails.

To effect the necessary communication when two or more intelligent devices are to be connected as nodes in a distributed system, some form of electronic highway must be provided between them that permits the exchange of information. Apart from data transfer, a certain amount of control information also has to be transferred. The main purpose of this control information is to make sure that the target device is ready to receive information before data transmission starts. This control information also prevents more than one device from trying to send information at the same time.

In modern installations, all communication and data transmission between processing nodes in a distributed instrumentation and control system is carried out digitally along some form of electronic highway. The highway can be either a parallel interface, a local area network (LAN), a digital fieldbus, or a combined LAN/fieldbus. A parallel interface protocol is commonly used to connect a small number of devices spread over a small geographical area, typically a single room. In the case of a large number of devices that are spread over larger geographical distances, typically a single building or site, an electronic highway in the form of either a LAN or a digital fieldbus is used. Instrumentation networks that are geographically larger than a single building or site can also be built, but these generally require transmission systems that include telephone lines as well as local networks at particular sites within the large system.

Manufacturers normally provide all of the hardware and software necessary to create an instrumentation network using the various intelligent devices in their product range. However, problems usually occur if the designer of an instrumentation network wishes to use components sourced from different manufacturers, in which case serious compatibility issues can arise. To overcome this, the Institute of Electronics and Electrical Engineers

(IEEE) developed [IEEE 1451](#), a series of smart device interface standards that allow components from different manufacturers to be connected onto the same network.

11.4.1 *Input–output interface*

An *I/O interface* is required to connect each intelligent device onto the electronic highway. Sensors with a digital output pose few interfacing problems. However, many intelligent devices still have an analogue output that uses the standard 4–20 mA transmission protocol and requires an analogue-to-digital converter in the *I/O interface*. For these, a protocol known as *Highway Addressable Remote Transducer (HART)* is the one most widely used to provide the necessary connection of such devices onto a digital network. HART is a bus-based networking protocol that has become a de facto standard for intelligent devices with an analogue sensor output. HART-compatible devices are provided by all major instrument manufacturers.

HART was always intended to be an interim network protocol to satisfy communication needs in the transitional period between the use of analogue communication with nonintelligent devices and fully digital communication with intelligent devices according to digital fieldbus protocol. Because of this need to support both old and new systems, HART supports two modes of use: hybrid and fully digital.

In *hybrid mode*, status/command signals are digital but data transmission takes place in analogue form (usually in 4–20 mA current loop format). One serious limitation of this mode is that it is impossible to transmit multiple measurement signals on a single bus, because the analogue signals would corrupt each other. Hence, when HART is used in hybrid mode, the network must be arranged in a star configuration using a separate line for each field device rather than a common bus.

In *fully digital mode*, data transmission and transmission of status/command signals are both digital. This enables one cable to carry signals for up to 15 intelligent devices. In practice, the fully digital mode of HART is rarely used, because the data transmission speed is limited compared with fieldbus protocols. Therefore, the main application of the HART protocol has been to provide a hybrid-mode communication capability with intelligent devices when existing analogue measurement signal transmission has to be retained because conversion to fully digital operation would be too expensive.

11.4.2 *Parallel data bus*

A number of different parallel data buses exist. All of these have the common feature of transmitting data in parallel (i.e., several bits are transmitted simultaneously). They also have separate data and control lines, which means that the data lines are used solely for data transmission and all control signals are routed onto dedicated control lines. This

optimizes data transmission speed. However, apart from having this common functionality, there is little compatibility among the different available parallel data buses; significant differences exist in the number of data lines, number of control lines, interrupt structure, data timing system, and logic levels used for operation. Equipment manufacturers tend to keep to the same parallel interface protocol for their entire range of devices, but different manufacturers use varying protocols. Thus, although it will normally be easy to connect a number of intelligent devices that all come from the same manufacturer, interfacing difficulties will likely be experienced if devices from different manufacturers are connected. Fortunately, the situation in the field is not as bad as it sounds, because the IEEE 488 bus has gained prominence as the preferred parallel data bus for instrumentation networks and has been adopted by a large number of manufacturers. Since it was introduced in 1975, the published standard for this bus has been revised on several occasions, including in 2004, when the IEEE published a standard jointly with the International Electrotechnical Commission (IEC) as standard [IEC 60488/IEEE488 IEC/IEEE, 2004](#). The bus provides a parallel interface that facilitates the connection of intelligent instruments, actuators, and controllers within a single room. Physically, the bus consists of a shielded, 24-conductor cable. For a standard IEEE 488 bus, the maximum length of bus allowable is 20 m, with no more than 15 instruments distributed along its length. However, this limit on the length and number of instruments can be overcome by using an active (i.e., with auxiliary power supply) bus extender. The maximum distance between two particular units on the bus should not exceed about 2 m. The maximum data transfer rate permitted by the bus is 1 megabit (Mbit)/s in theory, although the maximum data rate achieved in practice over a full 20-m length of bus is more likely to be in the range of 250–500 kilobits (kbit)/s.

11.4.3 Local area networks

LANs transmit data in digital format along serial transmission lines. Synchronous transmission is normally used because this allows relatively high transmission speeds by transmitting blocks of characters at a time. A typical data block consists of 80 characters: this is preceded by a synchronization sequence and followed by a stop sequence. The synchronization sequence causes the receiver to synchronize its clock with that of the transmitter. The two main standards for synchronous, serial transmission are RS422 and RS485. These are formally published by the American National Standards Institute (ANSI) Telecommunications Industry Association/Electronic Industries Alliance (TIA/EIA) with the codes ANSI/TIA/EIA-422-B and ANSI/TIA/EIA-485.

LANs are particularly valuable for monitoring and controlling systems that have a number of separate sensors, actuators, and control units that are dispersed over a large area. Indeed, for such large instrumentation systems, a LAN is the only viable transmission

medium in terms of performance and cost. Parallel data buses, which transmit data in analogue form, have signal attenuation and noise pickup over large distances, and the high cost of the long, multicore cables that they need is prohibitive.

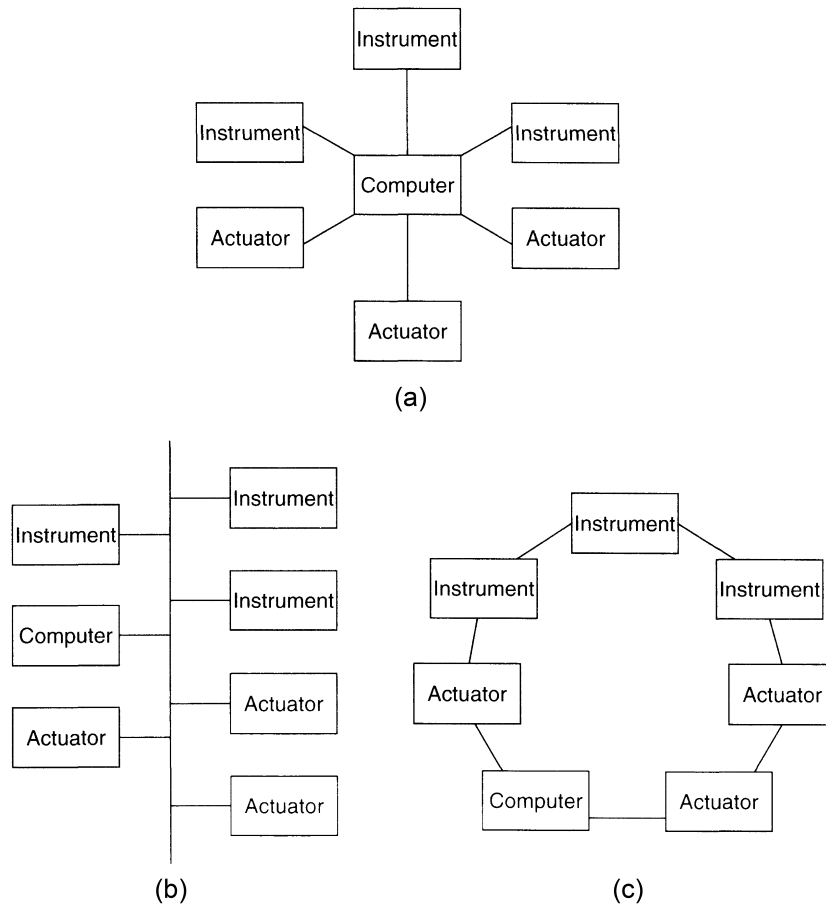
However, the development of instrumentation networks has problems. Careful design of the network is required to prevent corruption of data when two or more devices on the network try to access them simultaneously and perhaps put information onto the data bus at the same time. This problem is solved by designing a suitable network protocol that ensures that network devices do not access the network simultaneously, thus preventing data corruption.

In a LAN, the electronic highway can take the form of either copper conductors or fiber-optic cable. Copper conductors are the cheapest option and allow transmission speeds up to 10 Mbit/s, using either a simple pair of twisted wires or a coaxial cable. However, fiber optic cables are preferred in many networks for a number of reasons. The virtues of fiber optic cables as a data transmission medium have been expounded in Chapter 7. Apart from the high immunity of the signals to noise, a fiber optic transmission system can transfer data at speeds up to 240 Mbit/s. The reduction in signal attenuation during transmission also means that much longer transmission distances are possible without repeaters being necessary. For instance, the allowable distances between repeaters for a fiber-optic network are quoted as 1 km for half-duplex operation and up to 3.5 km for full-duplex operation. In addition, the bandwidth of fiber-optic transmission is higher than for electrical transmission. Some cost saving can be achieved by using plastic fiber-optic cables, but these cannot generally be used over distances greater than about 30 m because signal attenuation is too high.

There are many different protocols for LANs, but these are all based on one of three network structures known as star networks, bus networks, and ring networks, as shown in [Fig. 11.5](#). A LAN operates within a single building or site and can transmit data over distances up to about 500 m without signal attenuation being a problem. For transmission over greater distances, telephone lines are used in the network. Intelligent devices are interfaced to the telephone line used for data transmission via a modem. The **modem** converts the signal into a frequency-modulated analogue form. In this form, it can be transmitted over either the public switched telephone network or private lines rented from telephone companies. The latter, which are dedicated lines, allow higher data transmission rates.

Star networks

In a *star network*, each instrument and actuator is connected directly to the supervisory computer by its own signal cable. One apparent advantage of a star network is that data can be transferred if necessary using a simple serial communication protocol such as RS232. This is an industry standard protocol and so compatibility problems do not arise,

**Figure 11.5**

Network protocols: (A) Star. (B) Bus. (C) Ring.

but it represents old technology in which data transfer is slow. Because of this speed problem, parallel communication is usually preferred even for star networks.

Although star networks are simple in structure, the central supervisory computer node is a critical point in the system and failure of this means total failure of the whole system. When any device in the network needs to communicate with another device, a request has to be made to the central supervisory computer and all data transferred are routed through this central node. If the central node is out of operation for any reason, data communication in the network is stopped.

Ring and bus networks

In contrast to star networks, both ring and bus networks have a high degree of resilience in the face of one node breaking down. Hence, they are generally preferred to star networks.

If the processor in any node breaks down, the data transmission paths in the network are still maintained. Thus, the network can continue to operate, albeit at a degraded performance level, using the remaining computational power in the other processors. Most computer and intelligent instrument/actuator manufacturers provide standard conversion modules that allow their equipment to interface to one of these standard networks.

In a *ring network*, all intelligent devices are connected to a bus that is formed in a continuous ring. Ring protocol sends a special packet (or token) continuously around the ring to control access to the network. A station can send data only when it receives the token. During data transmission, the token is attached to the back of the message sent so that once the information has been safely received, the token can continue on its journey around the network.

A *bus network* is similar to a ring network, but the bus onto which the devices are connected is not continuous. Bus networks are also resilient to the breakdown of one node in the network. A *contention protocol* is normally used. This allows any station to have immediate access to the network unless another station is using it simultaneously, in which case the protocol manages the situation and prevents data loss or corruption. *Ethernet* is the most common form of bus network; this has gained a dominant position in the LAN marketplace.

11.4.4 Digital fieldbuses

“Fieldbus” is a generic word that describes a range of high-speed, bus-based network protocols that support two-way communication in digital format between a number of intelligent devices in a local area network. All forms of transmission are supported, including twisted pair, coaxial cable, fiber-optic, and radio links.

Intelligent devices in an automated system are composed of a range of control elements, actuators, information processing devices, storage systems, and operator displays as well as measurement devices. Hence, any fieldbus protocol must include provision for the needs of all system elements, and the communication requirements of field measurement devices cannot be viewed in isolation from these other elements. The design of a network protocol also has to cater to implementation in both large and small plants. A large plant may contain a number of processors in a distributed control system and have a large number of sensors and actuators. On the other hand, a small plant may be controlled by a single personal computer that provides an operator display on its monitor as well as communicating with plant sensors and actuators.

After fieldbus technology was introduced in 1988, there was no rapid move to develop an international standard; as a consequence, different manufacturers developed their own versions. This resulted in more than 50 different fieldbus protocols, the more prominent

ones of which were Foundation Fieldbus, Profibus, WorldFIP, ControlNet, P-net, CAN, and Interbus. Each fieldbus version supports all devices within the product range of one or several manufacturers, but there is little compatibility among the different protocols on offer. They differ in many major respects such as message format, access protocols, and rules for performance prediction. In recognition of the difficulties inherent in attempting to connect devices from different manufacturers that use a variety of incompatible interface standards and network protocols, the IEC set up a working party charged with defining a standard interface protocol. However, individual manufacturers continued to develop their own versions of fieldbus in parallel with the IEC initiative. The result of this was that when the IEC published its first fieldbus standard in 1999 (IEC 61158), the document had more than 4000 pages and covered eight different protocol sets, defining a standard for each of the 8 main fieldbus systems then in operation (Foundation Fieldbus, Profibus, ControlNet, P-Net, Foundation fieldbus High Speed Ethernet, SwiftNet, WorldFIP, and Interbus). In the period after 1999, addendums to IEC 61158 were published covering further fieldbus standards, particularly with respect to safety buses and high-speed, Ethernet-based fieldbuses.

Despite the failure of IEC 61158 to establish a single fieldbus standard in 1999 and the following years, a consortium of major international instrumentation manufacturers set up the Fieldbus Foundation in an attempt to move toward one worldwide fieldbus standard. This resulted in a version of Foundation Fieldbus, which provided a common standard for and interchangeability among all devices manufactured by members of the consortium. However, competing standards, in particular Profibus, remained.

The basic architecture of Foundation Fieldbus has two levels: upper and lower. The lower level provides for communication between field devices and field I/O devices whereas the upper level enables field I/O devices to communicate with controllers. These two levels have different characteristics. The lower level generally requires few connections, needs only a relatively slow data transfer rate, and must support intrinsically safe working. On the other hand, the upper level requires numerous connections and fast data transfer but does not have to satisfy intrinsic safety requirements. Three standard bus speeds are currently specified for the Foundation Fieldbus lower level of 31.25 kbit/s, 1 Mbit/s, and 2.5 Mbit/s. Maximum cable lengths allowed are 1900 m at 31.25 kbit/s, 750 m at 1 Mbit/s, and 500 m at 2.5 Mbit/s. For the upper Foundation Fieldbus layer, a high-speed ethernet protocol provides a data transfer rate up to 100 Mbit/s.

An attempt by the IEC to establish a fieldbus standard was published in 2014 ([IEC 61158, 2014](#)). This set general standards for specification of physical layers, data links, and application layers but gave up on earlier attempts to establish one common, worldwide fieldbus protocol. In the marketplace, Foundation Fieldbus and Profibus remain the dominant fieldbus versions.

11.5 Summary

The primary purpose of this chapter was to introduce the subject of intelligent devices. However, because computational power is the component that separates intelligent devices from their nonintelligent counterparts, we started the chapter by reviewing the main principles of digital computation. This led us to study the main elements in a computer, how computers operate particularly with respect to program execution, and how computers interface to outside components. We ended the introduction to digital computation with a review of practical issues that have to be considered when incorporating computers into measurement devices.

Moving to the subject of intelligent devices, we found that several terms are used to describe these. Prominent among these terms are names such as “intelligent instrument,” “intelligent sensor,” “smart sensor,” and “smart transmitter.” We learned that there are no industry-wide definitions of what these names mean, except that all are distinguished by incorporating some form of computational power. As a consequence, the same kind of device, even with similar attributes, may be known by two or more names. Therefore, the name that is used to describe a particular intelligent device is subject to the whims and style of its manufacturer.

Having explained this arbitrary nature in the way that intelligent devices are named, we described some commonly held views about the functions performed by devices known as smart sensors and those typically performed by devices known as smart transmitters. The conclusion drawn from this comparison of functions was that devices called smart transmitters tended to have a greater functionality than those called smart sensors.

Our investigation into the features of intelligent devices led us to conclude that these have significant advantages compared with nonintelligent devices. Perhaps the biggest single benefit is improved measurement accuracy. This is achieved by using a computer processor within each device that performs actions such as compensating for random errors, adjusting for measurement nonlinearities, and compensating for the loading effect of the measuring process on the measured system. Processing power also enables devices to perform functions such as remote self-calibration and self-diagnosis of faults. Smart transmitters typically have additional features such as incorporating multiple primary sensors covering different measurement ranges and allowing automatic selection of required range, including secondary sensors that can measure and compensate for environmental disturbances, plus incorporating signal conditioning and analogue-to-digital conversion functions. Sometimes smart transmitters also have redundant sensors, which can be used to replace failed sensors and so improve device reliability.

We then looked at the issues regarding communication between intelligent devices and other elements in a measurement/process control system. All communication and data

transmission between processing nodes in a distributed instrumentation and control system require the use of some form of electronic highway that can be a parallel interface, a LAN, a digital fieldbus, or a combined LAN/fieldbus. We then concluded the chapter with a look in broad detail at various features in these alternative forms of electronic highway, but observed that there was little point in studying the fine details of any particular form of highway because there were continuing developments in the format of highways, and particularly in the protocols used in LANs and digital fieldbuses. This means that the inclusion of a detailed study in the book would quickly be outdated.

11.6 Problems

- 11.1. Explain the principal components in the microprocessor contained within an intelligent instrument.
- 11.2. What are the two main types of computer memory.? Which type is predominantly used in intelligent instruments, and why?
- 11.3. What are the mechanisms for programming and program execution within an intelligent instrument?
- 11.4. Discuss the operation of the I/O interface within the processor of an intelligent instrument, mentioning particularly the mechanisms of address decoding and data transfer control.
- 11.5. What are the practical considerations involved in implementing a computer processor within a measurement system?
- 11.6. How does an intelligent instrument correct for environmentally induced errors in measurements? What preconditions must be satisfied to allow an intelligent instrument to correct for such errors? How are these preconditions satisfied?
- 11.7. Explain how adding intelligence to an instrument improves the accuracy of volume flow rate measurements.
- 11.8. What additional functions does an intelligent instrument typically perform apart from the correction of environmentally induced errors in measurements?
- 11.9. Describe the typical function of devices known as *smart sensors*.
- 11.10. Describe the mechanisms for communication between an intelligent sensor and the other components in a measurement system.
- 11.11. What is a smart transmitter? What are the main advantages of smart transmitters over analogue transmitters?
- 11.12. What problems do smart transmitters pose in terms of calibration, self-diagnosis, and fault detection functions?

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Measurement reliability and safety systems

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12.1 Introduction

In previous chapters of this book, we discussed the design of measurement systems, and we said a lot about how the performance of measurement systems with respect to

parameters such as accuracy can be improved. However, this earlier discussion was mainly about the attributes of a measurement system when it is new. We considered the effects of the passage of time only by noting that the characteristics of a measurement system degrade over time and have to be restored to their starting point by the process of recalibration. What we have not yet considered is the possibility of faults developing in measurement systems. At best, these faults impair the performance of the system, and at worst, they cause the system to stop working entirely. In safety-critical applications, measurement system faults can also have a serious adverse effect on the larger system of which the measurement system is a part.

It is therefore appropriate for us to devote this chapter to a study of measurement system reliability issues and their effect on safety. We will start by looking at how reliability is formally defined and will discuss its theoretical principles. This will then lead us on to consider ways in which reliability can be quantified. In particular, we will look at two laws that quantify the reliability of system components that are in series and in parallel with one another. This will enable us to examine how these laws can be applied to improve the reliability of measurement systems. We will also look at general precautions that can be taken to reduce the failure rate of instruments, including choosing instruments that can withstand operating conditions that are expected, protecting them adequately against damage during use, calibrating them at prescribed intervals to ensure that measurement inaccuracy remains within acceptable bounds, and duplicating critical measurement system components.

Because software is an important contributor to measurement system reliability, particularly with the widespread use of intelligent devices, we will extend our treatise on reliability to consider the reliability of software within a measurement system. The factors affecting reliability in software are fundamentally different from those affecting the reliability of hardware components. This is because software does not change with time. Therefore, the reliability of software has to be quantified in terms of the probability of the software failing because of some error within it that has existed since it was written but has remained undetected. This kind of failure usually occurs when some particular combination of input data is applied to the software and, in consequence, may not occur until the software has been in use for a considerable period of time. Having established a satisfactory way to quantify software reliability, we will consider what can be done to improve reliability.

The final consideration in this chapter is system safety. Measurement systems can have an impact on system safety in two main ways. First, failure of the measurement system may cause a dangerous situation to arise in a process because incorrect data are fed into the process control system. Second, the process itself may develop a dangerous fault that the measurement system fails to detect, thus preventing the operation of emergency responses such as the sounding of alarms or the opening of pressure relief valves and so on. To respond to potential safety problems associated with the malfunction of measurement systems, we will look at the main ways that are available to design safety systems.

12.2 Reliability

The reliability of measurement systems can be quantified as the mean time between faults occurring in the system. In this context, a fault means the occurrence of an unexpected condition in the system that causes the measurement output to either be incorrect or not to exist at all. The following sections summarize the principles of reliability theory that are relevant to measurement systems.

12.2.1 Principles of reliability

The reliability of a measurement system is defined as the ability of the system to perform its required function within specified working conditions for a stated period of time. Unfortunately, factors such as manufacturing tolerances in an instrument and varying operating conditions conspire to make the faultless operating life of a system impossible to predict. Such factors are subject to random variation and chance, and therefore reliability cannot be defined in absolute terms. The nearest one can get to an absolute quantification of reliability are quasiabsolute terms such as the mean time between failures (MTBF), which expresses the average time that the measurement system works without failure. Otherwise, reliability has to be expressed as a statistical parameter that defines the probability that no faults will develop over a specified interval of time.

In quantifying reliability for a measurement system, an immediate difficulty that arises is defining what counts as a fault. Total loss of a measurement output is an obvious fault, but a fault that causes a finite but incorrect measurement is more difficult to identify. The usual approach is to identify such faults by applying statistical process control techniques.

Reliability quantification in quasiabsolute terms

Although reliability is essentially probabilistic in nature, it can be quantified in quasiabsolute terms by the MTBF and the mean time to failure (MTTF) parameters. These two quantities are usually average values calculated over a number of identical instruments, and therefore actual values for any particular instrument may vary substantially from the average value.

The MTBF is a parameter that expresses the average time between faults occurring in an instrument, calculated over a given period of time. For example, suppose the history of an instrument is logged over a 360-day period and the time intervals in days between faults occurring is:

11 23 27 16 19 32 6 24 13 21 26 15 14 33 29 12 17 22

The mean interval is 20 days, which is therefore the MTBF. An alternative way to calculate MTBF is simply to count the number of faults occurring over a given period.

In this example, 18 faults were recorded over a period of 360 days, and so the MTBF can be calculated as: $MTBF = 360/18 = 20$ days.

Unfortunately, in the case of instruments that have a high reliability, such in-service calculation of reliability in terms of the number of faults occurring over a given period of time becomes grossly inaccurate because faults occur too infrequently. In this case, MTBF predictions provided by the instrument manufacturer can be used, because manufacturers have the opportunity to monitor the performance of a number of identical instruments installed in different companies. If there is a total of F faults recorded for N identical instruments in time T , the MTBF can be calculated as $MTBF = TN/F$. One drawback of this approach is that it does not consider the conditions of use, such as the operating environment.

The MTTF is an alternative way to quantify reliability that is normally used for devices such as thermocouples that are discarded when they fail for the first time. MTTF expresses the average time before failure occurs, calculated over a number of identical devices. Suppose a batch of 20 thermocouples is put through an accelerated-use test in the same environment and the simulated time before failure (in months) of each device is:

7 9 13 6 10 11 8 9 14 8 8 12 9 15 11 9 10 12 8 11

The mean of these 20 numbers is 10. Therefore, the simulated MTTF is 10 months.

The final reliability-associated term of importance in measurement systems is the *mean time to repair* (MTTR). This expresses the average time needed for repair of an instrument. MTTR can also be interpreted as the *mean time to replace*, because replacement of a faulty instrument by a spare one is usually preferable in manufacturing systems to losing production while an instrument is repaired. As an example, suppose the time in hours taken to repair an instrument over a history of 18 breakdowns is recorded, with the following times measured:

4 1 3 2 1 9 2 1 7 2 3 4 1 3 2 4 4 1

The mean of these values is 3 and the MTTR is therefore 3 hours.

The MTBF and MTTR parameters are often expressed in terms of a combined quantity known as the **availability** figure. This measures the proportion of the total time that an instrument is working (i.e., the proportion of the total time it is in an unfailed state). The availability is defined as the ratio:

$$\text{Availability} = \frac{MTBF}{MTBF + MTTR}$$

In measurement systems, the aim must always be to maximize the MTBF figure and minimize the MTTR figure, thereby maximizing the availability. As far as the MTBF and

MTTF figures are concerned, good design and high-quality control standards during manufacture are the appropriate means of optimizing these figures. Design procedures resulting in faults being easy to repair are also an important factor in reducing the MTTR figure.

■ Example 12.1

Data are collected by a manufacturer about a particular measuring instrument that is used 24 hours/day, 7 days per week, recording both the intervals in days between breakdowns and the time taken (in days) to repair each fault that causes a breakdown. The following data are collected:

Times before breakdown in days: 11.4 16.7 9.8 12.3 17.9 14.1 20.2 15.0 8.6 18.5

Time to repair faults (days): 0.2 0.7 1.4 0.1 0.6 3.4 0.5 0.2 1.3 0.8

Calculate the availability of the instrument.

■ Solution

$$\begin{aligned} \text{Mean time between failures (MTBF)} &= \frac{\sum(11.4 + 16.7 + 9.8 + 12.3 + 17.9 + 14.1 \\ &\quad + 20.2 + 15.0 + 8.6 + 18.5)}{10} \\ &= 14.45 \end{aligned}$$

$$\begin{aligned} \text{MTTR} &= \frac{\sum(0.2 + 0.7 + 1.4 + 0.1 + 0.6 + 3.4 + 0.5 + 0.2 \\ &\quad + 1.3 + 0.8)}{10} = 0.92 \end{aligned}$$

$$\text{Availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} = \frac{14.45}{14.45 + 0.92} = 0.94 = 94\%$$

Failure patterns

The pattern of failure in an instrument may increase, stay the same, or decrease over its life. In the case of *electronic components*, the failure rate typically changes with time in the manner shown in Fig. 12.1a. This form of characteristic is frequently known as a *bath tub curve*. Early in their life, electronic components can have a high rate of fault

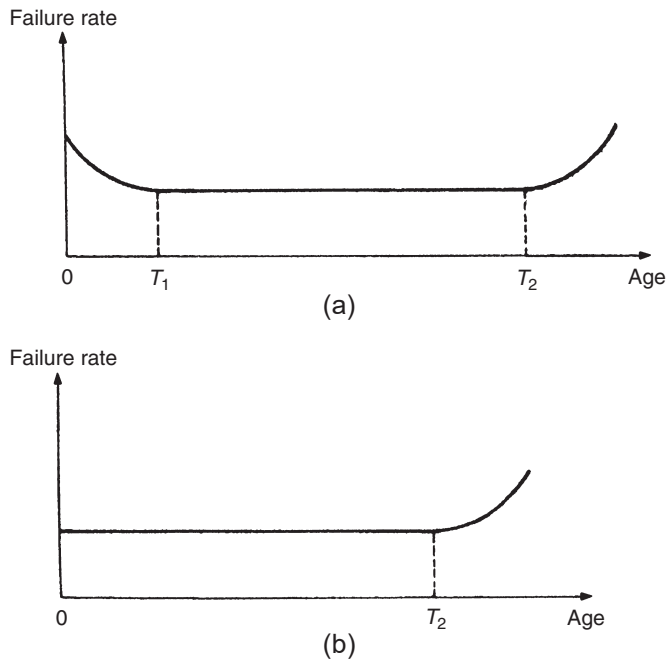


Figure 12.1

Typical variation of reliability with component age: (a) Electronic components (bathtub curve).
(b) Mechanical components.

incidence up to time T_1 (Fig. 12.1a). After this initial working period, the fault rate decreases to a low level and remains there until time T_2 when aging effects cause the fault rate to resume increasing. Instrument manufacturers often burn in electronic components for a length of time corresponding to time T_1 . This means that the components have reached the high-reliability phase of their life before they are supplied to customers.

Mechanical components usually have different failure characteristics, as shown in Fig. 12.1b. Material fatigue is a typical reason for the failure rate to increase over the life of a mechanical component. In the early part of their life, when all components are relatively new, many instruments exhibit a low incidence of faults. Then, at a later stage, time T_2 , when fatigue and other aging processes start to have a significant effect, the rate of faults increases and continues to increase thereafter.

Complex systems containing many different components often exhibit a constant pattern of failure over their lifetime. Each component within such systems has its own failure pattern in which the failure rate increases or decreases with time. The greater the number of such components within a system, the greater is the tendency for the failure patterns in the individual components to cancel out and for the rate of fault incidence to assume a constant value.

Reliability quantification in probabilistic terms

In probabilistic terms, the reliability $R(T)$ of an instrument X is defined as the probability that the instrument will not fail within a certain period of time T . The unreliability or likelihood of failure $F(T)$ is a corresponding term that expresses the probability that the instrument will fail within the specified time interval. $R(T)$ and $F(T)$ are related by the expression:

$$F(T) = 1 - R(T) \quad (12.1)$$

To calculate $R(T)$, accelerated lifetime testing¹ is carried out for a number (N) of identical instruments. Provided all instruments have similar conditions of use, the times of failure, t_1, t_2, \dots, t_n will be distributed about the MTTF t_m . If the probability density of the time-to-failure is represented by $f(t)$, the probability that a particular instrument will fail in time interval δt is given by $f(t)\delta t$, and the probability that the instrument will fail before time T is given by:

$$F(T) = \int_0^T f(t)dt$$

The probability that the instrument will fail in time interval ΔT after T , assuming that it has survived to time T , is given by:

$$\frac{F(T + \Delta T) - F(T)}{R(T)},$$

where $R(T)$ is the probability that the instrument will survive to time T . Dividing this expression by ΔT gives the average failure rate in the interval from T to $T + \Delta T$ as:

$$\frac{F(T + \Delta T) - F(T)}{\Delta T R(T)}.$$

In the limit as $\Delta T \rightarrow 0$, the instantaneous failure rate at time T is given by:

$$\theta_f = \frac{d[F(T)]}{dt} \frac{1}{R(T)} = \frac{F'(T)}{R(T)} \quad (12.2)$$

If it is assumed that the instrument is in the constant failure rate phase of its life, denoted by the interval between times T_1 and T_2 in Fig. 12.1, then the instantaneous failure rate at T is also the mean failure rate, which can be expressed as the reciprocal of the MTBF (i.e., mean failure rate = $\theta_f = 1/t_m$).

¹ Accelerated lifetime testing means subjecting an instrument to a much greater frequency of use than would normally be expected. If an instrument is normally used 10 times per day, 100 days of normal use can be simulated by using it 1000 times in a single day.

Differentiating (Eq. 12.1) with respect to time gives: $F'(T) = -R'(T)$. Hence, substituting for $F'(T)$ in (Eq. 12.2) gives:

$$\theta_f = -\frac{R'(T)}{R(T)}.$$

This can be solved (Johnson et al., 2017) to give the expression:

$$R(T) = \exp(-\theta_f T) \quad (12.3)$$

Examination of Eq. (12.3) shows that at time $t = 0$, the unreliability is 0. Also, because t tends to ∞ , the unreliability tends to a value of 1. This agrees with intuitive expectations that the value of unreliability should lie between values of 0 and 1. Another point of interest in Eq. (12.3) is to consider the unreliability when $T = \text{MTBF}$ (i.e., when $T = t_m$). Then, $F(T) = 1 - \exp(-1) = 0.63$: that is, the probability of a product failing after it has been operating for a length of time equal to the MTBF is 63%.

Further analysis of Eq. (12.3) shows that for $T/t_m \leq 0.1$:

$$F(T) \approx T/t_m \quad (12.4)$$

This is a useful formula for calculating (approximately) the reliability of a critical product that is used only for a time that is a small proportion of its MTBF.

■ Example 12.2

If the MTTF of an instrument is 50,000 hours, calculate the probability that it will not fail during the first 10,000 hours of operation. ■

■ Solution

From (Eq. 12.3),

$$R(T) = \exp(-\theta_f T) = \exp(-10,000 / 50,000) = 0.8187. \quad \blacksquare$$

■ Example 12.3

If the MTTF of an instrument is 80,000 hours, calculate the probability that it will not fail during the first 8000 hours of operation. ■

■ Solution

In this case, $T/t_m = 80,000/8000 = 0.1$, and so Eq. (12.4) can be applied, giving $R(T) = 1 - F(T) \approx 1 - T/t_m \approx 0.9$. To illustrate the small level of inaccuracy involved in using the approximate expression of (Eq. 12.4), if we calculate the probability according to (Eq. 12.3) we get $R(T) = \exp(-0.1) = 0.905$. Thus, there is a small but finite error in applying (Eq. 12.4) instead of (Eq. 12.3).

12.2.2 Laws of reliability in complex systems

Measurement systems are usually composed of a number of components connected together in series, and hence it is necessary to know how the reliabilities of individual components are aggregated into a reliability figure for the whole system. In some cases, identical measurement components are put in parallel to improve reliability, because the measurement system then fails only if all of the parallel components fail. These two cases are covered by particular laws of reliability.

Reliability of components in series

A measurement system consisting of several components in series fails when any one of the separate components develops a fault. The reliability of such a system can be quantified as the probability that none of the components will fail within a given interval of time. For a system of n series components, reliability R_S is the product of the separate reliabilities of the individual components according to the joint probability rule:

$$R_S = R_1 R_2 \cdots R_n \quad (12.5)$$

■ Example 12.4

A measurement system consists of a sensor, an output conversion element, and a signal processing circuit, for which the reliability figures are 0.9, 0.95, and 0.99, respectively. Calculate the reliability of the whole measurement system.

■ Solution

Applying (Eq. 12.5), $R_S = 0.9 \times 0.95 \times 0.99 = 0.85$.

Reliability of components in parallel

One way to improve the reliability of a measurement system is to connect two or more instruments in parallel. This means that the system fails only if every parallel instrument fails. For such systems, system reliability R_S is given by:

$$R_S = 1 - F_S \quad (12.6)$$

where F_S is the unreliability of the system. The equation for calculating F_S is similar to (Eq. 12.5). Thus, for n instruments in parallel, the unreliability is given by:

$$F_S = F_1 F_2 \cdots F_n \quad (12.7)$$

If all the instruments in parallel are identical, (Eq. 12.7) can be written in the simpler form:

$$F_S = (F_X)^n \quad (12.8)$$

where F_X is the unreliability of each instrument.

■ Example 12.5

In a particular safety-critical measurement system, three identical instruments are connected in parallel. If the reliability of each instrument is 0.95, calculate the reliability of the measurement system.

■ Solution

From (Eq. 12.1), the unreliability of each instrument F_X is given by: $F_X = 1 - R_X = 1 - 0.95 = 0.05$.

Applying (Eq. 12.8), $F_S = (F_X)^3 = (0.05)^3 = 0.000125$.

Thus, from (Eq. 12.6), $R_S = 1 - F_S = 1 - 0.000125 = 0.999875$.

12.2.3 Improving measurement system reliability

When designing a measurement system, the aim is always to reduce the probability of the system failing to as low a level as possible. An essential requirement in achieving this is to ensure that the system is replaced at or before time T_2 in its life, shown in Fig. 12.1, when the statistical frequency of failures starts to increase. Therefore, the initial aim should be to set lifetime T equal to T_2 and minimize the probability $F(T)$ of the system failing within this

specified lifetime. Once all measures to reduce $F(T)$ have been applied, the acceptability of the reliability $R(T)$ has to be assessed against the requirements of the measurement system. Inevitably, cost enters into this, because efforts to increase $R(T)$ usually increase the cost of buying and maintaining the system. Lower reliability is acceptable in some measurement systems in which the cost of failure is low, such as in manufacturing systems in which the cost of lost production or the loss due to making out-of-specification products is not serious. However, for other applications, such as when failure of the measurement system incurs high costs or causes safety problems, high reliability is essential. Some special applications for which human access is difficult or impossible, such as measurements in unmanned spacecraft, satellites, and nuclear power plants, demand especially high reliability because repair of faulty measurement systems is impossible.

The various means of increasing $R(T)$ are considered next. However, once all efforts to increase $R(T)$ have been exhausted, the only solution available if the reliability specified for a working period T is still not high enough is to reduce period T over which the reliability is calculated by replacing the measurement system earlier than time T_2 .

Choice of instrument

The type of components and instruments used within measuring systems has a large effect on system reliability. Of particular importance in choosing instruments is to consider conditions existing in the operating environment in which they will be used. In parallel with this, appropriate protection must be given (for example, enclosing thermocouples in sheaths) if it is anticipated that the environment may cause premature failure of an instrument. Some instruments are more affected than others and thus will be more likely to fail in certain environments. The knowledge necessary to make informed choices about the suitability of instruments for particular environments, and the correct protection to give them, requires many years of experience, although instrument manufacturers can give useful advice in most cases.

Instrument protection

Adequate protection of instruments and sensors from the effects of the operating environment is necessary. For example, thermocouples and resistance thermometers (RTDs) should be protected by a sheath under adverse operating conditions.

Regular calibration

The most common reason for faults occurring in a measurement system, in which the error in the measurement goes outside acceptable limits, is drift in the performance of the instrument away from its specified characteristics. Such faults can usually be avoided by ensuring that the instrument is recalibrated at the recommended time intervals. Types of intelligent instruments and sensors that perform self-calibration have clear advantages in this respect.

Redundancy

Redundancy means using two or more identical measuring instruments or measurement system components in parallel such that any one instrument can provide the required measurement. Eq. (12.5) showed the use of three identical instruments in parallel to make a particular measurement instead of a single instrument. This increased the reliability from 95% to 99.99%. Redundancy can also be applied in larger measurement systems in which particular components seriously degrade the overall reliability of the system. Consider the five-component measurement system shown in Fig. 12.2a, in which the reliabilities of the individual system components are $R_1 = R_3 = R_5 = 0.99$ and $R_2 = R_4 = 0.95$.

Using (Eq. 12.5), system reliability is given by
 $R_S = 0.99 \times 0.95 \times 0.99 \times 0.95 \times 0.99 = 0.876$.

Now, consider what happens if redundant instruments are put in parallel with the second and fourth system component, as shown in Fig. 12.2b. The reliabilities of these sections of the measurement system are modified to new values, R'_2 and R'_4 , which can be calculated using Eqs. (12.1), (12.6), and (12.8): $F_2 = 1 - R_2 = 0.05$. Hence, $F'_2 = (0.05)^2 = 0.0025$ and $R'_2 = 1 - F'_2 = 0.9975$. $R'_4 = R'_2$, because $R_4 = R_2$.

Using (Eq. 12.5) again, system reliability is now
 $R_S = 0.99 \times 0.9975 \times 0.99 \times 0.9975 \times 0.99 = 0.965$.

Thus, the redundant instruments have improved system reliability by a large amount. However, this improvement in reliability is achieved only at the cost of buying and maintaining the redundant components that have been added to the measurement system. If this practice of using redundant instruments to improve reliability is followed, provision must be made to replace failed components with the standby units. The most efficient way

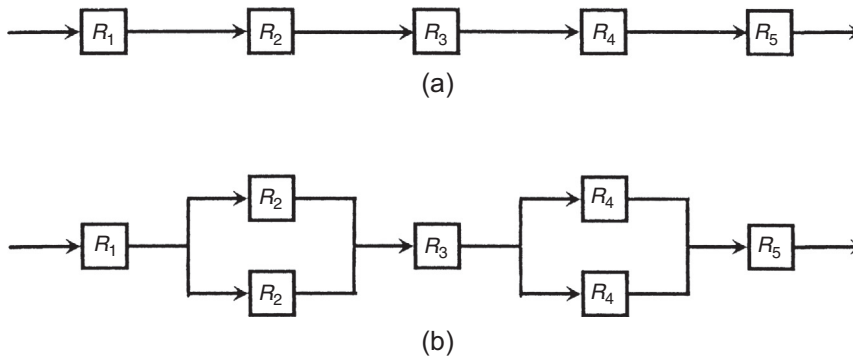


Figure 12.2

Improving measurement system reliability: (a) original system; (b) duplicating components that have poor reliability.

to do this is to use an automatic switching system, but manual methods of replacement can also work reasonably well in many cases.

The principle of increasing reliability by placing components in parallel is often extended to other aspects of measurement systems such as the connectors in electrical circuits, because bad connections frequently cause malfunctions. For example, two separate pairs of plugs and sockets are frequently used to make the same connection. The second pair is redundant (i.e., the system can usually function at 100% efficiency without it), but it becomes useful if the first pair of connectors fails.

12.2.4 Software reliability

Because computer processors and the software within them are increasingly found in most measurement systems, the issue of the reliability of such components has become important. Computer hardware behaves very much like electronic components in general, and the rules for calculating reliability given earlier can be applied. However, the factors affecting reliability in software are fundamentally different. Application of the general engineering definition of reliability to software is inappropriate because the characteristics of error mechanisms in software and in engineering hardware are fundamentally different. Hardware systems that work correctly when first introduced can develop faults at any time in the future, and so the MTBF is a sensible measure of reliability. However, software does not change with time: if it starts off being error-free, it will remain so. Therefore, what we need to know, in advance of its use, is whether faults will be found in the software after it has been put into use. Thus, for software, an MTBF reliability figure is of little value. Instead, we must somehow express the probability that errors will not occur in it.

Quantifying software reliability

A fundamental problem in predicting that errors will not occur in software is that, however exhaustive the testing, it is impossible to say with certainty that all errors have been found and eliminated. Errors can be quantified by three parameters, D , U , and T , in which D is the number of errors detected by testing the software, U the number of undetected errors, and T the total number of errors (both detected and undetected).

Hence,

$$U = T - D \quad (12.9)$$

Good program testing can detect most errors and so make D approach T so that U tends toward 0. However, because the value of T can never be predicted with certainty, it is difficult to predict that software is error-free, whatever degree of diligence is applied during testing procedures.

No matter what approach is taken to quantifying reliability, software testing is an essential prerequisite to the quantification methods available. Although it is never possible to detect all errors that might exist, the aim must always be to find and correct as many errors as possible by applying a rigorous testing procedure. Software testing is a particularly important aspect of the wider field of software engineering. However, because it is a subject of considerable complexity, the detailed procedures available are outside the scope of this book. A large number of books now cover good software engineering in general and software testing procedures in particular, and the reader requiring further information is referred to referenced texts such as [Fenton and Bieman \(2014\)](#), [Naik and Tripathy \(2008\)](#) and [Pfleeger and Atlee \(2009\)](#).

One approach to quantifying software reliability ([Fenton and Bieman, 2014](#)) is to monitor the rate of error discovery during testing and then extrapolate this into an estimate of the MTBF for the software once it has been put into use. Testing can then be extended until the predicted MTBF is greater than the projected time-horizon of use of the software. This approach is rather unsatisfactory because it accepts that errors in the software exist and predicts only that errors will not emerge frequently.

Confidence in the measurement system is much greater if we can say “There is a high probability that there are zero errors in the software” rather than “There are a finite number of errors in the software but they are unlikely to emerge within the expected lifetime of its usage.” One way to achieve this is to estimate the value of T (total number of errors) from initial testing and then carry out further software testing until the predicted value of T is 0, in a procedure known as *error seeding*. In this method, the programmer responsible for producing the software deliberately puts a number of errors, E , into the program, such that the total number of errors in the program increases from T to T' , in which $T' = T + E$. Testing is then carried out by a different programmer who will identify a number of errors given by D' , in which $D' = D + E'$ and E' is the number of deliberately inserted errors that are detected by this second programmer. Normally, real errors detected (D) will be less than T and seeded errors detected (E') will be less than E . However, on the assumption that the ratio of seeded errors detected to the total number of seeded errors will be the same as the ratio of the real errors detected to the total number of real errors, the following expression can be written:

$$\frac{D}{T} = \frac{E'}{E} \quad (12.10)$$

Because E' is measured, E is known, and D can be calculated from the number of errors, D' , detected by the second programmer according to $D = D' - E'$, the value of T can then be calculated as:

$$T = DE/E' \quad (12.11)$$

■ Example 12.6

The author of a digital signal-processing algorithm that forms a software component within a measurement system adds 12 deliberate faults to the program. The program is then tested by a second programmer, who finds 34 errors. Of these detected errors, the program author recognizes 10 of them as being seeded errors. Estimate the original number of errors present in the software (i.e., excluding the seeded errors).

■ Solution

The total number of errors detected (D') is 34 and the program author confirms that the number that were seeded errors within these (E') is 10 and that the total number of seeded errors (E) was 12. Because $D' = D + E'$ (see earlier), $D = D' - E' = 24$. Hence, from (Eq. 12.11), $T = DE/E' = 24 \times 12/10 = 28.8$.

One flaw in (Eq. 12.11) is the assumption that the seeded errors are representative of all the real (unseeded) errors in the software in both proportion and character. This assumption is never entirely valid in practice because if errors are unknown, their characteristics are also unknown. Thus, although this approach may be able to give an approximate indication of the value of the total number of errors, T , it can never predict its actual value with certainty.

An alternative to error seeding is the *double-testing* approach, in which two independent programmers test the same program (Fenton and Bieman, 2014). Suppose the number of errors detected by each programmer is D_1 and D_2 , respectively. Normally, errors detected by the two programmers will be partly common and partly different. Let C be the number of common errors that both programmers find. The error detection success of each programmer can be quantified as:

$$S_1 = D_1/T ; S_2 = D_2/T \quad (12.12)$$

It is reasonable to assume that the proportion of errors, D_1 , that programmer 1 finds out of the total number of errors, T , is the same proportion as the number of errors, C , that he or she finds out of the number, D_2 , found by programmer 2:

$$\frac{D_1}{T} = \frac{C}{D_2} = S_1 ,$$

and hence

$$D_2 = \frac{C}{S_1} \quad (12.13)$$

From (Eq. 12.12), $T = D_2/S_2$, and substituting in the value of D_2 obtained from (Eq. 12.13), the following expression for T is obtained:

$$T = C/S_1S_2 \quad (12.14)$$

From (Eq. 12.13), $S_1 = C/D_2$ and from (Eq. 12.12), $S_2 = D_2S_1/D_1 = C/D_1$. Thus, substituting for S_1 and S_2 in (Eq. 12.14):

$$T = D_1D_2/C \quad (12.15)$$

■ Example 12.7

A piece of software is tested independently by two programmers, and the number of errors found is 24 and 26, respectively. Of the errors found by programmer 1, 21 are the same as those found by programmer 2.

■ Solution

$D_1 = 24$, $D_2 = 26$, and $C = 21$. Hence, applying (Eq. 12.15), $T = D_1D_2/C = 24 \times 26/21 = 29.7$.

Program testing should continue until the number of errors that have been found is equal to the predicted total number of errors, T . In the case of Example 12.7, this means continuing testing until 30 errors have been found. However, the problem with doing this is that T is only an estimated quantity and there may actually be only 28 or 29 errors in the program. Thus, continuing testing until 30 errors have been found would mean testing forever! Hence, once 28 or 29 errors have been found and continued testing for a significant time after this has detected no more errors, the testing procedure should be terminated even though the program could still contain one or two errors. The approximate nature of the calculated value of T also means that its true value could be 31 or 32, and therefore the software may still contain errors if testing is stopped once 30 errors have been found. Thus, the fact that T is only an estimated value means the statement that a program is error-free once the number of errors detected is equal to T can be expressed only in probabilistic terms.

To quantify this probability, further testing of the program is necessary. The starting point for this further testing is the stage at which the total number of errors that are predicted, T , have been found (or when the number found is slightly less than T but further testing does not seem to find any more errors). The next step is to seed the program with W new errors and then test it until all W seeded errors have been found. Provided that no new errors have been found during this further testing phase, the probability that the program is error-free can then be expressed as:

$$P = W/(W + 1) \quad (12.16)$$

However, if any new error is found during this further testing phase, the error must be corrected and then the seeding and testing procedure must be repeated. Assuming that no new errors are detected, a value of $W = 10$ gives $P = 0.91$ (probability of 91% that the program is error-free). To get to 99% error-free probability, W has to be 99.

■ Example 12.8

A program is tested and an estimate of 24 is obtained for the total number of errors. The program is then seeded with 30 deliberate errors and further testing is carried out until all 30 seeded errors have been found. If no new (previously undetected) errors are found during this further testing to find the seeded errors, calculate the probability that the program is error-free after this further testing.

■ Solution

If no new errors (i.e., unseeded ones) are found during the second stage of testing, the probability that the program is error-free can be expressed by the expression for P in Eq. (12.16).

For the given value of $W = 30$, $P = W/(W + 1) = 30/31 = 0.968$.

Thus, the probability that the program is error-free is 0.968 (96.8%).

Improving software reliability

The a priori requirement in achieving high reliability in software is to ensure that it is produced according to sound software engineering principles. Formal standards for achieving high quality in software are set out in [BS/ISO/IEC 90003, 2014](#). Libraries and bookshops, especially academic ones, offer a number of texts on good software design procedures. These differ significantly in their style of approach, but all have the common

aim of encouraging the production of error-free software that conforms to the design specification. It is not within the scope of this book to enter into arguments about which software design approach is best, because the choice among different software design techniques largely depends on personal preference. However, it is essential that software contributing to a measurement system be produced according to good software engineering principles.

The second stage of reliability enhancement is the application of a rigorous testing procedure as described in the last section. This is a very time-consuming and hence expensive business, and so testing should continue only until the calculated level of reliability is the minimum needed for the requirements of the measurement system. However, if a very high level of reliability is demanded, such rigorous testing becomes extremely expensive and an alternative approach known as *N-version programming* is often used. *N-version programming* requires N different programmers to produce N different versions of the same software according to a common specification. Then, assuming that there are no errors in the specification itself, any difference in the output of one program compared with the others indicates an error in that program. Commonly, $N = 3$ is used: that is, three different versions of the program are produced, but $N = 5$ is used for measurement systems that are very critical. In this latter case, a voting system is used, which means that up to two of the five versions can be faulty without incorrect outputs being generated.

Unfortunately, although this approach reduces the chance of software errors in measurement systems, it is not foolproof, because the degree of independence between programs cannot be guaranteed. Different programmers, who may be trained in the same place and use the same design techniques, may generate different programs that have the same errors. Thus, this method has the best chance of success if the programmers are trained independently and use different design techniques.

Languages such as Ada also improve the safety of software because they contain special features designed to detect the kind of programming errors that are commonly made. Such languages have been specifically developed with safety-critical applications in mind.

12.3 Safety systems

Measurement system reliability is usually inexorably linked to safety issues, because measuring instruments to detect the onset of dangerous situations that may potentially compromise safety are a necessary part of all safety systems implemented. Statutory safety legislation now exists in all countries around the world. Although the exact content of legislation varies among countries, a common theme is to set out responsibilities for all personnel whose actions may affect the safety of themselves or others. Penalties are

prescribed for contravention of the legislation, which can include fines, custodial sentences, or both. Legislation normally sets out duties for both employers and employees.

Duties of employers include:

- To ensure that process plant is operated and maintained in a safe way so that the health and safety of all employees are protected.
- To provide such training and supervision as is necessary to ensure the health and safety of all employees.
- To provide a monitoring and shutdown system (safety system) for any process plant or other equipment that may cause danger if certain conditions arise.
- To ensure the health and safety, as far as is reasonably practical, of all persons who are not employees but who may reasonably be expected to be at risk from operations carried out by a company.

Duties of employees include:

- To take reasonable care for their own safety
- To take reasonable care for the safety of others
- To avoid misusing or damaging any equipment or system that is designed to protect people's safety.

The primary concern of measurement and instrumentation technologists with regard to safety legislation is, first, to ensure that all measurement systems are installed and operated in a safe way and second, to ensure that instruments and alarms installed as part of safety protection systems operate reliably and effectively.

Intrinsic safety

Intrinsic safety describes the ability of measuring instruments and other systems to operate in explosive or flammable environments with no risk of sparks or arcs causing an explosion or fire. The detailed design of systems to make them intrinsically safe is outside the scope of this book. However, the general principles are either to design electrical systems in a way that avoids the possibility of parts that may spark coming into contact with the operating environment or else to avoid using electrical components altogether. The latter point means that pneumatic sensors and actuators continue to find favor in some applications despite the advantages of electrical devices in most other respects.

Installation practice

Good installation practice is necessary to prevent the possibility of people getting electrical shocks from measurement systems. Instruments that have a mains power supply must be subject to normal rules about the condition of supply cables, clamping of wires, and earthing of all metal parts. However, most measurement systems operate at low

voltage and so pose no direct threat unless parts of the system come into contact with mains conductors. This should be prevented by applying codes of practice that require that all cabling for measurement systems to be kept physically separate from that used to carry mains voltages to equipment. Normally, this prohibits the use of the same trunking to house both signal wires and mains cables, although some special forms of trunking are available that have two separate channels separated by a metal barrier, thus allowing them to be used for both mains cables and signal wires. This subject is covered in depth in the many texts on electrical installation practice.

12.3.1 Introduction to safety systems

The purpose of safety systems is to monitor parameter values in manufacturing plant and other systems and to make an effective response when plant parameters vary from normal operating values and cause a potentially dangerous situation to develop. The response can be to generate an alarm for the plant operator to take action or else to take more direct action to shut down the plant automatically. The design and operation of safety systems are subject to guidelines set by international standard [IEC 61508](#).

IEC61508

IEC 61508 (2010–16) sets out a code of practice designed to ensure that safety systems work effectively and reliably. Although primarily concerned with electrical, electronic, and programmable electronic safety systems, the principles embodied by the standard can be applied as well to systems with other technologies, such as mechanical, pneumatic, and hydraulic devices.

The IEC 61508 standard is subdivided into three sets of requirements:

- Proper management of design, implementation, and maintenance of safety systems.
- Competence and training of personnel involved in designing, implementing, or maintaining safety systems.
- Technical requirements for the safety system itself.

A key feature of IEC61508 is the *safety integrity level* (SIL), which is expressed as the degree of confidence that a safety system will operate correctly and ensure that there is an adequate response to any malfunctions in the manufacturing plant, or other systems, that may cause a hazard and put humans at risk. The SIL value is set according to what the tolerable risk is in terms of the rate of failure for a process. The procedure for defining the required SIL value is known as *risk analysis*. What is tolerable depends on what the consequences of a dangerous failure are in terms of injury to one or more people or death to one or more people. The acceptable level of tolerance for particular industries and processes is set according to guidelines defined by safety regulatory authorities, expert

advice, and legal requirements. The table below gives the SIL value corresponding to various levels of tolerable risk for a continuously operating plant.

Safety integrity level	Probability of dangerous failure per hour	Probability of dangerous failure per year
4	10^{-9} to 10^{-8}	10^{-5} to 10^{-4}
3	10^{-8} to 10^{-7}	10^{-4} to 10^{-3}
2	10^{-7} to 10^{-6}	10^{-3} to 10^{-2}
1	10^{-6} to 10^{-5}	10^{-2} to 10^{-1}

The safety system is required to have sufficient reliability to match the rate of dangerous failures in a plant to the SIL value set. This reliability level is known as the *safety integrity* of the system. *Plant reliability* is calculated by principles identical to those set out in Section 12.2 for measurement systems and is based on a count of the number of faults that occur over a certain interval of time. However, it must be emphasized that the frequency of potentially dangerous failures is usually less than the rate of occurrence of faults in general. Thus, the reliability value for a plant cannot be used directly as a prediction of the rate of occurrence of dangerous failures. Hence, the total failures over a period of time must be analyzed and divided between faults that are potentially dangerous and those that are not.

Once risk analysis has been carried out to determine the appropriate SIL value, the required performance of the safety protection system can be calculated. For example, if the maximum allowable probability of dangerous failures per hour is specified as 10^{-8} and the actual probability of dangerous failures in a plant is calculated as 10^{-3} per hour, the safety system must have a minimum reliability of $10^{-8}/10^{-3}$, i.e., 10^{-5} failures for a one-hour period.

12.3.2 Design of a safety system

A typical safety system consists of a sensor, a trip amplifier, and either an actuator or alarm generator, as shown in Fig. 12.3. For example, in a safety system designed to protect against abnormally high pressures in a process, the sensor would be some form of pressure transducer, and the trip amplifier would be a device that amplifies the measured pressure signal and generates an output that activates either an actuator or an alarm if the measured pressure signal exceeded a preset threshold value. A typical actuator in this case would be a relief valve.

Software is increasingly embedded within safety systems to provide intelligent interpretation of sensor outputs, such as identifying trends in measurements. Safety systems that incorporate software and a computer processor are commonly known as *microprocessor-based protection systems*. In any system containing software, the

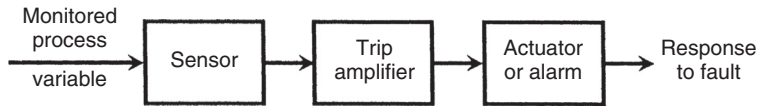


Figure 12.3
Elements of a safety system.

reliability of the software is crucial to the overall reliability of the safety system, and the reliability-quantification techniques described in Section 12.3 assume great importance.

To achieve the very high levels of reliability normally specified for safety systems, it is usual to guard against system failure by either triplicating the safety system and implementing two-out-of-three voting or, alternatively, by providing a switchable standby safety system. These techniques are considered next.

Two-out-of-three voting system

This system involves triplicating the safety system, as shown in Fig. 12.4. Shutdown action is taken, or an alarm is generated, if two of the three systems indicate the requirement for action. This allows the safety system to operate reliably if any one of the triplicated systems fails; it is often known as a two-out-of-three voting system. Reliability R_S is given by:

$$\begin{aligned}
 R_S &= \text{Probability of all three systems operating correctly} \\
 &+ \text{Probability of any two systems operating correctly} \\
 &= R_1R_2R_3 + (R_1R_2F_3 + R_1F_2R_3 + F_1R_2R_3)
 \end{aligned} \tag{12.17}$$

where R_1 , R_2 , R_3 , and F_1 , F_2 , and F_3 are the reliabilities and unreliabilities of the three systems, respectively. If all of the systems are identical (such that $R_1 = R_2 = R_3 = R$, etc):

$$R_S = R^3 + 3R^2F = R^3 + 3R^2(1 - R) \tag{12.18}$$

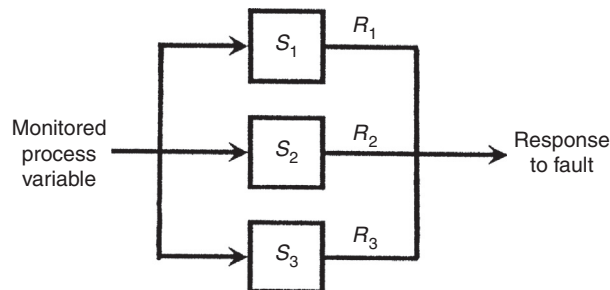


Figure 12.4
Two-out-of-three voting system.

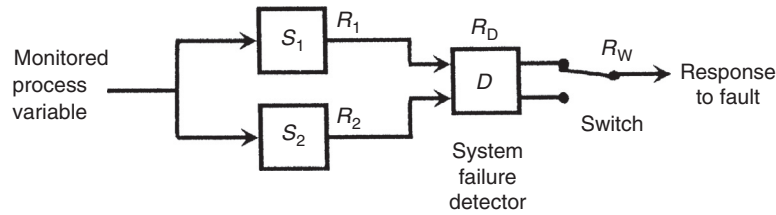


Figure 12.5
Standby system.

■ Example 12.9

In a particular protection system, three safety systems are connected in parallel and a two-out-of-three voting strategy is applied. If the reliability of each of the three systems is 0.95, calculate the overall reliability of the whole protection system.

■ Solution

Applying (Eq. 12.18), $R_S = 0.95^3 + [3 \times 0.95^2 \times (1 - 0.95)] = 0.993$.

Standby system

A standby system avoids the cost of providing and running three separate safety systems in parallel. Use of a standby system means that only two safety systems have to be provided. The first system is in continuous use but the second system is normally not operating and is switched into operation only if the first system develops a fault. The flaws in this approach are the necessity for faults in the primary system to be reliably detected and the requirement that the switch must always work correctly. The probability of failure, F_S , of a standby system of the form shown in Fig. 12.5, assuming no switch failures during normal operation, can be expressed as:

$$\begin{aligned} F_S &= \text{Probability of systems } S_1 \text{ and } S_2 \text{ both failing, given successful switching} \\ &\quad + \text{Probability of } S_1 \text{ and the switching system both failing at the same time} \\ &= F_1 F_2 R_D R_W + F_1 (1 - R_D R_W) \end{aligned}$$

System reliability is given by:

$$R_S = 1 - F_S = 1 - F_1 (1 + F_2 R_D R_W - R_D R_W) \quad (12.19)$$

where R_D is the reliability of the fault detector and R_W is the reliability of the switching system.

The derivation of (Eq. 12.19) assumes that there are no switch failures during normal operation of the system: that is, there are no switch failures during the time when the controlled process is operating satisfactorily and there is no need to switch over to the standby system. However, because the switch is subject to a continuous flow of current, its reliability cannot be assumed to be 100%. If the reliability of the switch in normal operation is represented by R_N , the expression in (Eq. 12.19) must be multiplied by R_N and the reliability of the system becomes:

$$R_S = R_N[1 - F_1(1 + F_2R_DR_W - R_DR_W)] \quad (12.20)$$

The problem of detecting faults in the primary safety system reliably can be solved by operating both safety systems in parallel. This enables faults in the safety system to be distinguished from faults in the monitored process. If only one of the two safety systems indicates a failure, this can be taken to indicate a failure of one of the safety systems rather than a failure of the monitored process. However, if both safety systems indicate a fault, this almost certainly means that the monitored process has developed a potentially dangerous fault. This scheme is known as *one-out-of-two voting*, but it is obviously inferior in reliability to the two-out-of-three scheme described earlier.

■ Example 12.10

In a particular protection system, a switchable standby safety system is used to increase reliability. If the reliability of the main system is 0.95, that of the standby system is 0.96*, that of the fault detection and switching system is 0.95, and the reliability of the switch in normal operation is 0.98, calculate the reliability of the protection system.

■ Solution

Applying (Eq. 12.20), the parameter values are $F_1 = 0.05$, $F_2 = 0.04$, $R_DR_W = 0.95$, and $R_N = 0.98$. Hence:

$$R_S = 0.98[1 - 0.05(1 + \{0.04 \times 0.95\} - 0.95)] = 0.976.$$

* Because the standby system is not subject to normal use, its reliability tends to be higher than the primary system even if the two systems consist of nominally identical components.

Actuators and alarms

The final element in a safety system is either an automatic actuator or an alarm that requires a human response. The reliability of the actuator can be calculated in the same way as all other elements in the system and incorporated into the calculation of the overall system reliability, as expressed in Eqs. (12.17–12.20). However, the reliability of alarms cannot be quantified in the same manner. Therefore, safety system reliability calculations have to exclude the alarm element. In consequence, the system designer needs to take steps to maximize the probability that the human operator will take the necessary response to alarms that indicate a dangerous plant condition.

An important criterion in system design is that alarms about dangerous conditions in the plant must be much more prominent than alarms about conditions that are not dangerous. Care should also be taken to ensure that the operator of a plant is not bombarded by too many alarms, because this leads the operator into the habit of ignoring alarms. Ignoring an alarm indicating that a fault is starting to occur may cause dangerous conditions in the plant to develop. Thus, alarms should be uncommon rather than routine, so that they attract the attention of the plant operator. This ensures, as far as possible, that the operator will take the proper action in response to an alarm about a potentially dangerous situation.

12.4 Summary

The topic that we studied in this chapter was the subject of measurement system reliability and the effect this can have on other systems associated with the measurement system. We started by studying the principles of reliability and looking at how the reliability of a measurement system can be quantified. We then looked at the principal laws of reliability and considered how these could be applied to improve the reliability of a system. We also looked at other precautions that we could take when designing and operating measurement systems to avoid failure or impaired performance. These precautions included choosing suitable instruments according to the expected operating conditions, protecting instruments appropriately when using them in adverse environments, recalibrating them at the recommended frequency, and building redundancy into critical parts of a measurement system.

We then considered the subject of software reliability. This was important in view of the widespread use of intelligent instruments containing software. This study showed us that there are substantial differences between mechanisms contributing to the reliability of software and those contributing to the reliability of hardware. The reliability of instrument hardware is related to the factors such as mechanical wear, degradation because of the effect of the operating environment and mechanical failure. These are faults that develop over a period of time. However, the mechanisms of software failure are fundamentally different. Software does not change with time, and if it is error-free when first written,

it will remain error-free forever. When software is used, errors that it has had all of its life suddenly cause a problem when particular conditions occur, usually when particular combinations of input values are applied. Thus, the usual rules of reliability quantification applied to hardware components in measurement systems are inappropriate for associated software. Instead, special procedures have to be applied to quantify software reliability in terms of the probability that it will not fail. Having established appropriate ways of quantifying software reliability, we went on to look at how reliability can be improved.

The final topic of study in the chapter was that of system safety issues. After an examination of the definition and quantification of safety levels and the associated IEC 61508 code of practice for safety systems, we looked at how safety systems could be designed to address safety issues. We then ended the chapter by looking in some detail at particular designs of safety system in terms of two-out-of-three voting systems using standby systems and the appropriate use of safety actuators and alarms.

12.5 Problems

- 12.1 How is the reliability of a measurement system defined? What is the difference between quantifying reliability in quasiabsolute terms and quantifying it probabilistic terms?
- 12.2 Explain the rules for calculating the overall reliability of system components that are connected:
 - (a) in series with each other and
 - (b) in parallel with one another.
- 12.3 Discuss ways in which the reliability of measurement systems can be improved.
- 12.4 Define the following terms and give examples that show how they are calculated: mean time before failure, mean time to failure, mean time to repair, and availability.
- 12.5 What is redundancy in a measurement system? Give an example that shows how redundancy can be used to improve the reliability of a measurement system.
- 12.6 How do the mechanisms affecting the reliability of software differ from those affecting the reliability of mechanical and electrical system components?
- 12.7 How can software reliability be quantified?
- 12.8 Discuss some ways in which the reliability of software components within measurement systems can be improved.
- 12.9 What are the principal duties of employers and employees with regard to safety? How do these affect the design and operation of measurement systems?
- 12.10 Explain the following terms that are met in the design of safety systems:
 - (a) two-out-of-three voting system
 - (b) standby system

- 12.11 The performance of a measuring instrument is monitored over a one-year (365-day) period and the intervals (in days) between faults that are recorded are:

27 6 18 41 54 29 46 14 49 38 17 26

Calculate the mean time between failures.

- 12.12 The days on which an instrument failed were recorded over a 12-month period such that day 1 = Jan 1, day 32 = Feb 1, etc.).

Number of the day on which a fault occurred: 18 72 111 173 184 227 286 309 356.

Calculate the mean time between failures.

- 12.13 A manufacturer monitors the performance of a new type of instrument that is installed at 20 different locations. If a total of 9 faults are recorded over a 100-day period, calculate the mean time between failures that should be specified for any one instrument.

- 12.14 The time before failure of each platinum resistance thermometer used in a particular location is recorded. The times before failure in days for 10 successive thermometers are recorded as: 405 376 433 425 388 402 445 412 397 366.

Calculate the mean time to failure.

- 12.15 The repair times in hours of an instrument over a history of 10 breakdowns are recorded as: 10.5 5.75 8.25 30.0 12.5 15.0 6.5 3.25 14.5 9.25.

Calculate the mean time to repair.

- 12.16 If the mean time between failures for an instrument is 247 days and the mean time to repair is 3 days, calculate its availability.

- 12.17 Data are collected by a manufacturer about a particular piece of machinery that is used 24 hours/day, 7 days per week, recording both the intervals in days between breakdowns and the time taken to repair each fault that causes a breakdown. The following data are collected:

Times before breakdown (in days): 13.3 4.3 12.7 17.8 14.2 20.1 15.6 8.9 8.1 11.5

Time to repair faults (days): 0.8 1.5 0.2 0.7 4.0 0.6 0.9 1.4 0.8 0.1

Calculate the mean time between failures, the mean time to repair, and the availability of the machine.

- 12.18 (a) How do the two similarly sounding terms “mean time between failures” and “mean time to-failure” differ, and in what sense do they mean the same?

- (b) The time before failure of each thermistor in a particular location is recorded. The times before failure in days for 10 successive thermistors are recorded as:

304 282 325 319 291 302 336 309 298 274.

Calculate the mean time to failure.

- 12.19 Data are collected by a manufacturer about an industrial robot that is used 24 hours/day, 7 days per week, recording both the intervals in days between breakdowns and the time taken to repair each fault that causes a breakdown. The following data are collected:

Times before breakdown (in days): 21.4 18.5 36.7 19.8 22.3 27.9 24.1

30.2 25.0 8.6

Time to repair faults (days): 0.4 0.7 2.0 0.1 0.6 3.5 0.5 0.2 1.3 0.8

Calculate the mean time before failure, the mean time to repair, and the availability of the machine.

- 12.20 If the mean time to failure of an instrument is 100,000 hours, calculate the probability that it will not fail in the first 50,000 hours.
- 12.21 Four measurement components connected in series have the following reliabilities: 0.98 0.93 0.95 0.99. Calculate the reliability of the whole measurement system.
- 12.22 If the mean time to failure of a light bulb is 100,000 hours, calculate the probability that it will not fail in the first 5000 hours.
- 12.23 Five components connected in series in a manufacturing system have the following reliabilities: 0.99 0.97 0.98 0.99 0.98. Calculate the reliability of the whole manufacturing system.
- 12.24 If the mean time to failure of a hydraulic seal is 482 hours, calculate the probability that it will not fail in the first 360 hours.
- 12.25 Four components connected in series in a production system have the following reliabilities: 0.98 0.94 0.96 0.97.
- (a) Calculate the reliability of the whole production system.
- (b) If a better-quality component could be purchased to replace Component 2 in the system (the one with a reliability of 94%), what would its reliability have to be to increase the overall system reliability to 90%?
- 12.26 In a particular measurement system, two instruments with individual reliability of 0.95 are connected together in parallel. Calculate the reliability of the measurement system if it can continue to function as long as both of the instruments do not fail at the same time.

- 12.27 Five components connected in series in a production system have the following reliabilities: 0.97 0.99 0.98 0.94 0.96.
- Calculate the reliability of the whole production system.
 - If a better-quality component could be purchased to replace Component 4 in the system (the one with a reliability of 94%), what would its reliability have to be to increase the overall system reliability to 88%?

- 12.28 In a particular measurement system, three identical instruments with an individual reliability of 0.88 are connected together in parallel.
- Calculate the reliability of the measurement system if it can continue to function as long as all of the instruments do not fail at the same time.
 - What would the reliability of each instrument have to be to achieve an overall system reliability of 99.9%?

- 12.29 Calculate the reliability of the measurement system shown in Fig. 12.2b if the reliabilities of the individual components are:

$$R_1 = R_3 = R_5 = 0.95 \quad ; \quad R_2 = R_4 = 0.88$$

- 12.30 In a particular measurement system, two instruments with individual reliability of 0.80 are connected together in parallel.
- Calculate the reliability of the measurement system if it can continue to function as long as both of the instruments do not fail at the same time.
 - How many instruments with this same individual reliability of 80% would have to be connected together in parallel to achieve an overall system reliability of 99.5% (assuming that the system can continue to function as long as at least one instrument is still working)?
 - How many instruments in parallel are needed to achieve an overall system reliability of 99.95%?

- 12.31 Calculate the reliability of the measurement system shown in Fig. 12.2b if the reliabilities of the individual components are:

$$R_1 = R_3 = R_5 = 0.98 \quad ; \quad R_2 = R_4 = 0.90$$

- 12.32 (a) Calculate the reliability of the measurement system shown in Fig. 12.2a if the reliabilities of the individual components are:

$$R_1 = R_3 = R_5 = 0.95 \quad ; \quad R_2 = R_4 = 0.80$$

- (b) Calculate the new reliability if two components in the system (R_2 and R_4) are duplicated as in Fig. 12.2b.

- 12.33 To estimate the number of errors in a new piece of software by applying the error-seeding approach, a programmer puts 10 deliberate (seeded) faults into the program. A second programmer then tests the program and finds 27 errors, 8 of which are

- confirmed by the first programmer to be seeded errors. Estimate the original number of faults in the program (i.e., excluding the seeded errors).
- 12.34 The double-testing approach is applied to test a new computer program, and the two programmers who do the testing find 31 and 34 errors, respectively. If 27 of the errors found by Programmer 1 are the same as errors in the list produced by Programmer 2, estimate the actual number of errors in the program.
- 12.35 A program is tested and the total number of errors is estimated as 16. The program is then seeded with 20 deliberate errors and further testing is then carried out until all 20 seeded errors have been found.
- If no new (previously undetected) errors are found during this further testing to find all seeded errors, calculate the probability that the program is error-free after this further testing.
 - How many seeded errors would have to be put into the program and then detected to achieve a 98% probability that the program is error-free?
- 12.36 The double-testing approach is applied to test a new computer program, and the two programmers who do the testing find 38 and 42 errors, respectively. If 33 of the errors found by Programmer 1 are the same as errors in the list produced by Programmer 2, estimate the actual number of errors in the program.
- 12.37 To estimate the number of errors in a new piece of software by applying the error-seeding approach, a programmer puts 15 deliberate (seeded) faults into the program. A second programmer then tests the program and finds 35 errors, 12 of which are confirmed by the first programmer to be seeded errors. Estimate the original number of faults in the program (i.e., excluding the seeded errors).
- 12.38 The double-testing approach is applied to test a new computer program, and the two programmers who do the testing find 47 and 43 errors, respectively. If 40 of the errors found by Programmer 1 are the same as errors in the list produced by Programmer 2, estimate the actual number of errors in the program.
- 12.39 The total number of errors estimated after testing a program is 19. The program is then seeded with 25 deliberate errors and further testing is then carried out until all 25 seeded errors have been found.
- If no new (previously undetected) errors are found during this further testing to find all seeded errors, calculate the probability that the program is error-free after this further testing.
 - How many seeded errors would have to be put into the program and then detected to achieve a 97.5% probability that the program is error-free?
- 12.40 Three safety systems are connected in parallel in a protection system and a two-out-of-three voting strategy is applied. If the reliability of each of the three systems is 0.90, calculate the overall reliability of the whole protection system.

- 12.41 A switchable standby safety system is used to increase reliability in a protection system. If the reliability of the main system is 0.90, that of the standby system is 0.91, that of the fault detector/switching system is 0.90, and the reliability of the switch in normal operation is 0.96, calculate the reliability of the protection system.
- 12.42 Three safety systems are connected in parallel in a protection system and a two-out-of-three voting strategy is applied. If the reliability of each of the three systems is 0.95, calculate the overall reliability of the whole protection system.
- 12.43 A switchable standby safety system is used to increase reliability in a protection system. If the reliability of the main system is 0.94, that of the standby system is 0.95, that of the fault detector/switching system is 0.96, and the reliability of the switch in normal operation is 0.98, calculate the reliability of the protection system.
- 12.44 Three safety systems are connected in parallel in a protection system and a two-out-of-three voting strategy is applied. If the reliability of each of the three systems is 0.975, calculate the overall reliability of the whole protection system.

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Sensor technologies

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13.1 Introduction

We are now moving into a set of chapters where we will look in detail at the range of sensors available for measuring various physical quantities. As we study these sensors, we will quickly come to realize that a wide range of different physical principles are involved in their operation. It will also become apparent that the physical principles on which they operate are often important factors in choosing a sensor for a given application, since a sensor using a particular principle may perform much better than one using a different principle in given operating conditions. It is therefore prudent to devote this chapter to a study of the various physical principles that are exploited in measurement sensors before going on to the separate chapters devoted to measurement of various physical quantities. The physical principles that we shall examine are capacitance change, resistance change, magnetic phenomena (inductance, reluctance, and eddy currents), the Hall effect, properties of piezoelectric materials, resistance change in stretched/strained wires (strain gauges), properties of piezoresistive materials, light transmission (both along an air path and along a fiber-optic cable), properties of ultrasound, transmission of radiation, and properties of micromachined structures (microsensors and nanosensors). It should be noted that the chosen order of presentation of these is arbitrary and does not imply anything about the relative popularity of these various principles. It must also be pointed out that the list of technologies covered in this chapter is not a full list of all the technologies that are used in sensors but rather a list of technologies that are common to several different sensors that measure different physical quantities. Many other technologies are used in the measurement of single physical quantities. Temperature measurement is a good example of this, as several of the sensors used are based on technologies that are not covered in this chapter.

13.2 Capacitive sensors

Capacitive sensors consist of two parallel metal plates in which the dielectric between the plates is either air or some other medium. The capacitance C is given by $C = \epsilon_o \epsilon_r A / d$, where ϵ_o is the absolute permittivity, ϵ_r is the relative permittivity of the dielectric medium between the plates, A is the area of the plates, and d is the distance between them. Two forms of capacitive device exist, which differ according to whether the distance between the plates is fixed or not.

Capacitive devices in which the distance between the plates is variable are primarily used as displacement sensors. Motion of the moveable capacitive plate relative to a fixed one changes the capacitance. Such devices can be used directly as a displacement sensor by applying the motion to be measured to the moveable capacitor plate. Capacitive displacement sensors commonly form part of instruments measuring pressure, sound, or acceleration, as explained in later chapters.

In the alternative form of capacitor, the distance between the plates is fixed. Variation in capacitance is achieved by changing the dielectric constant of the material between the plates in some way. One application is where the dielectric medium is air and the device is used as a humidity sensor by measuring the moisture content of the air. The same principle is used in the capacitive moisture meter, which can measure the moisture content of both solids and liquids. Another common application is as a liquid level sensor, where the dielectric is part air and part liquid according to the level of the liquid that the device is inserted in. Both of these applications are discussed in greater detail in later chapters.

13.3 Resistive sensors

Resistive sensors rely on the variation of the resistance of a material when the measured variable is applied to it. This principle is most commonly applied in temperature measurement using resistance thermometers or thermistors. It is also used in displacement measurement using strain gauges or piezoresistive sensors. In addition, some moisture meters work on the resistance-variation principle. All of these applications are considered further in later chapters.

13.4 Magnetic sensors

Magnetic sensors utilize the magnetic phenomena of inductance, reluctance, and eddy currents to indicate the value of the measured quantity.

Inductive sensors translate movement into a change in the mutual inductance between magnetically coupled parts. One example of this is the inductive displacement transducer shown in Fig. 13.1. In this, the single winding on the central limb of an “E”-shaped ferromagnetic body is excited with an alternating voltage. The displacement to be

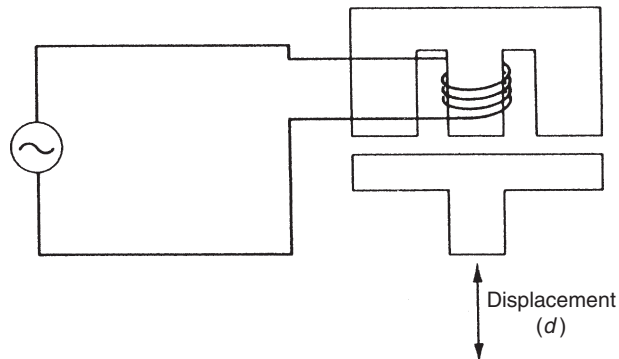


Figure 13.1
Inductive displacement sensor.

measured is applied to a ferromagnetic plate in close proximity to the “E” piece. Movements of the plate alter the flux paths and hence cause a change in the current flowing in the winding. By Ohm’s law, the current flowing in the winding is given by: $I = V/\omega L$. For fixed values of w and V , this equation becomes: $I = 1/KL$, where K is a constant. The relationship between L and the displacement, d , applied to the plate is a nonlinear one, and hence the output-current/displacement characteristic has to be calibrated.

The inductance principle is also used in differential transformers to measure translational and rotational displacements.

In **variable reluctance sensors**, a coil is wound on a permanent magnet rather than on an iron core as in variable inductance sensors. Such devices are commonly used to measure rotational velocities. Fig. 13.2 shows a typical instrument in which a ferromagnetic gearwheel is placed next to the sensor. As the tip of each tooth on the gearwheel moves toward and away from the pickup unit, the changing magnetic flux in the pickup coil causes a voltage to be induced in the coil whose magnitude is proportional to the rate of change of flux. Thus, the output is a sequence of positive and negative pulses whose frequency is proportional to the rotational velocity of the gearwheel.

Eddy current sensors consist of a probe containing a coil, as shown in Fig. 13.3, that is excited at a high frequency, which is typically 1 MHz. This is used to measure the displacement of the probe relative to a moving metal target. Because of the high frequency of excitation, eddy currents are induced only in the surface of the target, and the current magnitude reduces to almost zero a short distance inside the target. This allows the sensor to work with very thin targets, such as the steel diaphragm of a pressure sensor. The eddy currents alter the inductance of the probe coil, and this change can be translated into a d.c. voltage output that is proportional to the distance between the probe and the target. Measurement resolution as high as $0.1 \mu\text{m}$ can be achieved. The sensor can also work with a nonconductive target if a piece of aluminum tape is fastened to it.

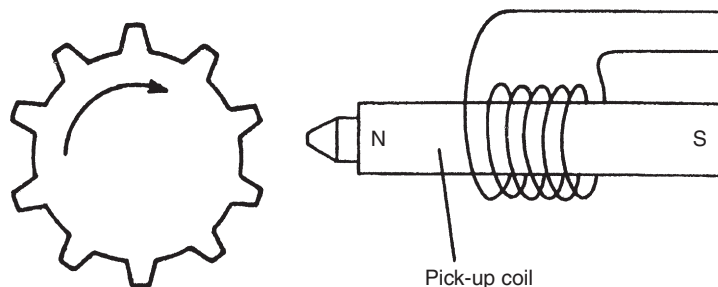


Figure 13.2
Variable reluctance sensor.

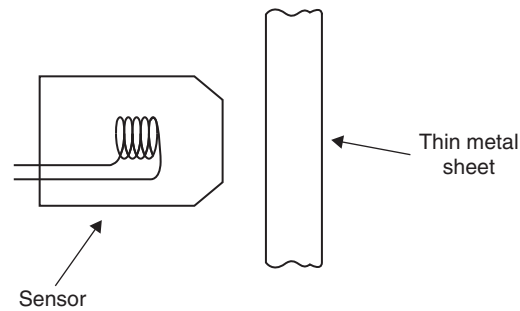


Figure 13.3
Eddy current sensor.

13.5 Hall-effect sensors

Basically, a Hall-effect sensor is a device that is used to measure the magnitude of a magnetic field. It consists of a conductor carrying a current that is aligned orthogonally with the magnetic field, as shown in Fig. 13.4. This produces a transverse voltage difference across the device that is directly proportional to the magnetic field strength. For an excitation current I and magnetic field strength B , the output voltage is given by $V = KIB$, where K is known as the Hall constant.

The conductor in Hall-effect sensors is usually made from a semiconductor material as opposed to a metal, because a larger voltage output is produced for a magnetic field of a given size. In one common use of the device as a proximity sensor, the magnetic field is provided by a permanent magnet that is built into the device. The magnitude of this field changes when the device becomes close to any ferrous metal object or boundary. The Hall

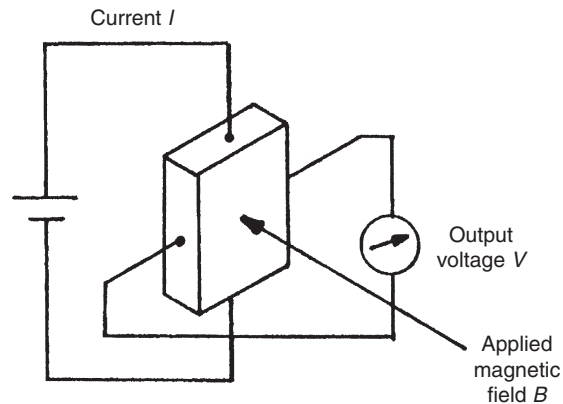


Figure 13.4
Principles of Hall-effect sensor.

effect is also commonly used in computer keyboard push buttons. When a button is depressed, a magnet attached underneath the button moves past a Hall-effect sensor. This generates an induced voltage in the sensor, which is converted by a trigger circuit into a digital output. Such push button switches can operate at high frequencies without contact bounce.

13.6 Piezoelectric transducers

Piezoelectric transducers produce an output voltage when a force is applied to them. They can also operate in the reverse mode where an applied voltage produces an output force. They are frequently used as ultrasonic transmitters and receivers. They are also used as displacement transducers, particularly as part of devices measuring acceleration, force, and pressure. In ultrasonic receivers, the sinusoidal amplitude variations in the ultrasound wave received are translated into sinusoidal changes in the amplitude of the force applied to the piezoelectric transducer. In a similar way, the translational movement in a displacement transducer is caused by mechanical means to apply a force to the piezoelectric transducer. Piezoelectric transducers are made from piezoelectric materials. These have an asymmetrical lattice of molecules that distorts when a mechanical force is applied to it. This distortion causes a reorientation of electric charges within the material, resulting in a relative displacement of positive and negative charges. The charge displacement induces surface charges on the material of opposite polarity between the two sides. By implanting electrodes into the surface of the material, these surface charges can be measured as an output voltage. For a rectangular block of material, the induced voltage is given by:

$$V = \frac{kFd}{A} \quad (13.1)$$

where F is the applied force in g, A is the area of the material in mm, d is the thickness of the material, and k is the piezoelectric constant. The polarity of the induced voltage depends on whether the material is compressed or stretched.

The input impedance of the instrument used to measure the induced voltage must be chosen carefully. Connection of the measuring instrument provides a path for the induced charge to leak away. Hence, the input impedance of the instrument must be very high, particularly where static or slowly varying displacements are being measured.

Materials exhibiting piezoelectric behavior include natural ones such as quartz, synthetic ones such as lithium sulfate, and ferroelectric ceramics such as barium titanate. The piezoelectric constant varies widely between different materials. Typical values of k are 2.3 for quartz and 140 for barium titanate. Applying Eq. (13.1) for a force of 1 g applied to a crystal of area 100 mm² and thickness 1 mm gives an output of 23 μV for quartz and 1.4 mV for barium titanate.

Certain polymeric films such as polyvinylidene also exhibit piezoelectric properties. These have a higher voltage output than most crystals and are very useful in many applications where displacement needs to be translated into a voltage. However, they have very limited mechanical strength and are unsuitable for applications where resonance might be generated in the material.

The piezoelectric principle is invertible, and therefore distortion in a piezoelectric material can be caused by applying a voltage to it. This is commonly used in ultrasonic transmitters, where the application of a sinusoidal voltage at a frequency in the ultrasound range causes sinusoidal variations in the thickness of the material and results in a sound wave being emitted at the chosen frequency. This is considered further in the section below on ultrasonic transducers.

13.7 Strain gauges

Strain gauges are devices that experience a change in resistance when they are stretched or strained. They are able to detect very small displacements, usually in the range of 0–50 μm , and are typically used as part of other transducers, for example, diaphragm pressure sensors that convert pressure changes into small displacements of the diaphragm. Measurement inaccuracies as low as $\pm 0.15\%$ of full-scale reading are achievable, and the quoted life expectancy is usually three million reversals. Strain gauges are manufactured to various nominal values of resistance, of which 120, 350, and 1000 Ω are very common. The typical maximum change of resistance in a 120 Ω device would be 5 Ω at maximum deflection.

The traditional type of strain gauge consists of a length of metal resistance wire formed into a zigzag pattern and mounted onto a flexible backing sheet, as shown in Fig. 13.5A. The wire is nominally of circular cross section. As strain is applied to the gauge, the shape of the cross section of the resistance wire distorts, changing the cross-sectional area. As

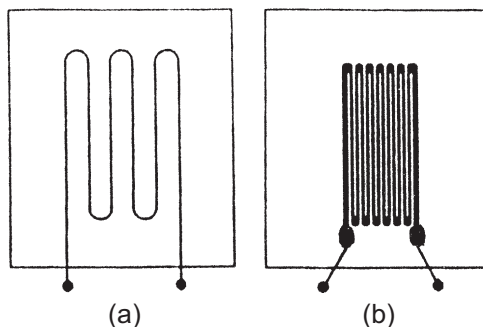


Figure 13.5

Strain gauges: (a) wire type; (b) foil type.

the resistance of the wire per unit length is inversely proportional to the cross-sectional area, there is a consequential change in resistance. The input-output relationship of a strain gauge is expressed by the **gauge factor**, which is defined as the change in resistance (R) for a given value of strain (S), i.e., $\text{gauge factor} = \delta R / \delta S$.

In recent years, wire-type gauges have largely been replaced, either by metal-foil types as shown in Fig. 13.5B, or by semiconductor types. Metal-foil types are very similar to metal-wire types except the active element consists of a piece of metal foil cut into a zigzag pattern. Cutting a foil into the required shape is much easier than forming a piece of resistance wire into the required shape, and this makes the devices cheaper to manufacture. A popular material in metal strain gauge manufacture is a copper-nickel-manganese alloy, which is known by the trade name of “Advance”. Semiconductor types have piezoresistive elements, which are considered in greater detail in the next section. Compared with metal gauges, semiconductor types have a much superior gauge factor (up to 100 times better) but they are more expensive. Also, whilst metal gauges have an almost zero temperature coefficient, semiconductor types have a relatively high temperature coefficient.

In use, strain gauges are bonded to the object whose displacement is to be measured. The process of bonding presents a certain amount of difficulty, particularly for semiconductor types. The resistance of the gauge is usually measured by a d.c. bridge circuit and the displacement is inferred from the bridge output measured. The maximum current that can be allowed to flow in a strain gauge is in the region of 5–50 mA depending on the type. Thus, the maximum voltage that can be applied is limited, and consequently, as the resistance change in a strain gauge is typically small, the bridge output voltage is also small and amplification has to be carried out. This adds to the cost of using strain gauges.

13.8 Piezoresistive sensors

A piezoresistive sensor is made from semiconductor material in which a p-type region has been diffused into an n-type base. The resistance of this varies greatly when the sensor is compressed or stretched. This is frequently used as a strain gauge, where it produces a significantly higher gauge factor than that given by metal wire or foil gauges. Also, measurement uncertainty can be reduced down to $\pm 0.1\%$. It is also used in semiconductor-diaphragm pressure sensors and in semiconductor accelerometers.

It should also be mentioned that the term “piezoresistive sensor” is sometimes used to describe all types of strain gauge, including metal types. However, this is incorrect since only about 10% of the output from a metal strain gauge is generated by piezoresistive effects, with the remainder arising out of the dimensional cross-section change in the wire or foil. Proper piezoelectric strain gauges, which are alternatively known as **semiconductor**

strain gauges, produce most (about 90%) of their output through piezoresistive effects, and only a small proportion of the output is due to dimensional changes in the sensor.

13.9 Optical sensors

Optical sensors are based on the transmission of light between a light source and a light detector, as shown in Fig. 13.6. The transmitted light can travel along either an air-path or a fiber-optic cable. Either form of transmission gives immunity to electromagnetically induced noise, and also provides greater safety than electrical sensors when used in hazardous environments.

13.9.1 Optical sensors (Air-path)

Air-path optical sensors are commonly used to measure proximity, translational motion, rotational motion, and gas concentration. These uses are discussed in more detail in later chapters. A number of different types of light sources and light detectors are used.

Light sources

Light sources suitable for transmission across an air path include tungsten-filament lamps, laser diodes, and light-emitting diodes (LEDs). However, as the light from tungsten lamps is usually in the visible part of the light-frequency spectrum, it is prone to interference from the sun and other sources. Hence, infrared LEDs or infrared laser diodes are usually preferred. These emit light in a narrow frequency band in the infrared region and are not affected by sunlight.

Light detectors

The main forms of light detectors used with optical systems are photoconductors (photoresistors), photovoltaic devices (photocells), phototransistors, and photodiodes.

Photoconductive devices are sometimes known by the alternative name of **photoresistors**. They convert changes in incident light into changes in resistance, with the resistance

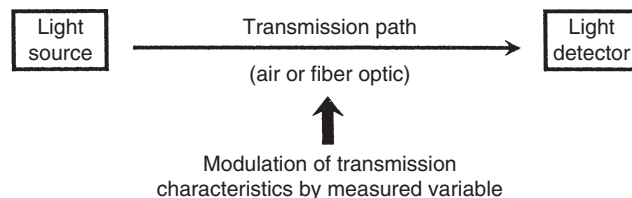


Figure 13.6
Operating principles of optical sensors.

reducing according to the intensity of light to which they are exposed. They are made from various materials such as cadmium sulfide, lead sulfide, and indium antimonide.

Photovoltaic devices are often called **photocells**. They are also commonly known as **solar cells** when a number of them are used in an array as a means of generating energy from sunlight. They are made from various types of semiconductor material. Their basic mode of operation is to generate an output voltage whose magnitude is a function of the magnitude of the incident light that they are exposed to.

Photodiodes are devices where the output current is a function of the amount of incident light. Again, they are made from various types of semiconductor material.

A **phototransistor** is effectively a standard bipolar transistor with a transparent case that allows light to reach its base-collector junction. It has an output in the form of an electrical current and could be regarded as a photodiode with an internal gain. This gain makes it more sensitive to light than a photodiode, particularly in the infrared region, but it has a slower response time. It is an ideal partner for infrared LED and laser diode light sources.

13.9.2 Optical sensors (Fiber-optic)

Instead of using air as the transmission medium, optical sensors can use fiber-optic cable to transmit light between a source and a detector. Fiber-optic cables can be made from either plastic or glass fibers or a combination of the two, though it is now rare to find cables made only from glass fibers since these are very fragile. Cables made entirely from plastic fibers have particular advantages for sensor applications because they are cheap and have a relatively large diameter of 0.5–1.0 mm, making connection to the transmitter and receiver easy. However, plastic cables cannot be used in certain hostile environments where they would be severely damaged. The cost of the fiber-optic cable itself is insignificant for sensing applications, as the total cost of the sensor is dominated by the cost of the transmitter and receiver.

Fiber-optic sensors characteristically enjoy long life. For example, the life expectancy of reflective fiber-optic switches is quoted at 10 million operations. Their accuracy is also good, with $\pm 1\%$ of full-scale reading being quoted as a typical inaccuracy level for a fiber-optic pressure sensor. Further advantages are their simplicity, low cost, small size, high reliability, and capability of working in many kinds of hostile environment. The only significant difficulty in designing a fiber-optic sensor is in ensuring that the proportion of light entering the cable is maximized. This is the same difficulty that was described earlier in chapter 7 when we were discussing the use of fiber-optic cables for signal transmission.

Two major classes of fiber-optic sensor exist, intrinsic sensors and extrinsic sensors. In **intrinsic sensors**, the fiber-optic cable itself is the sensor, whereas in **extrinsic sensors**, the fiber-optic cable is only used to guide light to/from a conventional sensor.

Intrinsic sensors

In intrinsic sensors, the physical quantity being measured causes some measurable change in the characteristics of the light transmitted by the cable. The modulated light parameters are one or more of the following:

- intensity
- phase
- polarization
- wavelength
- transit time

Sensors that modulate light intensity tend to use mainly multimode fibers, but only monomode cables are used to modulate other light parameters. A particularly useful feature of intrinsic fiber-optic sensors is that they can, if required, provide distributed sensing over distances of up to 1 meter.

Light intensity is the simplest parameter to manipulate in intrinsic sensors because only a simple source and detector are required. The various forms of switches shown in [Fig. 13.7](#) are perhaps the simplest form of these, as the light path is simply blocked and unblocked as the switch changes state. Modulation of the intensity of transmitted light also takes place in various simple forms of proximity, displacement, pressure, pH, and smoke sensors. Some of these are sketched in [Fig. 13.8](#). In proximity and displacement sensors (the latter are sometimes given the special name *Fotonic sensors*), the amount of reflected light varies with the distance between the fiber ends and a boundary. In pressure sensors, the refractive index of the fiber, and hence the intensity of light transmitted, varies according to the mechanical deformation of the fibers caused by pressure. In the pH probe, the amount of light reflected back into the fibers depends on the pH-dependent color of the chemical indicator in the solution around the probe tip. Finally, in a form of smoke detector, two fiber-optic cables placed either side of a space detect any reduction in the intensity of light transmission between them caused by the presence of smoke.

A simple form of accelerometer can be made by placing a mass subject to the acceleration on a multimode fiber. The force exerted by the mass on the fiber causes a change in the intensity of light transmitted, hence allowing the acceleration to be determined. The typical inaccuracy quoted for this device is ± 0.02 g in the measurement range of ± 5 g and $\pm 2\%$ in the measurement range up to 100 g.

A similar principle is used in probes that measure the internal diameter of tubes. The probe consists of eight strain-gauged cantilever beams that track changes in diameter, giving a measurement resolution of 20 μm .

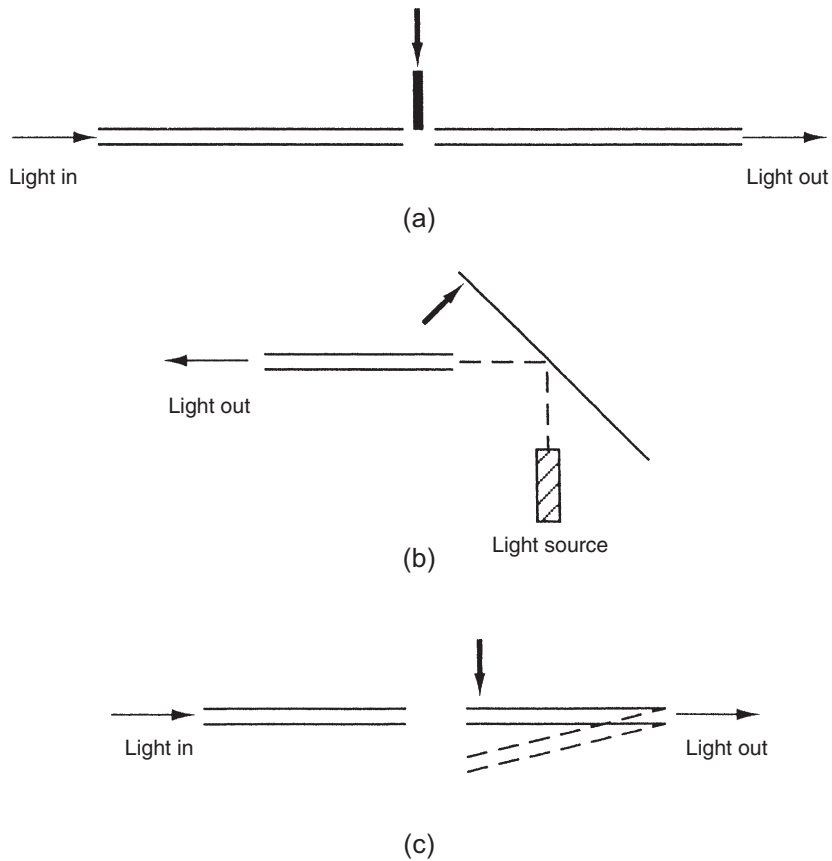


Figure 13.7

Intrinsic fiber-optic sensors. (a) Shutter switch; (b) reflective switch; (c) optical microswitch.

A slightly more complicated method of effecting light-intensity modulation is the variable shutter sensor shown in Fig. 13.9. This consists of two fixed fibers with two collimating lenses and a variable shutter between them. Movement of the shutter changes the intensity of light transmitted between the fibers. This is used to measure the displacement of various devices such as Bourdon tubes, diaphragms, and bimetallic thermometers.

Yet another type of intrinsic sensor uses cable where the core and cladding have similar refractive indices but different temperature coefficients. This is used as a temperature sensor. Temperature rises cause the refractive indices to become even closer together and losses from the core to increase, thus reducing the quantity of light transmitted.

Refractive index variation is also used in a form of intrinsic sensor used for cryogenic leak detection. The fiber used for this has a cladding whose refractive index becomes greater than that of the core when it is cooled to cryogenic temperatures. The fiber-optic cable is

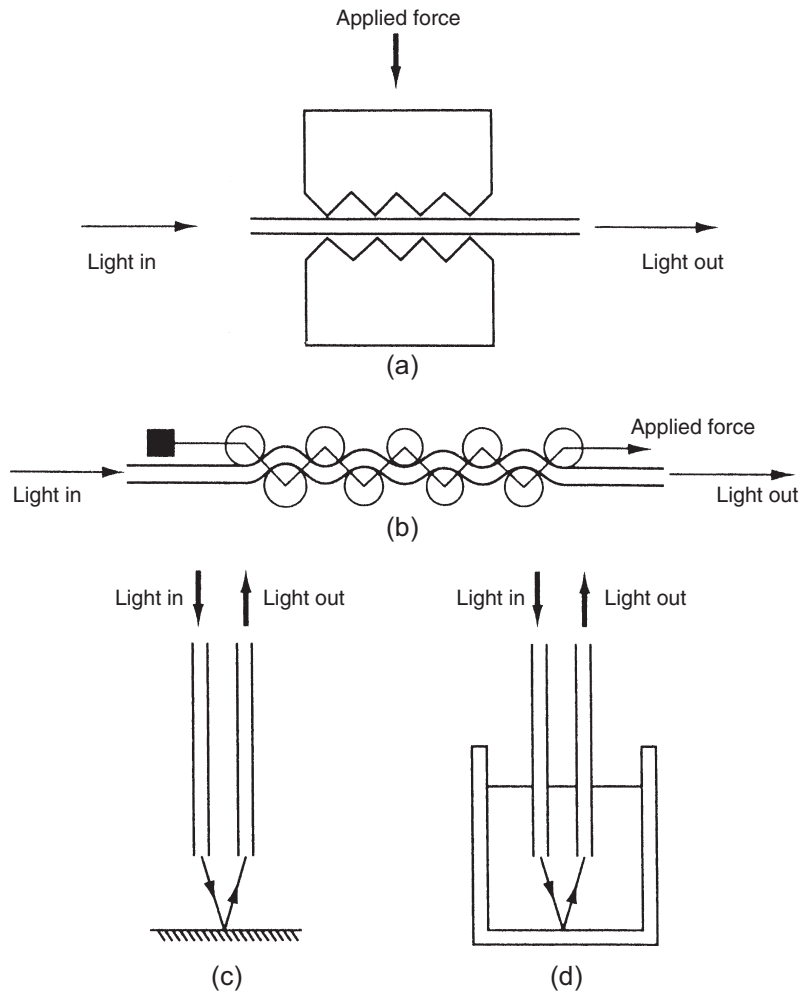


Figure 13.8

Intensity modulating sensors. (a) simple pressure switch; (b) roller-chain pressure sensor (microbend sensor); (c) proximity sensor; (d) pH sensor.

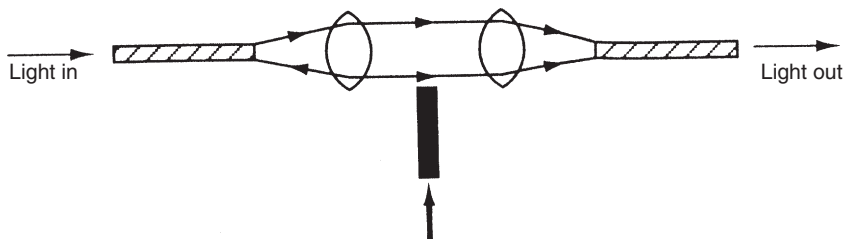


Figure 13.9

Variable shutter sensor.

laid in the location where cryogenic leaks might occur. If any leaks do occur, light traveling in the core is transferred to the cladding, where it is attenuated. Cryogenic leakage is thus indicated by monitoring the light transmission characteristics of the fiber.

A further use of refractive index variation is found in devices that detect oil in water. These use a special form of cable where the cladding used is sensitive to oil. Any oil present diffuses into the cladding and changes the refractive index, thus increasing light losses from the core. Unclad fibers are used in a similar way. In these, any oil present settles on the core and allows light to escape.

The *cross-talk sensor* measures several different variables by modulating the intensity of light transmitted. It consists of two parallel fibers that are close together and where one or more short lengths of adjacent cladding are removed from the fibers. When immersed in a transparent liquid, there are three different effects that each causes a variation in the intensity of light transmitted. Thus, the sensor can perform three separate functions. Firstly, it can measure temperature according to the temperature-induced variation in the refractive index of the liquid. Secondly, it can act as a level detector, as the transmission characteristics between the fibers change according to the depth of the liquid. Thirdly, it can measure the refractive index of the liquid itself when used under controlled-temperature conditions.

The refractive index of a liquid can be measured in an alternative way by using an arrangement where light travels across the liquid between two cable ends that are fairly close together. The angle of the cone of light emitted from the source cable, and hence the amount of light transmitted into the detector, is dependent on the refractive index of the liquid.

The use of materials where the fluorescence varies according to the value of the measurand can also be used as part of intensity-modulating intrinsic sensors. Fluorescence-modulating sensors can give very high sensitivity and are potentially very attractive in biomedical applications where requirements exist to measure very small quantities such as low-oxygen and carbon monoxide concentrations, low blood pressure levels, etc. Similarly, low concentrations of hormones, steroids, etc. may be measured.

As mentioned previously, the phase, polarization, wavelength, and transit time can be modulated as well as intensity in intrinsic sensors. Monomode cables are used almost exclusively in these types of intrinsic sensor.

Phase modulation normally requires a coherent (laser) light source. It can provide very high sensitivity in displacement measurement but cross sensitivity to temperature and strain degrades its performance. Additional problems are maintaining frequency stability of the light source and manufacturing difficulties in coupling the light source to the fiber. Various versions of this class of instrument exist to measure temperature, pressure,

strain, magnetic fields, and electric fields. Field-generated quantities such as electric current and voltage can also be measured. In each case, the measurand causes a phase change between a measuring and a reference light beam that is detected by an interferometer.

The principle of phase modulation has also been used in the fiber-optic accelerometer (where a mass subject to acceleration rests on a fiber), and in fiber strain gauges (where two fibers are fixed on the upper and lower surfaces of a bar under strain). The fiber-optic gyroscope described in Chapter 20 is a further example of a phase-modulating device.

Devices using polarization modulation require special forms of fiber that maintain polarization. Polarization changes can be effected by electrical fields, magnetic fields, temperature changes, and mechanical strain. Each of these parameters can therefore be measured by polarization modulation.

Various devices that modulate the wavelength of light are used for special purposes. However, the only common wavelength-modulating fiber-optic device is the form of laser Doppler flowmeter that uses fiber-optic cables, as described in Chapter 15.

Fiber-optic devices using modulation of the transit time of light are uncommon because of the speed of light. Measurement of the transit time for light to travel from a source, be reflected off an object, and travel back to a detector is only viable for extremely large distances. However, a few special arrangements have evolved that use transit-time modulation. These include instruments such as the optical resonator, which can measure both mechanical strain and temperature. Temperature-dependent wavelength variation also occurs in semiconductor crystal beads (e.g., aluminum gallium arsenide). This is bonded to the end of a fiber-optic cable and excited from an LED at the other end of the cable. Light from the LED is reflected back along the cable by the bead at a different wavelength. Measurement of the wavelength change allows temperatures in the range up to 200°C to be measured accurately. A particular advantage of this sensor is its small size, typically 0.5 mm diameter at the sensing tip. Finally, to complete the catalog of transit-time devices, the frequency modulation in a piezoelectric quartz crystal used for gas sensing can also be regarded as a form of time-domain modulation.

Extrinsic sensors

Extrinsic fiber-optic sensors use a fiber-optic cable, normally a multimode one, to transmit modulated light from a conventional sensor such as a resistance thermometer. A major feature of extrinsic sensors, which makes them so useful in such a large number of applications, is their ability to reach places that are otherwise inaccessible. One example of this is the insertion of fiber-optic cables into the jet engines of aircraft to measure temperature by transmitting radiation into a radiation pyrometer located remotely from the

engine. Fiber-optic cable can be used in the same way to measure the internal temperature of electrical transformers, where the extreme electromagnetic fields present make other measurement techniques impossible.

An important advantage of extrinsic fiber-optic sensors is the excellent protection against noise corruption that they give to measurement signals. Unfortunately, the output of many sensors is not in a form that can be transmitted by a fiber-optic cable, and conversion into a suitable form must therefore take place prior to transmission. For example, a platinum resistance thermometer (PRT) translates temperature changes into resistance changes. The PRT therefore needs electronic circuitry to convert the resistance changes into voltage signals and thence into a modulated light format, and this in turn means that the device needs a power supply. This complicates the measurement process and means that low-voltage power cables must be routed with the fiber-optic cable to the transducer. One particular adverse effect of this is that the advantage of intrinsic safety is lost. One solution to this problem is to use a power source in the form of electronically generated pulses driven by a lithium battery. Alternatively, power can be generated by transmitting light down the fiber-optic cable to a photocell. Both of these solutions provide intrinsically safe operation.

Piezoelectric sensors lend themselves particularly to use in extrinsic sensors because the modulated frequency of a quartz crystal can be readily transmitted into a fiber-optic cable by fitting electrodes to the crystal that are connected to a low-power LED. Resonance of the crystal can be created either by electrical means or by optical means using the photothermal effect. The photothermal effect describes the principle where, if light is pulsed at the required oscillation frequency and directed at a quartz crystal, the localized heating and thermal stress caused in the crystal results in it oscillating at the pulse frequency. Piezoelectric extrinsic sensors can be used as part of various pressure, force, and displacement sensors. At the other end of the cable, a phase-locked loop is typically used to measure the transmitted frequency.

Fiber-optic cables are also now commonly included in digital encoders, where the use of fibers to transmit light to and from the discs allows the light source and detectors to be located remotely. This allows the devices to be smaller, which is a great advantage in many applications where space is at a premium.

Distributed sensors

A number of discrete sensors can be distributed along a fiber-optic cable to measure different physical variables along its length. Alternatively, sensors of the same type, which are located at various points along a cable, provide distributed sensing of a single measured variable.

13.10 Ultrasonic transducers

Ultrasonic devices are used in many fields of measurement, particularly for measuring fluid flow rates, liquid levels, and large translational displacements (in range sensors). Details of such applications can be found in later chapters.

Ultrasound is a band of frequencies in the range above 20 KHz, that is, above the sonic range that humans can usually hear. Measurement devices that use ultrasound consist of one device that transmits an ultrasound wave and another device that receives the wave. Changes in the measured variable are determined either by measuring the change in time taken for the ultrasound wave to travel between the transmitter and receiver, or alternatively, by measuring the change in phase or frequency of the transmitted wave.

The most common form of ultrasonic element is a piezoelectric crystal contained in a casing, as illustrated in [Fig. 13.10](#). Such elements can operate interchangeably as either a transmitter or receiver. These are available with operating frequencies that vary between 20 KHz and 15 MHz. The principles of operation, by which an alternating voltage generates an ultrasonic wave and vice versa, have already been covered in the section above on piezoelectric transducers.

For completeness, mention should also be made of capacitive ultrasonic elements. These consist of a thin, dielectric membrane between two conducting layers. The membrane is stretched across a backplate and a bias voltage is applied. When a varying voltage is applied, it behaves as an ultrasonic transmitter and an ultrasound wave is produced. The system also works in the reverse direction as an ultrasonic receiver. Elements with resonant frequencies in the range between 30 KHz and 3 MHz can be obtained.

13.10.1 Transmission speed

The transmission speed of ultrasound varies according to the medium through which it travels. Transmission speeds for some common media are given in [Table 13.1](#).

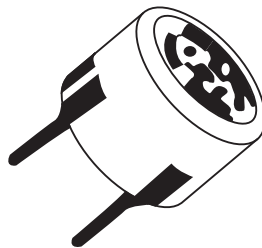


Figure 13.10
Ultrasonic sensor.

Table 13.1: Transmission speed of ultrasound through different media.

Medium	Velocity (m/s)
Air	331.6
Water	1440
Wood (pine)	3320
Iron	5130
Rock (granite)	6000

When transmitted through air, the speed of ultrasound is affected by environmental factors such as temperature, humidity, and air turbulence. Of these, temperature has the largest effect. The velocity of sound through air varies with temperature according to:

$$V = 331.6 + 0.6T \text{ m/s} \quad (13.2)$$

where T is the temperature in $^{\circ}\text{C}$. Thus, even for a relatively small temperature change of 20 degrees from 0°C to 20°C , the velocity changes from 331.6 m/s to 343.6 m/s.

Humidity changes have a much smaller effect on speed. If the relative humidity increases by 20%, the corresponding increase in the transmission velocity of ultrasound is 0.07% (corresponding to an increase from 331.6 m/s to 331.8 m/s at 0°C).

Changes in air pressure itself have negligible effect on the velocity of ultrasound. Similarly, air turbulence normally has no effect. However, if turbulence involves currents of air at different temperatures, then random changes in ultrasound velocity occur according to Eq. (13.2).

13.10.2 Directionality of ultrasound waves

An ultrasound element emits a spherical wave of energy, although the peak energy is always in a particular direction. The magnitude of energy emission in any direction is a function of the angle made with respect to the direction that is normal to the face of the ultrasonic element. The peak emission occurs along a line that is normal to the transmitting face of the ultrasonic element, and this is loosely referred to as the “direction of travel.” At any angle other than the “normal” one, the magnitude of transmitted energy is less than the peak value. Fig. 13.11 shows the characteristics of the emission for a range of ultrasonic elements. This is shown in terms of the attenuation of the transmission magnitude (measured in dB) as the angle with respect to the “normal” direction increases. For many purposes, it is useful to treat the transmission as a conical volume of energy, with the edges of the cone defined as the transmission angle where the amplitude of the energy in the transmission is -6 dB compared with the peak value (i.e., where the amplitude of the energy is half that in the normal direction). Using this definition, a

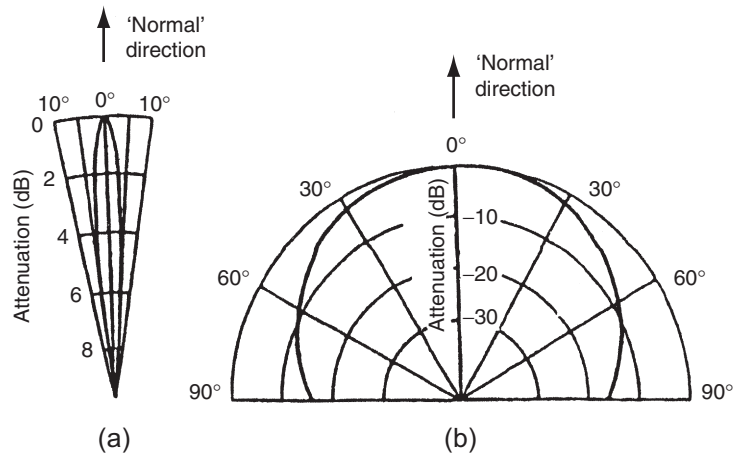


Figure 13.11

Ultrasonic emission characteristics. (a) narrow angle of peak emission; (b) wide angle of peak emission.

40 kHz ultrasonic element has a transmission cone of ± 50 degrees and a 400 kHz one element has a transmission cone of ± 3 degrees.

It should be noted that air currents can deflect ultrasonic waves such that the peak emission is no longer normal to the face of the ultrasonic element. It has been shown experimentally that an air current moving with a velocity of 10 km/h deflects an ultrasound wave by 8 mm over a distance of 1 m.

13.10.3 Relationship between wavelength, frequency and directionality of ultrasound waves

The frequency and wavelength of ultrasound waves are related according to:

$$\lambda = v/f \quad (13.3)$$

where λ is the wavelength, v is the velocity, and f is the frequency of the ultrasound waves.

This shows that the relationship between λ and f depends on the velocity of the ultrasound and hence varies according to the nature and temperature of the medium through which it travels. Table 13.2 compares the typical nominal frequencies, wavelengths, and transmission cones (-6 dB limits) for three different types of ultrasonic transmitter elements.

Table 13.2: Comparison of frequency, wavelength, and cone angle for various ultrasonic transmitters.

Nominal frequency (kHz)	23	40	400
Wavelength (in air at 0°C)	14.4	8.3	0.83
Cone angle of transmission (-6 dB limits)	± 80 degrees	± 50 degrees	± 3 degrees

It is clear from Table 13.2 that the directionality (cone angle of transmission) reduces as the nominal frequency of the ultrasound transmitter increases. However, the cone angle also depends on factors other than the nominal frequency, particularly on the shape of the transmitting horn in the element, and different models of ultrasonic element with the same nominal frequency can have substantially different cone angles.

13.10.4 Attenuation of ultrasound waves

Ultrasound waves suffer attenuation in the amplitude of the transmitted energy according to the distance traveled. The amount of attenuation also depends on the nominal frequency of the ultrasound and the adsorption characteristics of the medium through which it travels. The amount of adsorption depends not only on the type of transmission medium but also on the level of humidity and dust in the medium.

The amplitude X_d of the ultrasound wave at a distance d from the emission point can be expressed as:

$$\frac{X_d}{X_0} = \frac{\sqrt{e^{-\alpha d}}}{fd} \quad (13.4)$$

where X_0 is the magnitude of the energy at the point of emission, f is the nominal frequency of the ultrasound, and α is the attenuation constant that depends on the ultrasound frequency, the medium that the ultrasound travels through and any pollutants in the medium such as dust or water particles.

13.10.5 Ultrasound as a range sensor

The basic principles of an ultrasonic range sensor are to measure the time between transmission of a burst of ultrasonic energy from an ultrasonic transmitter and receipt of that energy by an ultrasonic receiver. Then, the distance d can be calculated from:

$$d = vt \quad (13.5)$$

where v is the ultrasound velocity and t is the measured energy transit time. An obvious difficulty in applying this equation is the variability of v with temperature according to Eq. (13.2). One solution to this problem is to include an extra ultrasonic transmitter/receiver pair in the measurement system in which the two elements are positioned a known distance apart. Measurement of the transmission time of energy between this fixed pair provides the necessary measurement of velocity and hence compensation for any environmental temperature changes.

The degree of directionality in the ultrasonic elements used for range measurement is unimportant as long as the receiver and transmitter are positioned carefully so as to face

each other exactly (i.e., such that the “normal” lines to their faces are coincident). Thus, directionality imposes no restriction on the type of element suitable for range measurement. However, element choice is restricted by the attenuation characteristics of different types of element, and relatively low-frequency elements have to be used for the measurement of large ranges.

Measurement resolution and accuracy

The best measurement resolution that can be obtained with an ultrasonic ranging system is equal to the wavelength of the transmitted wave. As wavelength is inversely proportional to frequency, high-frequency ultrasonic elements would seem to be preferable. For example, whilst the wavelength and hence resolution for a 40 kHz element is 8.6 mm at room temperature (20°C), it is only 0.86 mm for a 400 kHz element. However, choice of element also depends on the required range of measurement. The range of higher-frequency elements is much reduced compared with low-frequency ones due to the greater attenuation of the ultrasound wave as it travels away from the transmitter. Hence, choice of element frequency has to be a compromise between measurement resolution and range.

The best measurement accuracy obtainable is equal to the measurement resolution value, but this is only achieved if the electronic counter used to measure the transmission time starts and stops at exactly the same point in the ultrasound cycle (usually the point in the cycle corresponding to peak amplitude is used). However, the sensitivity of the ultrasonic receiver also affects measurement accuracy. The amplitude of the ultrasound wave that is generated in the transmitter ramps up to full amplitude in the manner shown in Fig. 13.12. The receiver has to be sensitive enough to detect the peak of the first cycle, which can usually be arranged. However, if the range of measurement is large, attenuation of the ultrasound wave may cause the amplitude of the first cycle to become less than the threshold level that the receiver is set to detect. In this case, only the second cycle will be

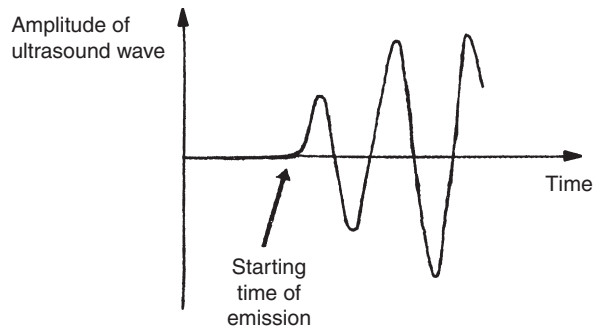


Figure 13.12

Ramp-up of ultrasonic wave after emission.

detected and there will be an additional measurement error equal to one wavelength. For large transmission distances, even the second cycle may be undetected, meaning that the receiver only “sees” the third cycle.

13.10.6 Effect of noise in ultrasonic measurement systems

Signal levels at the output of ultrasonic measurement systems are usually of low amplitude and are therefore prone to contamination by electromagnetic noise. Because of this, it is necessary to use special precautions such as making ground (earth) lines thick, using shielded cables for transmission of the signal from the ultrasonic receiver, and locating the signal amplifier as close to the receiver as possible.

Another potentially serious form of noise is background ultrasound produced by manufacturing operations in the typical industrial environment that many ultrasonic range measurement systems operate. Analysis of industrial environments has shown that ultrasound at frequencies up to 100 kHz is generated by many operations and some operations generate ultrasound at higher frequencies up to 200 kHz. There is not usually any problem if ultrasonic measurement systems operate at frequencies above 200 kHz, but these often have insufficient range for the needs of the measurement situation. In these circumstances, any objects that are likely to generate energy at ultrasonic frequencies should be covered in sound-absorbing material such that interference with ultrasonic measurement systems is minimized. The placement of sound-absorbing material around the path that the measurement ultrasound wave travels along contributes further toward reducing the effect of background noise. A natural solution to the problem is also partially provided by the fact that the same processes of distance traveled and adsorption that attenuate the amplitude of ultrasound waves traveling between the transmitter and receiver in the measurement system also attenuate ultrasound noise that is generated by manufacturing operations.

Because ultrasonic energy is emitted at angles other than the direction that is normal to the face of the transmitting element, a problem arises with respect to energy that is reflected off some object in the environment around the measurement system and back into the ultrasonic receiver. This has a longer path than the direct one between the transmitter and receiver and can cause erroneous measurements in some circumstances. One solution to this is to arrange for the transmission-time counter to stop as soon as the receiver first detects the ultrasound wave. This will usually be the wave that has traveled along the direct path, and so no measurement error is caused as long as the rate at which ultrasound pulses are emitted is such that the next burst is not emitted until all reflections from the previous pulse have died down. However, in circumstances where the direct path becomes obstructed by some obstacle, the counter will only be stopped when the reflected signal is detected by the receiver, giving a potentially large measurement error.

13.10.7 Exploiting Doppler shift in ultrasound transmission

The Doppler effect is evident in all types of wave motion and describes the apparent change in frequency of the wave when there is relative motion between the transmitter and receiver. If a continuous ultrasound wave with velocity v and frequency f takes t seconds to travel from a source S to a receiver R , then R will receive ft cycles of sound during time t (see Fig. 13.13).

Suppose now that R moves towards S at velocity r (with S stationary). R will receive rt/λ extra cycles of sound during time t , increasing the total number of sound cycles received to $(ft + rt/\lambda)$. With $(ft + rt/\lambda)$ cycles received in t seconds, the apparent frequency f' is given by:

$$f' = \frac{ft + rt/\lambda}{t} = f + r/\lambda = f + \frac{rf}{v} = \frac{f(v+r)}{v} \text{ [using the relation } \frac{1}{\lambda} = \frac{f}{v} \text{ from Eq. 13.3]}$$

The frequency difference Δf can be expressed as: $\Delta f = f' - f = \frac{f(v+r)}{v} - f = \frac{fr}{v}$, from which the velocity of the receiver r can be expressed as: $r = v\Delta f/f$.

Similarly, it can be shown that, if R moves away from S with velocity r , f' is given by:

$$f' = \frac{f(v-r)}{v} \text{ and } \Delta f = -\frac{fr}{v}.$$

If the ultrasound source moves towards the stationary receiver at velocity s , it will move a distance st in time t and the ft cycles that are emitted during time t will be compressed into a distance $(vt-st)$.

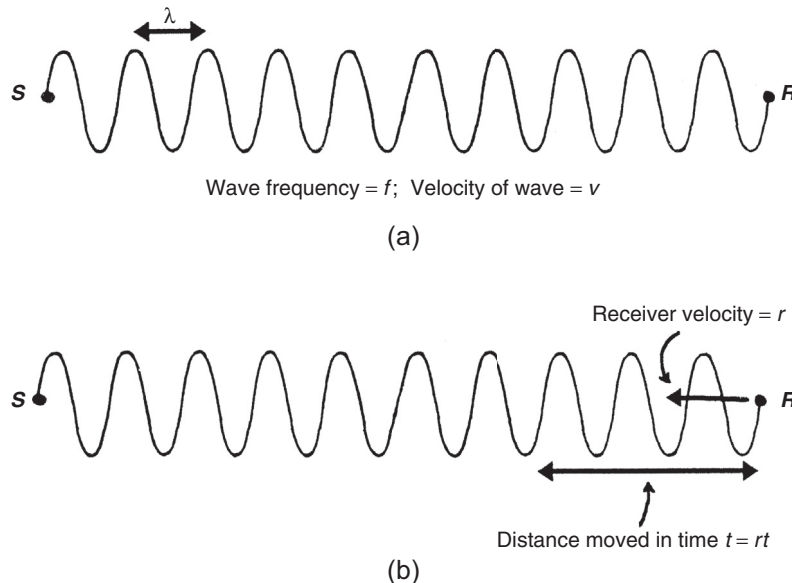


Figure 13.13

Illustration of Doppler effect. (a) Source (S) and Receiver (R) stationary; (b) Receiver (R) moving toward Source (S).

Hence, the apparent wavelength λ' will be given by:

$$\lambda' = \frac{vt - st}{ft} = \frac{v - s}{f}.$$

Using Eq. 13.3, this can be expressed alternatively as $f' = \frac{v}{\lambda'} = \frac{vf}{v-s}$.

Similarly, with S moving away from R, it can be shown that $f' = \frac{vf}{v+s}$.

Thus, the velocity of an ultrasound receiver moving with respect to an ultrasound source can be calculated from the measured ratio between the real and apparent frequencies of the wave. This is used in devices like the Doppler shift flowmeter.

13.11 Nuclear sensors

Nuclear sensors are uncommon measurement devices, partly because of the strict safety regulations that govern their use and partly because they are usually expensive. Some very low-level radiation sources are now available that largely overcome the safety problems, but measurements are then prone to contamination by background radiation. The principle of operation of nuclear sensors is very similar to optical sensors in that radiation is transmitted between a source and a detector through some medium in which the magnitude of transmission is attenuated according to the value of the measured variable. Caesium-137 is commonly used as a gamma-ray source, and a sodium iodide device is commonly used as a gamma-ray detector. The latter gives a voltage output that is proportional to the radiation incident upon it. One current use of nuclear sensors is in a noninvasive technique for measuring the level of liquid in storage tanks (see Chapter 17). They are also used in mass flow rate measurement (see Chapter 16) and in medical scanning applications ([Webster, 2009](#)).

13.12 Microsensors (MEMS sensors)

Microsensors are two- and three-dimensional micromachined structures that have smaller size, improved performance, better reliability, and lower production costs than many alternative forms of sensor. They are part of the wider class of *micro-electro-mechanical-systems (MEMS) devices* that also includes microactuators. Typical sizes of microsensors range from 10 μm (0.01 mm or 10^{-5} m) up to 5 mm. The defining feature of any MEMS device is an element with some sort of mechanical functionality integrated with microelectronics. Microsensors can be regarded as miniature transducers, since they convert energy in the form of a measured mechanical signal into energy in electrical form. Individual devices vary from simple ones where the mechanical part does not move to much more complex ones involving several moving elements. Currently, devices to measure temperature, pressure, torque, acceleration, rotational velocity, humidity, sound,

magnetic fields, radiation, and optical, biological, biomedical, and chemical parameters are either in production or at advanced stages of research.

Microsensors are usually constructed from a silicon-semiconductor material but are sometimes fabricated from other materials such as metals, plastics, polymers, glasses, and ceramics that are deposited on a silicon base. Silicon is an ideal material for sensor construction because of its excellent mechanical properties. Its tensile strength and Young's modulus is comparable to that of steel, whilst its density is less than that of aluminum. Sensors made from a single crystal of silicon remain elastic almost to the breaking point, and mechanical hysteresis is very small. In addition, silicon has a very low coefficient of thermal expansion and can be exposed to extremes of temperature and to most gases, solvents, and acids without deterioration. This means that silicon-based sensors suffer very little fatigue and consequently have a very long life (service life in terms of billions of operations are often quoted).

One downside of silicon is the relatively large expense in producing it owing to the complexity of the crystal silicon structure. This is why other materials are often used as an alternative to silicon. **Polymer-based microsensors** can be produced much more cheaply using either embossing, injection molding, or stereolithography, and these are used in applications like disposable blood-testing devices. **Ceramic-based microsensors** made from various ceramic materials are also cheaper than silicon devices. **Metal-based microsensors** are made from various metals including aluminum, chromium, copper, gold, nickel, platinum, silver, titanium, and tungsten, using manufacturing techniques that include electroplating, sputtering, and evaporation processes. These are cheaper than silicon devices, and whilst their mechanical properties are inferior to those of silicon, they are still quite reliable.

Microengineering techniques are an essential enabling technology for microsensors, which are designed so that their electromechanical properties change in response to a change in the measured parameter. Many of the techniques used for integrated circuit (IC) manufacture are also used in sensor fabrication, common techniques being crystal growing and polishing, thin-film deposition, ion implantation, wet and dry chemical and laser etching, and photolithography. However, apart from standard IC production techniques, some special techniques are also needed in addition to produce the 3D structures that are unique to some types of microsensor. The various manufacturing techniques are also used to form sensors directly in silicon crystals and films. Typical structures have forms such as thin diaphragms, cantilever beams, and bridges.

Whilst the small size of a microsensor is of particular benefit in many applications, it also leads to some problems that require special attention. For example, microsensors typically have very low capacitance. This makes the output signals very prone to noise contamination. Hence, it is usually necessary to integrate microelectronic circuits that perform signal processing in the device, which therefore becomes a **smart microsensor**.

Another problem is that microsensors generally produce output signals of very low magnitude. This requires the use of special types of analog-to-digital converter that can cope with such low-amplitude input signals. One suitable technique is sigma-delta conversion. This is based on charge-balancing techniques and gives better than 16 bit accuracy in less than 20 ms. Special designs can reduce conversion time down to less than 0.1 ms if necessary. The latest trend is to incorporate both analog-to-digital conversion and amplification within the microsensor device. The very latest generations of microsensors currently under development include digital intelligence to provide linearization, calibration, and temperature compensation functions. Even smaller-scale devices are also being developed, which have been given the name of *nanosensors* (see next section).

Microsensors are currently used most commonly for measuring pressure, acceleration, torque, and chemical parameters. They are used in particularly large numbers in the automotive industry, where unit prices can be very low. Typical applications are in air bag actuation and vehicle stability control. Microsensors are also widely used in medical applications, particularly for blood pressure measurement.

Mechanical microsensors transform measured variables such as torque, pressure, and acceleration into a displacement. The displacement is usually measured by capacitive or piezoresistive techniques, although some devices use other technologies such as resonant frequency variation, resistance change, inductance change, the piezoelectric effect, and changes in magnetic or optical coupling. The design of a cantilever beam microaccelerometer is shown in Figure 19.15. The proof mass within this is about 100 μm across, and the typical deflection measured is of the order of 1 micron (10^{-3} mm).

An alternative capacitive microaccelerometer provides a calibrated, compensated, and amplified output. It has a capacitive silicon microsensor to measure displacement of the proof mass. This is integrated with a signal-processing chip and protected by a plastic enclosure. The capacitive element has a 3D structure, which gives a higher measurement sensitivity than surface-machined elements.

Microsensors to measure many other physical variables are either in production or at advanced stages of research. Microsensors measuring magnetic field are based on a number of alternative technologies such as Hall effect, magnetoresistors, magnetodiodes, and magnetotransistors. Radiation microsensors are made from silicon p-n diodes or avalanche photodiodes and can detect radiation over wavelengths from the visible spectrum to infrared. Microsensors in the form of a microthermistor, a p-n thermodiode, or a thermotransistor are used as digital thermometers. Microsensors have also enabled measurement techniques that were previously laboratory-based ones to be extended into field instruments. Examples are spectroscopic instruments and devices to measure viscosity.

As well as current research expanding the applications and functionality of microsensors, there is also work going on that is looking at ways of integrating microactuators with microsensors. The aim of this next stage of research is to fabricate a microsensor, a microactuator, and controlling microelectronics onto a single silicon substrate as a smart MEMS device. When this is achieved, this entire control system on a single chip will represent a very important technological breakthrough. These will have application in areas like environmental control, where microsensors measure environmental parameters and microactuators effect control action in the form of pumping, filtering, regulating, or positioning operations.

13.13 Nanosensors (NEMS sensors)

The most recent advance in sensor miniaturization is the development of nanosensors based on nanotechnology. These are part of the wider class of *nano-electro-mechanical-system (NEMS) devices* that includes nanoactuators as well as nanosensors. Nanosensors vary from 1 to 100 nm in size (10^{-9} to 10^{-7} m) and are made from thin layers of either metal films or semiconductors. These are fabricated using similar techniques to MEMS sensors, but employing more advanced forms of etching, optical lithography, or electron beam lithography. They are typically used as accelerometers, biological sensors, and sensors for airborne chemicals. In time, they are expected to replace MEMS devices in many applications, since NEMS devices have advantages in terms of lower production costs and reduced power consumption, apart from their smaller size. As mentioned earlier in Chapter 11, smart NEMS devices are also currently being researched, in which computing power is being integrated with a sensor in a nanoscale chip. A typical proposed application is a smart hearing aid.

13.14 Summary

This chapter has revealed 12 different physical principles that are used in measurement sensors. As noted in the introduction to the chapter, the chosen order of presentation of these principles has been arbitrary and is not intended to imply anything about the relative popularity of these various principles.

The first principle covered was capacitance change, which we found was based on two capacitor plates with either a variable or fixed distance between them. We learned that sensors with a variable distance between the plates are primarily used for displacement measurement, either as displacement sensors in their own right or to measure the displacement within certain types of pressure, sound, and acceleration sensor. The alternative type of capacitive sensor where the distance between the plates is fixed is typically used to measure moisture content, humidity values, and liquid level.

Moving on to the resistance-change principle, we found that this is used in a wide range of devices for temperature measurement (resistance thermometers and thermistors) and displacement measurement (strain gauges and piezoresistive sensors). We also noted that some moisture meters work on the resistance-variation principle.

We then looked at sensors that use the magnetic phenomena of inductance, reluctance, and eddy currents. We saw that the principle of inductance change was mainly used to measure translational and rotational displacements, reluctance change was commonly used to measure rotational velocities, and the eddy current effect was typically used to measure the displacement between a probe and a very thin metal target such as the steel diaphragm of a pressure sensor.

Next we looked at the Hall effect. This measures the magnitude of a magnetic field and is commonly used in a proximity sensor. It is also employed in computer keyboard push buttons.

We then moved on to piezoelectric transducers. These generate a voltage when a force is applied to them. Alternatively, if a voltage is applied to them, an output force is produced. A common application is in ultrasonic transmitters and receivers. They are also used as displacement transducers, particularly as part of devices measuring acceleration, force, and pressure.

Our next subject of study was strain gauges. These devices exploit the physical principle of a change in resistance when the metal wire that they are made from is stretched or strained. They detect very small displacements and are typically used within devices like diaphragm pressure sensors to measure the small displacement of the diaphragm when a pressure is applied to it. We looked into some of the science involved in strain gauge design, particularly with respect to the alternative materials used for the active element.

Moving on, we then looked at piezoresistive sensors. We saw that these could be regarded as a semiconductor strain gauge, since they consist of a semiconductor material whose resistance varies when it is compressed or stretched. They are commonly used to measure the displacement in diaphragm pressure sensors where the resistance change for a given amount of diaphragm displacement is much greater than is obtained in metal strain gauges, thus leading to better measurement accuracy. They are also used as accelerometers. Before concluding this discussion, we also observed that the term “piezoresistive sensor” is sometimes (but incorrectly) used to describe metal strain gauges as well as semiconductor ones.

In our discussion of optical sensors that followed, we observed first of all that these could involve both transmission of light through air and transmission along a fiber-optic cable. Air-path optical sensors exploit the transmission of light from a source to a detector across

an open air path and are commonly used to measure proximity, translational motion, rotational motion, and gas concentration.

Sensors that involve the transmission of light along a fiber-optic cable are commonly called fiber-optic sensors. Their principle of operation is to translate the measured physical quantity into a change in either the intensity, phase, polarization, wavelength, or transmission time of the light carried along the cable. We went on to see that two kinds of fiber-optic sensors can be distinguished, known as intrinsic sensors and extrinsic sensors. In intrinsic sensors, the fiber-optic cable itself is the sensor, whereas in extrinsic sensors, the fiber-optic cable is merely used to transmit the light to/from a conventional sensor. Our look at intrinsic sensors revealed that different forms of these are used to measure a very wide range of physical variables including proximity, displacement, pressure, pH, smoke intensity, acceleration, temperature, cryogenic leakage, oil content in water, liquid level, refractive index of a liquid, parameters in biomedical applications (oxygen concentration, carbon monoxide concentrations, blood pressure level, hormone concentration, steroid concentration), mechanical strain, magnetic field strength, electric field strength, electrical current, electrical voltage, angular position and acceleration in gyroscopes, liquid flow rate, and gas presence. By comparison, the range of physical variables measured by extrinsic sensors is much smaller, being limited mainly to the measurement of temperature, pressure, force, and displacement (both linear and angular).

This then led to a discussion of ultrasonic sensors. These are commonly used to measure range (large displacements), small translational displacements, fluid flow rate, and liquid level. We learned that ultrasonic sensors work in one of two ways, either by measuring the change in time taken for an ultrasound wave to travel between a transmitter and receiver, or by measuring the change in phase or frequency of the transmitted wave. Whilst both of these principles are simple in concept, we went on to see that the design and use of ultrasonic sensors suffers from a number of potential problems. Firstly, the transmission speed can be affected by environmental factors, temperature changes being a particular problem and humidity changes to a lesser extent. The nominal operating frequency of ultrasonic elements also has to be carefully chosen according to the intended application as this affects the effective amount of spread of the transmitted energy either side of the direction normal to the face of the transmitting element. Attenuation of the transmitted wave can cause problems. This is particularly so when ultrasonic elements are used as range sensors. This follows from the start-up nature of a transmitted ultrasonic wave, which exhibits an increasing amplitude over the first two or three cycles of the emitted energy wave. Attenuation of the wave as it travels over a distance may mean that the detector fails to detect the first or even second cycle of the transmitted wave, causing an error that is equal to one or two times the ultrasound wavelength. Noise can also cause significant problems in the operation of ultrasonic sensors, since they are easily contaminated by electromagnetic noise and are particularly affected by noise generated by

manufacturing operations at a similar frequency to that of the ultrasonic measuring system. Since there is some emission of ultrasonic energy at angles other than the normal direction to the face of the ultrasonic element, stray reflections of transmissions in these nonnormal directions by structures in the environment around the ultrasonic system may interfere with measurements.

The next type of sensor discussed was nuclear sensors. We learned that these did not enjoy widespread use, with the main applications being in noninvasive measurement of liquid level, mass flow rate measurement, and in some medical scanning applications. This limited number of applications is partly due to the health dangers posed to users by the radiation source that they use and partly due to their relatively high cost. Danger to users can largely be overcome by using low-level radiation sources, but this makes measurements prone to contamination by background radiation.

Next, we looked at microsensors (part of MEMS devices), which we learned were millimeter-sized, two- and three-dimensional micromachined structures that are usually made from silicon semiconductor materials but can sometimes be made from other materials. These types of sensors have smaller size, improved performance, better reliability, and lower production costs than many alternative forms of sensors and are used to measure temperature, pressure, torque, acceleration, humidity, magnetic fields, radiation, chemical parameters, and some parameters in medical applications such as blood pressure.

Finally, we briefly looked at the ongoing research into nanosensors, which are the next level of miniaturization down from microsensors and are part of the class of devices called NEMS. At the time of publication of this book, there were few if any commercial applications of these, but their advantages in terms of even smaller size, lower cost, and reduced power consumption mean that they will replace MEMS sensors in time in many applications.

13.15 Problems

- 13.1 Describe the general working principles of capacitive sensors and discuss some applications of them.
- 13.2 Discuss some applications of resistive sensors.
- 13.3 What types of magnetic sensors exist and what are they mainly used for? Describe the mode of operation of each.
- 13.4 What are Hall-effect sensors? How do they work and what are they used for?
- 13.5 How does a piezoelectric transducer work and what materials are typically used in their construction? Discuss some common applications of this type of device.
- 13.6 What is a strain gauge and how does it work? What are the problems in making and using a traditional metal-wire strain gauge and how have these problems been overcome in new types of strain gauge?

- 13.7 Discuss some applications of strain gauges.
- 13.8 What are piezoresistive sensors and what are they typically used for?
- 13.9 What is the principal advantage of an optical sensor? Discuss the mode of operation of the two main types of optical sensor.
- 13.10 What are air-path optical sensors? Discuss their mode of operation, including details of light sources and light detectors used.
- 13.11 How do fiber-optic sensors work? Discuss their use in intrinsic and extrinsic sensors.
- 13.12 Explain the basic principles of operation of ultrasonic sensors and discuss what they are typically used for.
- 13.13 What factors govern the transmission speed and directionality of ultrasonic waves? How do these factors affect the application of ultrasonic sensors?
- 13.14 Discuss the use of ultrasonic sensors in range-measuring systems, mentioning the effect of attenuation of the wave as it travels. How can the measurement resolution and accuracy be optimized?
- 13.15 Discuss the effects of extraneous noise in ultrasonic measurement systems. How can these effects be reduced?
- 13.16 Discuss the phenomenon of Doppler shift in ultrasound transmission and explain how this can be used in sensors.
- 13.17 Why are nuclear sensors not in common use?
- 13.18 What are microsensors (MEMS sensor)? How are they made and what applications are they used in?
- 13.19 List the various light parameters that are modulated in different forms of intrinsic fiber-optic sensors. Discuss the characteristics of each type of light modulation and give examples of sensors that are based on each type.
- 13.20 What are extrinsic fiber-optic sensors? What features of them make them particularly useful in many applications? Give examples of such applications. What implementation problems do extrinsic fiber-optic sensors pose?
- 13.21 What is the relationship between wavelength, frequency, and directionality of ultrasound waves? What is the relationship between the attenuation of an ultrasound wave, its frequency, and the distance traveled from the ultrasonic wave source?

Reference

Webster, J.G., 2009. *Medical Instrumentation Application and Design*. John Wiley.

Temperature measurement

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14.1 Introduction

We are probably well aware that temperature measurement is important in all spheres of life. In engineering applications, it is particularly important in the process industries, where it is the most commonly measured process variable. It is therefore appropriate for us to devote this first chapter on the measurement of individual physical variables to the subject of temperature measurement.

Unfortunately, temperature measurement poses some interesting theoretical difficulties because of its rather abstract nature. These difficulties become especially apparent when we consider the calibration of temperature-measuring instruments, particularly when we look for a primary reference standard at the top of the calibration chain. Foremost among these difficulties is the fact that any given temperature cannot be related to a fundamental standard of temperature in the same way that the measurement of other quantities can be related to the primary standards of mass, length, and time. If two bodies of lengths l_1 and l_2 are connected together end to end, the result is a body of length $l_1 + l_2$. A similar relationship exists between separate masses and separate times. However, if two bodies at the same temperature are connected, the joined body has the same temperature as each of the original bodies.

This is a root cause of the fundamental difficulties that exist in establishing an absolute standard for temperature in the form of a relationship between it and other measurable quantities for which a primary standard unit exists. In the absence of such a relationship, it is necessary to establish fixed, reproducible reference points for temperature in the form of freezing and triple points of substances in which the transitions among solid, liquid, and gaseous states are sharply defined. The *International Practical Temperature Scale (IPTS)*¹ uses this philosophy and defines a number of *fixed points* for reference temperatures. Three examples are:

- the triple point of hydrogen: -259.35°C
- the freezing point of zinc: 419.53°C
- the freezing point of gold: 1064.18°C

A full list of fixed points defined in the IPTS can be found in [Section 14.14](#).

If we start writing down the physical principles that are affected by temperature, we will get a relatively long list. Many of these physical principles form the basis for temperature-measuring instruments. It is therefore reasonable for us to study temperature measurement by dividing the instruments used to measure temperature into separate classes according to

¹The IPTS is subject to periodic review and improvement as research produces more precise fixed reference points. The latest version was published in 1990.

the physical principle on which they operate. This will give us nine classes of instrument based on the following principles:

- (i) The thermoelectric effect
- (ii) Resistance change
- (iii) Sensitivity of semiconductor device
- (iv) Radiative heat emission
- (v) Thermography
- (vi) Thermal expansion
- (vii) Sensitivity of fiber-optic devices
- (viii) Color change
- (ix) Change of state of material (pyrometric cones)

We will consider each of these in the following sections. The final section in this list covers microelectromechanical system (MEMS) sensors, which are miniature sensors currently based on measuring radiative heat emission.

14.2 Thermoelectric effect sensors (*thermocouples*)

Thermoelectric effect sensors rely on the physical principle that when any two different metals are connected, an electromagnetic field (emf), which is a function of the temperature, is generated at the junction between the metals. The general form of this relationship is:

$$e = a_1T + a_2T^2 + a_3T^3 + \cdots + a_nT^n \quad (14.1)$$

where e is the emf generated and T is the absolute temperature.

This is clearly nonlinear, which is inconvenient for measurement applications. Fortunately, for certain pairs of materials, the terms involving squared and higher powers of T (a_2T^2 , a_3T^3 , etc.) are approximately 0 and the emf–temperature relationship is approximately linear according to:

$$e \approx a_1T \quad (14.2)$$

Wires of such pairs of materials are connected at one end, and in this form they are known as *thermocouples*. Thermocouples are an important class of device because they provide the most commonly used method of measuring temperatures in industry.

Thermocouples are manufactured from various combinations of the base metals copper and iron, various base–metal alloys (alumel, constantan, chromel, nicrosil, and nisil), the noble metals platinum, gold, and tungsten, and various noble–metal alloys. Only certain combinations of these are used as thermocouples and most standard combinations are known by internationally recognized type letters: for example, Type K is chromel-alumel.

The emf-temperature characteristics for some of these standard thermocouples are shown in Fig. 14.1. These show reasonable linearity over at least part of their temperature-measuring ranges.

The structure of a typical thermocouple, made from one chromel wire and one constantan wire, is shown in Fig. 14.2(a). For purposes of analysis, it is useful to represent the thermocouple by its equivalent electrical circuit, shown in Fig. 14.2(b). The emf generated at the point where the different wires are connected is represented by a voltage source, E_1 , and the point is known as the **hot junction**. The temperature of the hot junction is customarily shown as T_h on the diagram. The emf generated at the hot junction is measured at the open ends of the thermocouple, which is known as the **reference junction**.

To make a thermocouple conform to some precisely defined emf-temperature characteristic, it is necessary for all metals that are used to be refined to a high degree of purity and all alloys to be manufactured to an exact specification. This makes the materials expensive, and consequently thermocouples are typically only a few centimeters long. It is clearly impractical to connect a voltage-measuring instrument at the open end of the thermocouple to measure its output in such close proximity to an environment whose temperature is being measured, and therefore **extension leads** up to several meters long are normally connected between the thermocouple and the measuring instrument. This modifies the equivalent circuit to that shown in Fig. 14.3(a). There are thus three junctions in the system and

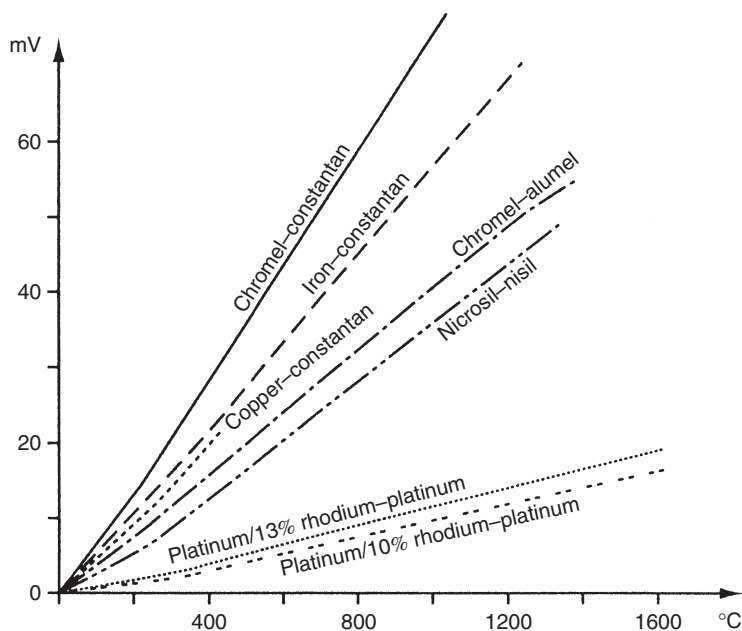


Figure 14.1

Electromagnetic field temperature characteristics for some standard thermocouple materials.

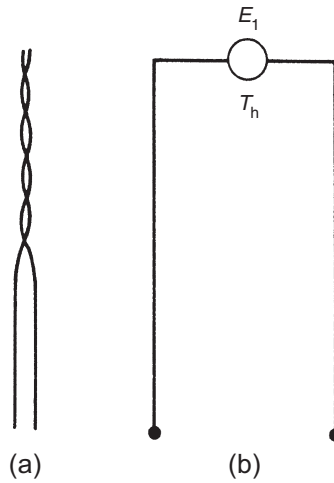


Figure 14.2
 (a) Thermocouple. (b) Equivalent circuit.

consequently three voltage sources, E_1 , E_2 , and E_3 , with the point of measurement of the emf (still called the reference junction) moved to the open ends of the extension leads.

The measuring system is completed by connecting the extension leads to the voltage-measuring instrument. Because the connection leads will normally be of materials different to those of the thermocouple extension leads, this introduces two further

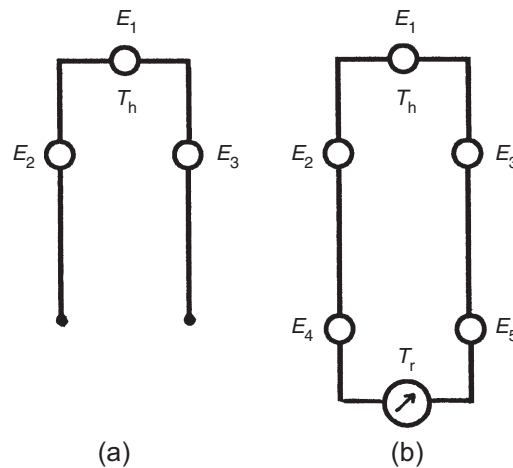


Figure 14.3
 (a) Equivalent circuit for thermocouple with extension leads. (b) Equivalent circuit for thermocouple and extension leads connected to a meter.

emf-generating junctions, E_4 and E_5 , into the system, as shown in Fig. 14.3(b). The net output emf measured (E_m) is then given by:

$$E_m = E_1 + E_2 + E_3 + E_4 + E_5 \quad (14.3)$$

and this can be reexpressed in terms of E_1 as:

$$E_1 = E_m - E_2 - E_3 - E_4 - E_5 \quad (14.4)$$

To apply Eq. (14.1) to calculate the measured temperature at the hot junction, E_1 has to be calculated from Eq. (14.4). To do this, it is necessary to calculate the values of E_2 , E_3 , E_4 , and E_5 .

It is usual to choose materials for the extension lead wires such that the magnitudes of E_2 and E_3 are approximately 0 irrespective of the junction temperature. This avoids the difficulty that would otherwise arise in measuring the temperature of the junction between the thermocouple wires and the extension leads, and also in determining the emf–temperature relationship for the thermocouple–extension lead combination.

A near-zero junction voltage is most easily achieved by choosing the extension leads to be of the same basic materials as the thermocouple, but in which their cost per unit length is greatly reduced by manufacturing them to a lower specification. As an alternative to using lower-specification materials of the same basic type as the thermocouple, copper compensating leads are also sometimes used with certain types of base metal thermocouples. In this case, the law of intermediate metals has to be applied to compensate for the emf at the junction between the thermocouple and compensating leads.

Unfortunately, the use of extension leads of the same basic materials as the thermocouple but manufactured to a lower specification is still prohibitively expensive in the case of noble metal thermocouples. It is necessary in this case to search for base metal extension leads that have thermoelectric behavior similar to that of the noble metal thermocouple. In this form, the extension leads are usually known as *compensating leads*. A typical example of this is the use of nickel/copper-copper extension leads connected to a platinum/rhodium-platinum thermocouple. The approximately equivalent thermoelectric behavior of compensating leads is valid for only a limited range of temperatures that is considerably less than the measuring range of the thermocouple that they are connected to.

To analyze the effect of connecting the extension leads to the voltage-measuring instrument, a thermoelectric law known as the *law of intermediate metals* can be used. This states that the emf generated at the junction between two metals or alloys A and C is equal to the sum of the emf generated at the junction between metals or alloys A and B and the emf generated at the junction between metals or alloys B and C , in which all junctions are at the same temperature. This can be expressed more simply as:

$$e_{AC} = e_{AB} + e_{BC} \quad (14.5)$$

Suppose we have an iron-constantan thermocouple connected by copper leads to a meter. We can express E_4 and E_5 in Fig. 14.4 as:

$$E_4 = e_{\text{iron-copper}} \quad ; \quad E_5 = e_{\text{copper-constantan}}$$

The sum of E_4 and E_5 can be expressed as: $E_4 + E_5 = e_{\text{iron-copper}} + e_{\text{copper-constantan}}$

Applying Eq. (14.5): $e_{\text{iron-copper}} + e_{\text{copper-constantan}} = e_{\text{iron-constantan}}$.

Thus, the effect of the connecting the thermocouple extension wires to the copper leads to the meter is canceled out, and the actual emf at the reference junction is equivalent to that arising from an iron-constantan connection at the reference junction temperature, which can be calculated according to Eq. (14.1). Hence, the equivalent circuit in Fig. 14.3(b) becomes simplified to that shown in Fig. 14.4. The emf E_m measured by the voltage-measuring instrument is the sum of only two emfs, consisting of the emf generated at the hot junction temperature E_1 and the emf generated at the reference junction temperature E_{ref} . The emf generated at the hot junction can then be calculated as:

$$E_1 = E_m + E_{ref}$$

E_{ref} can be calculated from Eq. (14.1) if the temperature of the reference junction is known. In practice, this is often achieved by immersing the reference junction in an ice bath to maintain it at a reference temperature of 0°C . However, as discussed in the following section on thermocouple tables, it is very important that the ice bath remains exactly at 0°C if this is to be the reference temperature assumed; otherwise, significant measurement errors can arise. For this reason, refrigeration of the reference junction at a temperature of 0°C is often preferred.

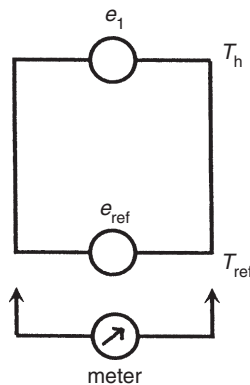


Figure 14.4

Effective electromagnetic field (voltage) sources in a thermocouple measurement system.

14.2.1 Thermocouple tables

Although the preceding discussion suggested that the unknown temperature T can be evaluated from the calculated value of emf E_1 at the hot junction using Eq. (14.1), this is difficult to do in practice because Eq. (14.1) is a high-order polynomial expression. An approximate translation between the value of E_1 and temperature can be achieved by expressing Eq. (14.1) in graphical form as in Fig. 14.1. However, this is not usually sufficiently accurate and it is normal practice to use tables of emf and temperature values known as *thermocouple tables*. These include compensation for the effect of the emf generated at the reference junction (E_{ref}), which is assumed to be at 0°C . Thus, the tables are valid only when the reference junction is exactly at this temperature. Compensation for the case in which the reference junction temperature is not at 0 is considered later in this section.

Tables for a range of standard thermocouples are given in Appendix 3.² In these tables, a range of temperatures is given in the left-hand column and the emf output for each standard type of thermocouple is given in the columns to the right. In practice, any general emf output measurement taken at random will not be found exactly in the tables, and interpolation will be necessary between the values shown in the table. This is illustrated by Example 14.2.

■ Example 14.1

If the emf output measured from a chromel-constantan thermocouple is 13.419 mV with the reference junction at 0°C , the appropriate column in the tables shows that this corresponds to a hot junction temperature of 200°C .

■ Example 14.2

Suppose the measured output emf for a chromel-constantan thermocouple (reference junction at 0°C) is 10.72 mV. Looking at the table for a chromel-constantan thermocouple, the value of 10.72 mV does not appear in the table. The closest emf values in the table are 10.501 mV corresponding to a temperature of 160°C and 11.222 mV corresponding to a temperature of 170°C . It is necessary to carry out interpolation

² The tables given in Appendix 3 are computed according to the standard thermocouple equations corresponding to the reference fixed point temperatures given in the 1990 IPTS. However, because thermocouples available from manufacturers can sometimes differ slightly in the chemical composition of alloys and the purity of metals, it is always preferable to use thermocouple tables supplied in manufacturers' data sheets rather than rely on tables published elsewhere.

between these values to determine the temperature corresponding to the emf value of 10.72 mV. The process of interpolation is explained in detail in Appendix 4.

The situation is illustrated in Fig. 14.5. The relationship between the thermocouple output voltage and temperature is a curve (curvature exaggerated in the figure), and the exact temperature corresponding to an output of 10.72 mV would be determined by drawing a horizontal line through the 10.72 point on the vertical axis as far as the thermocouple curve and then drawing a vertical line from the point of intersection with the curve down to the horizontal axis, where the point of intersection would indicate the temperature. In practice, we cannot do this because we do not know the nature of the curve between these two points given by the thermocouple table (10.501 mV at 160°C and 11.222 mV at 170°C). The best that we can do is to draw a straight line between these two points that approximates the actual curve between the points. Thus, the horizontal line through the 10.72 point on the vertical axis is drawn as far as the straight line between the points given in the table. A vertical line is then drawn from the point of intersection with this straight line down to the horizontal axis, where the point of intersection best indicates the temperature that we can obtain.

Mathematically, this process can be explained as follows:

The emf value of 10.72 mV is 0.219 mV above the value of 10.501 mV ($10.72 - 10.501 = 0.219$).

The difference between the values of 10.501 and 11.222 mV is 0.721 mV.

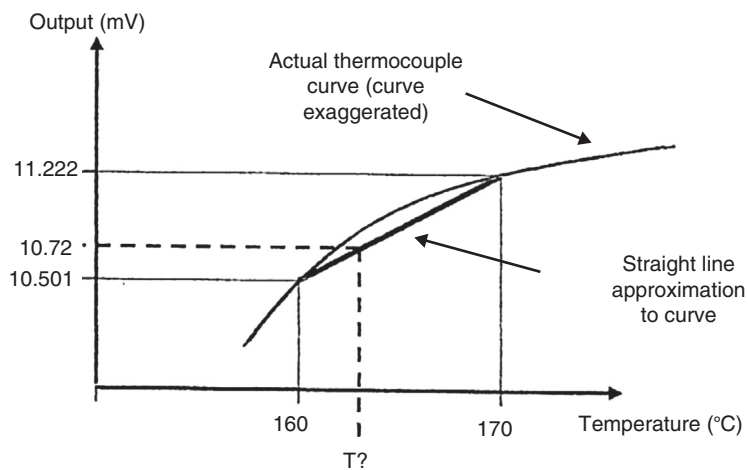


Figure 14.5

Procedure of interpolation between data points of a thermocouple table.

Thus, the value of 10.72 mV is $\frac{0.216}{0.721} = 0.304$ of the distance between the emf values of 10.501 and 11.222. Because of the straight line approximation to the thermocouple curve, it follows that the temperature is 0.304 of the distance between the temperatures of 160°C and 170°C.

As an equation, the temperature can be expressed as:

$$\text{Temperature} = \left(\frac{10.72 - 10.501}{11.222 - 10.501} \times 10 \right) + 160 = 163.04$$

14.2.2 Nonzero reference junction temperature

If the reference junction is immersed in an ice bath to maintain it at a temperature of 0°C so that thermocouple tables can be applied directly, the ice in the bath must be in a state of just melting. This is the only state in which ice is exactly at 0°C; otherwise, it will be either colder or hotter than this temperature. Thus, maintaining the reference junction at 0°C is not straightforward, particularly if the environmental temperature around the measurement system is relatively hot. In consequence, it is common practice in many practical applications of thermocouples to maintain the reference junction at a nonzero temperature by putting it into a controlled environment maintained by an electrical heating element. To be able to continue to apply thermocouple tables, correction then has to be made for this nonzero reference junction temperature using a second thermoelectric law known as the *law of intermediate temperatures*, which states that:

$$E_{(T_h, T_0)} = E_{(T_h, T_r)} + E_{(T_r, T_0)} \quad (14.6)$$

where $E_{(T_h, T_0)}$ is the emf with the junctions at temperatures T_h and T_0 , $E_{(T_h, T_r)}$ is the emf with the junctions at temperatures T_h and T_r , and $E_{(T_r, T_0)}$ is the emf with the junctions at temperatures T_r and T_0 , T_h is the hot-junction measured temperature, T_0 is 0°C, and T_r is the nonzero reference junction temperature that is somewhere between T_0 and T_h .

■ Example 14.3

Suppose the reference junction of a chromel-constantan thermocouple is maintained at a temperature of 80°C and the output emf measured is 40.102 mV when the hot junction is immersed in a fluid.

The quantities given are $T_r = 80^\circ\text{C}$ and $E_{(T_h, T_r)} = 40.102$ mV.

From the tables, $E_{(T_r, T_0)} = 4.983$ mV.

Now applying Eq. (14.6), $E_{(T_h, T_0)} = 40.102 + 4.983 = 45.085$ mV.

Referring to the tables, this indicates a fluid temperature of 600°C.

In using thermocouples, it is essential that they are connected correctly. Large errors can result if they are connected incorrectly, such as by interchanging the extension leads or by using incorrect extension leads. Such mistakes are particularly serious because they do not prevent some sort of output from being obtained that may look sensible although it is incorrect, and so the mistake may go unnoticed for a long time. The following examples illustrate the sort of errors that may arise.

■ Example 14.4

This example is an exercise in the use of thermocouple tables, but it also serves to illustrate the large errors that can arise if thermocouples are used incorrectly. In a particular industrial situation, a chromel-alumel thermocouple with chromel-alumel extension wires is used to measure the temperature of a fluid. In connecting this measurement system, the instrumentation engineer responsible has inadvertently interchanged the extension wires from the thermocouple. The ends of the extension wires are held at a reference temperature of 0°C and the output emf measured is 14.1 mV . If the junction between the thermocouple and extension wires is at a temperature of 40°C , what temperature of fluid is indicated and what is the true fluid temperature?

■ Solution

The initial step necessary to solve a problem of this type is to draw a diagrammatical representation of the system and to mark on this the emf sources, temperatures, and so on, as shown in Fig. 14.6. The first part of the problem is solved simply by looking

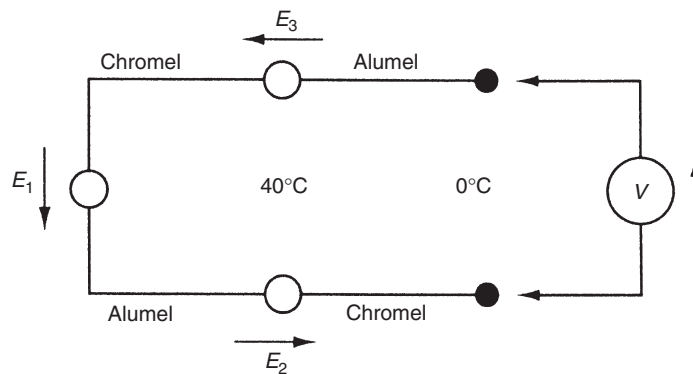


Figure 14.6
Diagram for solution of Example 14.4.

up in thermocouple tables what temperature the emf output of 12.1 mV indicates for a chromel-alumel thermocouple. This is 297.4°C. Then, summing emfs around the loop:

$$V = 12.1 = E_1 + E_2 + E_3 \quad \text{or} \quad E_1 = 12.1 - E_2 - E_3$$

$$E_2 = E_3 = \text{Emf}_{(\text{alumel}-\text{chromel})_{40}} = -\text{Emf}_{(\text{chromel}-\text{alumel})_{40}}^* = -1.611 \text{ mV.}$$

Hence, $E_1 = 12.1 + 1.611 + 1.611 = 15.322 \text{ mV}$.

Interpolating from the thermocouple tables, this indicates that the true fluid temperature is 374.5°C.

■ Example 14.5

This example also illustrates the large errors that can arise if thermocouples are used incorrectly. An iron-constantan thermocouple measuring the temperature of a fluid is connected by mistake with copper-constantan extension leads (such that the two constantan wires are connected and the copper extension wire is connected to the iron thermocouple wire). If the fluid temperature was actually 200°C, and the junction between the thermocouple and extension wires was at 50°C, what emf would be measured at the open ends of the extension wires if the reference junction is maintained at 0°C? What fluid temperature would be deduced from this (assuming that the connection mistake was not recognized)?

■ Solution

Again, the initial step necessary is to draw a diagram showing the junctions, temperatures, and emfs, as shown in Fig. 14.7. The various quantities can then be calculated:

$$E_2 = \text{Emf}_{(\text{iron}-\text{copper})_{50}}$$

* The thermocouple tables quote emfs using the convention that going from chromel to alumel is positive. Hence, the emf going from alumel to chromel is minus the emf going from chromel to alumel.

By the law of intermediate metals:

$$\begin{aligned} \text{Emf}_{(\text{iron-copper})_{50}} &= \text{Emf}_{(\text{iron-constantan})_{50}} - \text{Emf}_{(\text{copper-constantan})_{50}} \\ &= 2.585 - 2.035 \text{ [from thermocouple tables]} = 0.55 \text{ mV} \\ E_1 &= \text{Emf}_{(\text{iron-constantan})_{200}} = 10.777 \text{ [from thermocouple tables]} \\ V &= E_1 - E_2 = 10.777 - 0.55 = 10.227 \end{aligned}$$

Using tables and interpolating, 10.227 mV indicates a temperature of:

$$\left(\frac{10.227 - 10.222}{10.777 - 10.222} \right) 10 + 190 = 190.1^\circ \text{C}$$

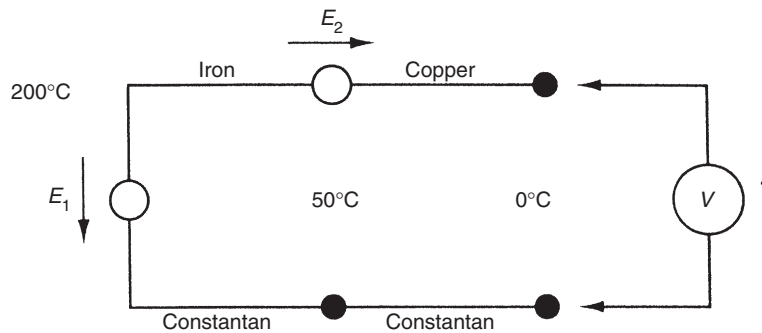


Figure 14.7

Diagram for solution of Example 14.5.

14.2.3 Thermocouple types

Thermocouples can be divided between those made from base metals and those made from noble metals.

Base metal thermocouples

The five standard base metal thermocouples are chromel-constantan (Type E), iron-constantan (Type J), chromel-alumel (Type K), nicrosil-nisil (Type N), and copper-constantan (Type T). There is also a sixth base metal one, nickel molybdenum-nickel cobalt (Type M), but this is used only in special applications. All base metal thermocouples are relatively cheap to manufacture, but they become inaccurate with age and have a short life. In many applications, performance is also affected through contamination by the working environment. To overcome this, the thermocouple can be enclosed in a **protective sheath**, but this has the adverse effect of introducing a significant

time constant, making the thermocouple slow to respond to temperature changes. Therefore, as far as possible, thermocouples are used without protection.

Chromel-constantan thermocouples (Type E) give the highest measurement sensitivity of $68 \mu\text{V}/^\circ\text{C}$, with an inaccuracy of $\pm 0.5\%$ and a useful measuring range of -200°C up to 900°C . Unfortunately, although they can operate satisfactorily in oxidizing environments when unprotected, their performance and life are seriously affected by reducing atmospheres.

Iron-constantan thermocouples (Type J) have a sensitivity of $55 \mu\text{V}/^\circ\text{C}$ and are the preferred type for general purpose measurements in the temperature range of -40°C to $+750^\circ\text{C}$, where the typical measurement inaccuracy is $\pm 0.75\%$. Their performance is little affected by either oxidizing or reducing atmospheres.

Copper-constantan thermocouples (Type T) have a measurement sensitivity of $43 \mu\text{V}/^\circ\text{C}$ and find their main application in measuring subzero temperatures down to -200°C , with an inaccuracy of $\pm 0.75\%$. They can also be used in both oxidizing and reducing atmospheres to measure temperatures up to 350°C .

Chromel-alumel thermocouples (Type K) are a widely used, general purpose device with a measurement sensitivity of $41 \mu\text{V}/^\circ\text{C}$. Their output characteristic is particularly linear over the temperature range of $700-1200^\circ\text{C}$, and this is therefore their main application, although their full measurement range is -200°C to $+1300^\circ\text{C}$. Like chromel-constantan devices, they are suitable for oxidizing atmospheres but not for reducing ones unless protected by a sheath. Their measurement inaccuracy is $\pm 0.75\%$.

Nicrosil-nisil thermocouples (Type N) were developed with the specific intention of improving on the lifetime and stability of chromel-alumel thermocouples. They therefore have thermoelectric characteristics similar to those of the latter, but their long-term stability and life are at least three times better. This allows them to be used in temperatures up to 1300°C . Their measurement sensitivity is $39 \mu\text{V}/^\circ\text{C}$ and they have a typical measurement uncertainty of $\pm 0.75\%$.

Nickel molybdenum-nickel cobalt thermocouples (Type M) have one wire made from a nickel-molybdenum alloy with 18% molybdenum and the other wire made from a nickel-cobalt alloy with 0.8% cobalt. They can measure at temperatures up to 1400°C , which is higher than other types of base metal thermocouples. Unfortunately, they are damaged in both oxidizing and reducing atmospheres. This means that they are rarely used except for special applications such as temperature measurement in vacuum furnaces.

Noble metal thermocouples

Thermocouples made from noble metals are expensive, but they enjoy high stability and long life even when used at high temperatures, although they cannot be used in reducing atmospheres. Unfortunately, their measurement sensitivity is relatively low. Because of

this, their use is mainly restricted to measuring high temperatures unless the operating environment is particularly aggressive in low-temperature applications. Various combinations of noble metals and noble metal alloys are used.

Platinum thermocouples (Type B) have one wire made from a platinum-rhodium alloy with 30% rhodium and the other wire made from a platinum-rhodium alloy with 6% rhodium. Their quoted measuring range is $+50^{\circ}\text{C}$ to $+1800^{\circ}\text{C}$, with a measurement sensitivity of $10\ \mu\text{V}/^{\circ}\text{C}$.

Platinum thermocouples (Type R) have one wire made from pure platinum and the other wire made from a platinum-rhodium alloy with 13% rhodium. Their quoted measuring range is 0°C to $+1700^{\circ}\text{C}$, with a measurement sensitivity of $10\ \mu\text{V}/^{\circ}\text{C}$ and quoted inaccuracy of $\pm 0.5\%$.

Platinum thermocouples (Type S) have one wire made from pure platinum and the other wire made from a platinum-rhodium alloy with 10% rhodium. They have characteristics similar to those of Type R devices, with a quoted measuring range of 0°C to $+1750^{\circ}\text{C}$, measurement sensitivity of $10\ \mu\text{V}/^{\circ}\text{C}$, and inaccuracy of $\pm 0.5\%$.

Tungsten thermocouples (Types C, D, and G) have one wire made from pure tungsten and the other wire made from a tungsten-rhenium alloy. Their measurement sensitivity of around $20\ \mu\text{V}/^{\circ}\text{C}$ is double that of platinum thermocouples and they can also operate at temperatures up to 2300°C . Unfortunately, they are damaged in both oxidizing and reducing atmospheres. Therefore, their main application is temperature measurement in vacuum furnaces. The main difference among C, D, and G types is the proportion of rhenium in the tungsten-rhenium alloy. The reader should consult manufacturers' catalogs if fuller details are required of the differences among C, D, and G types.

Chromel-gold/iron thermocouples have one wire made from chromel and the other wire made from a gold-iron alloy, which is in fact almost pure gold but with a very small iron content (typically 0.15%). These are rare, special-purpose thermocouples with a typical measurement sensitivity of $15\ \mu\text{V}/^{\circ}\text{K}$ designed specifically for cryogenic (very-low temperature) applications. The lowest temperature measurable is 1.2°K . Several versions are available which differ according to the iron content and consequent differences in the measurement range and sensitivity. Because of this variation in iron content, and also because of their rarity, these do not have an international type letter.

Platinel II (Type P) thermocouples have one wire made from a palladium-platinum-gold alloy and the other wire made from a gold-palladium alloy. They have a sensitivity and measurement range similar to those of base metal Type K thermocouples but are more resistant to corrosion when used in oxidizing atmospheres.

Iridium-rhodium alloy thermocouples have the two wires made from different alloys of iridium and rhodium. They can measure up to 2000°C in inert atmospheres.

Gold-platinum thermocouples are more accurate than pure platinum thermocouples (Types B, R, and S) but can be used only up to 1000°C.

Platinum-palladium thermocouples are also more accurate than pure platinum thermocouples and can be used up to 1500°C.

14.2.4 Thermocouple protection

Thermocouples are delicate devices that must be treated carefully if their specified operating characteristics are to be maintained. One major source of error is induced strain in the hot junction. This reduces the emf output, and precautions are normally taken to minimize induced strain by mounting the thermocouple horizontally rather than vertically. It is usual to cover most of the thermocouple wire with thermal insulation, which also provides mechanical protection, although the tip is left exposed if possible to maximize the speed of response to changes in the measured temperature. However, thermocouples are prone to contamination in some operating environments. This means that their emf–temperature characteristic varies from that published in standard tables. Contamination also makes them brittle and shortens their life.

Where they are prone to contamination, thermocouples have to be protected by enclosing them entirely in an insulated sheath, often called a probe. Fig. 14.8 shows a photograph of a typical probe encasing a thermocouple. Some common sheath materials and their maximum operating temperatures are shown in Table 14.1. Although the thermocouple is a device that has a naturally first-order type of step response characteristic, the time constant is usually so small as to be negligible when the thermocouple is used unprotected.



Figure 14.8
Typical probe encasing a thermocouple.

Table 14.1: Common sheath materials for thermocouples.

Material	Maximum operating temperature (°C) ^a
Mild steel	900
Nickel-chromium	900
Fused silica	1000
Special steel	1100
Mullite	1700
Recrystallized alumina	1850
Beryllia	2300
Magnesia	2400
Zirconia	2400
Thoria	2600

^aThe maximum operating temperatures quoted assume oxidizing or neutral atmospheres. For operation in reducing atmospheres, the maximum allowable temperature is usually reduced.

However, when enclosed in a sheath, the time constant of the combination of thermocouple and sheath is significant. The size of the thermocouple and hence the diameter required for the sheath has a large effect on the importance of this. The time constant of a thermocouple in a 1-mm-diameter sheath is only 0.15 s, which has little practical effect in most measurement situations, whereas a larger sheath of 6 mm diameter gives a time constant of 3.9 s that cannot be ignored so easily.

14.2.5 Thermocouple manufacture

Thermocouples are manufactured by connecting two wires of different materials, in which each material is produced so as to conform precisely with some defined composition specification. This ensures that its thermoelectric behavior accurately follows that for which standard thermocouple tables apply. The connection between the two wires is effected by welding, soldering, or, in some cases, just by twisting the wire ends together. Welding is the most common technique used generally; silver soldering is reserved for copper-constantan devices.

The diameter of wire used to construct thermocouples is usually in the range between 0.4 and 2 mm. Larger diameters are used when ruggedness and long life are required, although these advantages are gained at the expense of increasing the measurement time constant. In the case of noble metal thermocouples, the use of large-diameter wire incurs a substantial cost penalty. Some special applications have a requirement for a very fast response time in the measurement of temperature, and in such cases wire diameters as small as 0.1 μm can be used.

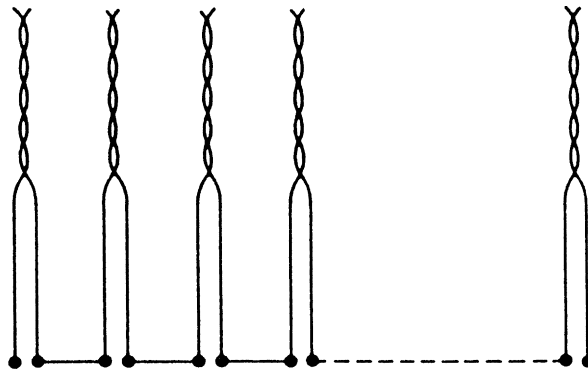


Figure 14.9
Thermopile.

14.2.6 The thermopile

The thermopile is the name given to a temperature-measuring device that consists of several thermocouples connected in series, such that all reference junctions are at the same cold temperature and all hot junctions are exposed to the temperature being measured, as shown in Fig. 14.9. The effect of connecting n thermocouples together in series is to increase the measurement sensitivity by a factor of n . A typical thermopile manufactured by connecting together 25 chromel-constantan thermocouples gives a measurement resolution of 0.001°C .

14.2.7 Digital thermometer

Thermocouples are also used in digital thermometers, of which both simple and intelligent versions exist (see Section 14.11 for a description of the latter). A simple digital thermometer is the combination of a thermocouple, a battery-powered, dual-slope digital voltmeter to measure the thermocouple output, and an electronic display. This provides a low-noise digital output that can resolve temperature differences as small as 0.1°C . The accuracy achieved depends on the accuracy of the thermocouple element, but reduction of measurement inaccuracy to $\pm 0.5\%$ is achievable.

14.2.8 The continuous thermocouple

The continuous thermocouple is one of a class of devices that detect and respond to heat. Other devices in this class include the *line-type heat detector* and *heat-sensitive cable*. The basic construction of all of these devices consists of two or more strands of wire separated by insulation within a long thin cable. Although they sense temperature, they do not provide an output measurement of temperature. Their function is to respond to abnormal temperature rises and thus prevent fires, equipment damage, and so forth.

The advantages of continuous thermocouples become more apparent if problems with other types of heat detectors are considered. The insulation in the line-type heat detector and heat-sensitive cable consists of plastic or ceramic material with a negative temperature coefficient (i.e., the resistance falls as the temperature rises). An alarm signal can be generated when the measured resistance falls below a certain level. Alternatively, in some versions, the insulation is allowed to break down completely, in which case the device acts as a switch. The major limitation of these devices is that the temperature change has to be relatively large, typically 50–200°C above ambient temperature, before the device responds. Also, it is generally impossible for such devices to give an output that indicates that an alarm condition is developing before it actually happens, and thus allow preventative action. Furthermore, after the device has generated an alarm, it usually has to be replaced. This is particularly irksome because there is a large variation in the characteristics of detectors coming from different batches, and so replacement of the device requires extensive on-site recalibration of the system.

In contrast, the continuous thermocouple suffers from very few of these problems. It differs from other types of heat detectors in that the two strands of wire inside it are a pair of thermocouple materials³ separated by special patented mineral insulation and contained within a stainless-steel protective sheath. If any part of the cable is subjected to heat, resistance of the insulation at that point is reduced and a hot junction is created between the two wires of dissimilar metals. An emf is generated at this hot junction according to normal thermoelectric principles.

The continuous thermocouple can detect temperature rises as small as 1°C above normal. Unlike other types of heat detector, it can also monitor abnormal rates of temperature rise and provide a warning of alarm conditions developing before they actually happen. Replacement is necessary only if a great degree of insulation breakdown has been caused by a substantial hot spot at some point along the detector's length. Even then, the use of thermocouple materials of standard characteristics in the detector means that recalibration is not needed if it is replaced. Calibration is also unaffected by cable length, and so a replacement cable may be of a different length from the one it is replacing. One further advantage of continuous thermocouples over earlier forms of heat detector is that no power supply is needed, which significantly reduces installation costs.

14.3 Varying-resistance devices

Varying-resistance devices rely on the physical principle of the variation of resistance with temperature. The devices are known as either resistance temperature devices (RTDs) or thermistors, according to whether the material used for their construction is a metal or a

³ Normally Type E, chromel-constantan, or Type K, chromel-alumel.

semiconductor, and both are common measuring devices. RTDs are also known by the name of *resistance thermometers*. The normal method of measuring resistance is to use a direct current (DC) bridge. The excitation voltage of the bridge has to be chosen carefully because, although a high value is desirable for achieving high measurement sensitivity, the self-heating effect of high currents flowing in the temperature transducer creates an error by increasing the temperature of the device and so changing the resistance value.

14.3.1 Resistance temperature device (resistance thermometer)

The resistance temperature device (commonly abbreviated to **RTD**), which is alternatively known as a *resistance thermometer*, is more accurate than a thermocouple and is usually preferred over thermocouples for measurements below 600°C. The RTD relies on the principle that the resistance of a metal varies with temperature according to the relationship:

$$R = R_0(1 + a_1T + a_2T^2 + a_3T^3 + \cdots + a_nT^n) \quad (14.7)$$

This equation is nonlinear and so is inconvenient for measurement purposes. The equation becomes linear if all terms in a_2T^2 and higher powers of T are negligible such that the resistance and temperature are related according to:

$$R \approx R_0(1 + a_1T)$$

This equation is approximately true over a limited temperature range for some metals, notably platinum, copper, and nickel, whose characteristics are summarized in [Fig. 14.10](#). Platinum has the most linear resistance-temperature characteristic and it also has good chemical inertness. It is therefore far more common than copper or nickel RTDs. Its resistance-temperature relationship is linear within $\pm 0.4\%$ over the temperature range between -200°C and $+40^\circ\text{C}$. Even at $+1000^\circ\text{C}$, the quoted inaccuracy is only $\pm 1.2\%$. Platinum thermometers are made in three forms: as a film deposited on a ceramic substrate, as a coil mounted inside a glass or ceramic probe, or as a coil wound on a mandrel, although the last of these is becoming rare. Nominal resistance at 0°C is typically $100\ \Omega$ or $1000\ \Omega$, although 200 and $500\ \Omega$ versions exist. Sensitivity is $0.385\ \Omega/^\circ\text{C}$ ($100\ \Omega$ type) or $3.85\ \Omega/^\circ\text{C}$ ($1000\ \Omega$ type). A high nominal resistance is advantageous in terms of higher measurement sensitivity, and the resistance of connecting leads has less effect on measurement accuracy. However, cost goes up as the nominal resistance increases. A photograph of a typical thin-film RTD is shown in [Fig. 14.11](#).

Besides having a less linear characteristic, both nickel and copper are inferior to platinum in terms of their greater susceptibility to oxidation and corrosion. This seriously limits their accuracy and longevity. However, because platinum is expensive compared with nickel and copper, the latter are used in RTDs when cost is important. The metal tungsten is used in RTDs under some circumstances, particularly for high-temperature

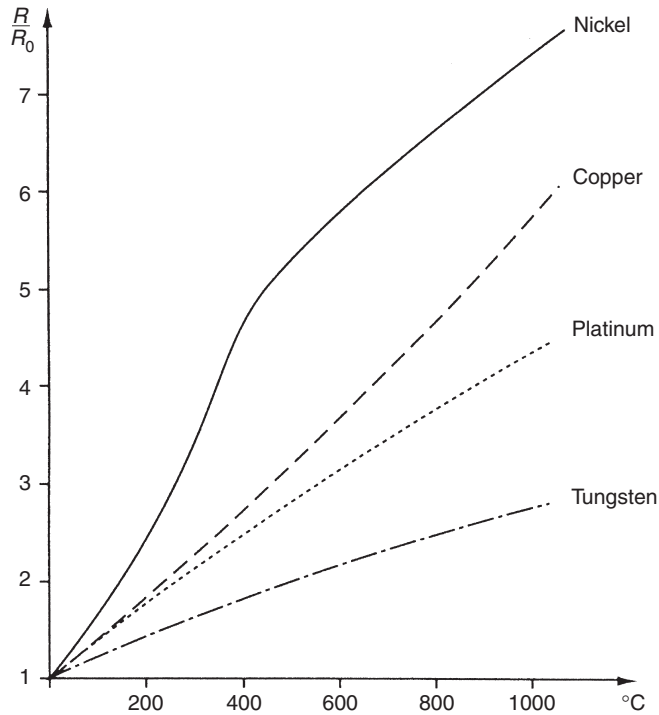


Figure 14.10

Typical resistance-temperature characteristics of metals.

measurements. Carbon resistor elements are also used at low temperatures because of their relatively low cost. The working ranges for each of these five types of RTD are:

Platinum: -270°C to $+1000^{\circ}\text{C}$ (although use above 650°C is uncommon)

Copper: -200°C to $+260^{\circ}\text{C}$

Nickel: -200°C to $+430^{\circ}\text{C}$



Figure 14.11

Photograph of a thin-film resistance temperature device.

Tungsten: -270°C to $+1100^{\circ}\text{C}$

Carbon: -273°C to -173°C

In the case of noncorrosive and nonconducting environments, RTDs are used without protection. In all other applications, they are protected inside a sheath. As in the case of thermocouples, such protection reduces the speed at which the system responds to rapid changes in temperature. A typical time constant for a sheathed platinum RTD is 0.4 seconds. Moisture buildup within the sheath can also impair measurement accuracy.

The frequency at which an RTD should be calibrated depends on the material from which it is made and on the operating environment. Practical experimentation is therefore needed to determine the necessary frequency; this must be reviewed if the operating conditions change.

14.3.2 Thermistors

Thermistors are manufactured from beads of semiconductor material. They can have a negative or positive temperature coefficient. In the negative temperature coefficient version, the resistance decreases as the temperature increases, according to:

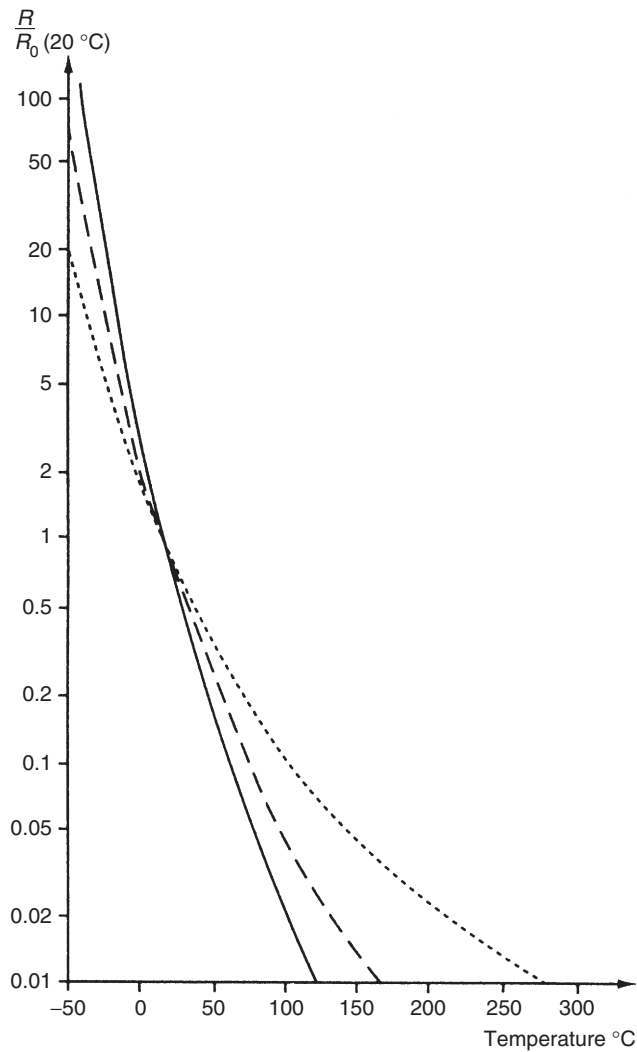
$$R = R_0 e^{\beta(1/T - 1/T_0)} \quad (14.8)$$

This relationship is illustrated in Fig. 14.12 and it is clearly nonlinear. Positive temperature coefficient versions also have a similar nonlinear characteristic, except that the resistance increases rather than decreases with temperature.

The form of the resistance—temperature curve is such that it is impossible to make a linear approximation to the curve over even a small temperature range, and hence the thermistor is definitely a nonlinear sensor. However, major advantages of thermistors are their relatively low cost and small size. This size advantage means that the time constant of thermistors operated in sheaths is small, although the size reduction also decreases its heat dissipation capability and so makes the self-heating effect greater. In consequence, thermistors have to be operated at generally lower current levels than RTDs and so the measurement sensitivity is less.

Negative coefficient thermistors are prepared from oxides of the iron group of metals such as chromium, cobalt, iron, manganese, and nickel. Positive coefficient thermistors are prepared from barium, strontium, or lead titanates.

As in the case of RTDs, some practical experimentation is needed to determine the necessary frequency at which a thermistor should be calibrated, and this must be reviewed if the operating conditions change.

**Figure 14.12**

Typical resistance/temperature characteristics of thermistor materials.

14.4 Semiconductor devices

Semiconductor devices to measure temperature consist of diodes or integrated circuit transistors. They have the advantage of being relatively inexpensive, but one difficulty that affects their use is the need to provide an external power supply to the sensor.

Integrated circuit transistors produce an output proportional to the absolute temperature. Different types are configured to give an output in the form of either a varying current (typically $1\ \mu\text{A/K}$) or varying voltage (typically $10\ \text{mV/K}$). Current forms are normally

used with a digital voltmeter that detects the current output in terms of the voltage drop across a 10-K Ω resistor. Although the devices have a low cost (typically a few dollars) and better linearity than either thermocouples or RTDs, they only have a limited measurement range from -50°C to $+150^{\circ}\text{C}$. Their inaccuracy is typically $\pm 3\%$, which limits their range of application. However, they are widely used to monitor pipes and cables, and their low cost means that it is feasible to mount multiple sensors along the length of the pipe or cable to detect hot spots.

In diodes, the forward voltage across the device varies with temperature. Output from a typical diode package is in the microamp range. Diodes have a small size, with good output linearity and a typical inaccuracy of only $\pm 0.5\%$. Silicon diodes cover the temperature range of -50 to $+200^{\circ}\text{C}$, and germanium ones of -270 to $+40^{\circ}\text{C}$.

14.5 Radiation thermometers

All objects emit electromagnetic radiation as a function of their temperature above absolute zero, and radiation thermometers (also known as radiation pyrometers) measure this radiation to calculate the temperature of the object. The total rate of radiation emission per second is given by:

$$E = KT^4 \quad (14.9)$$

The power spectral density of this emission varies with temperature in the manner shown in Fig. 14.13. The major part of the frequency spectrum lies within the band of wavelengths between 0.3 and 40 μm , which corresponds to the visible (0.3–0.72 μm) and infrared (0.72–1000 μm) ranges. Because the magnitude of the radiation varies with temperature, measurement of the emission from a body allows the temperature of the body to be calculated. The choice of the best method of measuring the emitted radiation depends on the temperature of the body. At low temperatures, the peak of the power spectral density function (Fig. 14.13) lies in the infrared region, whereas at higher temperatures it moves toward the visible part of the spectrum. This phenomenon is observed as the red glow that a body begins to emit as its temperature is increased beyond 600°C .

Different versions of radiation thermometers are capable of measuring temperatures between -100°C and $+10,000^{\circ}\text{C}$ with measurement inaccuracy as low as $\pm 0.05\%$ in the more expensive versions (although this level of accuracy is not obtained when measuring very high temperatures). Portable, battery-powered, handheld versions are also available and are particularly easy to use. The important advantage that radiation thermometers have over other types of temperature-measuring instrument is that there is no contact with the hot body while its temperature is being measured. Thus, the measured system is not disturbed in any way. Furthermore, there is no possibility of contamination, which is particularly important in the food, drug, and many other process industries. They are especially suitable

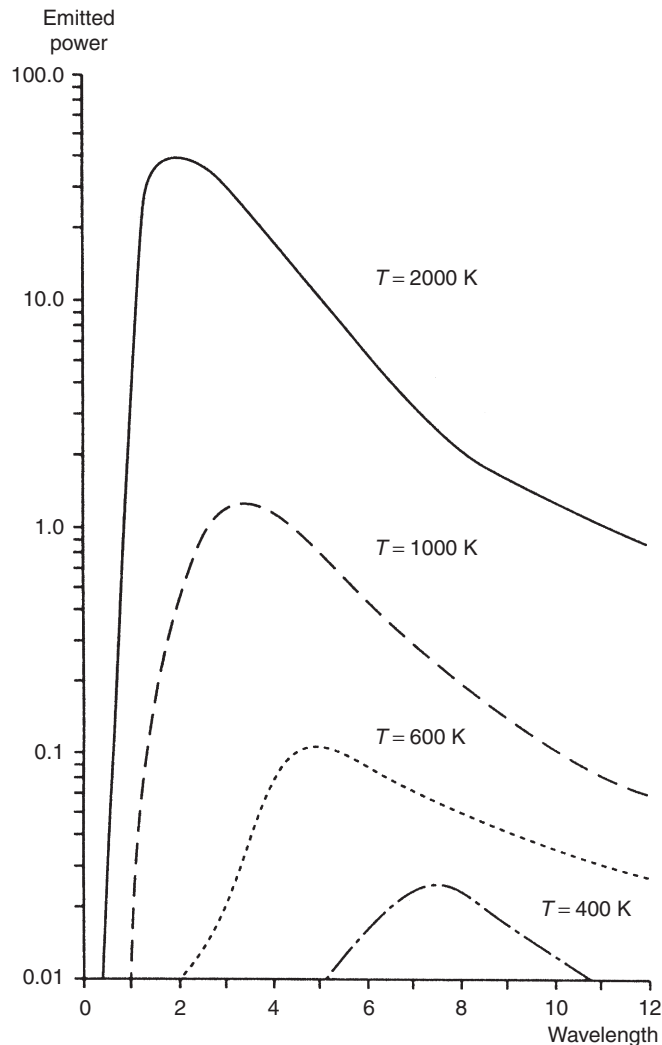


Figure 14.13

Power spectral density of radiated energy emission at various temperatures.

for measuring high temperatures that are beyond the capabilities of contact instruments such as thermocouples, RTDs, and thermistors. They are also capable of measuring moving bodies, such as the temperature of steel bars in a rolling mill. However, their use is not as straightforward as the discussion so far might have suggested, because the radiation from a body varies with the composition and surface condition of the body as well as with temperature. This dependence on surface condition is quantified by the *emissivity* of the body. The use of radiation thermometers is further complicated by absorption and scattering of the energy between the emitting body and the radiation detector. Energy is scattered by

atmospheric dust and water droplets and absorbed by carbon dioxide, ozone, and water vapor molecules. Therefore, all radiation thermometers have to be carefully calibrated for each particular body whose temperature they are required to monitor.

Various types of radiation thermometer exist, as described below. The optical pyrometer can be used only to measure high temperatures, but various types of radiation pyrometers are available that among them cover the whole temperature spectrum. Intelligent versions (see Section 14.12) also provide full or partial solutions to many of the problems described below for nonintelligent pyrometers.

14.5.1 Optical pyrometer

The optical pyrometer, illustrated in Fig. 14.14, is obsolete regarding the availability of new instruments, but some older instruments remain in use in industrial applications such as measuring the temperature of furnaces. It is designed to measure temperatures at which the peak radiation emission is in the red part of the visible spectrum (i.e., where the measured body glows a certain shade of red according to the temperature). This limits the instrument to measuring temperatures above 600°C . The instrument contains a heated tungsten filament within its optical system. The current in the filament is increased until its color is the same as the hot body: under these conditions, the filament apparently disappears when viewed against the background of the hot body. Temperature measurement is therefore obtained in terms of the current flowing in the filament. Because the brightness of different materials at any particular temperature varies according to the emissivity of the material, the calibration of the optical pyrometer must be adjusted according to the emissivity of the target. Manufacturers provide tables of standard material emissivities to assist with this.

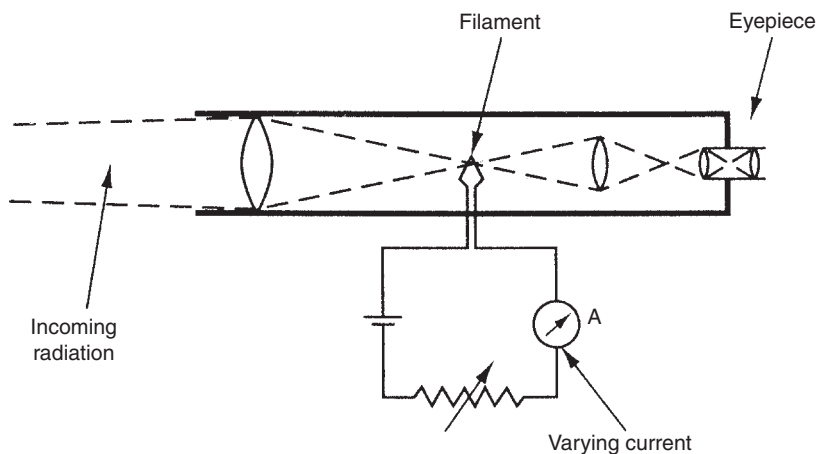


Figure 14.14
Optical pyrometer.

The inherent measurement inaccuracy of an optical pyrometer is $\pm 5^{\circ}\text{C}$. However, in addition to this error, there can be a further operator-induced error of $\pm 10^{\circ}\text{C}$ arising from the difficulty of judging the moment when the filament just disappears. Measurement accuracy can be improved by employing an optical filter within the instrument that passes a narrow band of frequencies with a wavelength of around $0.65\ \mu\text{m}$ corresponding to the red part of the visible spectrum. This also extends the upper temperature measurable from 5000°C in unfiltered instruments up to $10,000^{\circ}\text{C}$.

The instrument cannot be used in automatic temperature control schemes because the eye of the human operator is an essential part of the measurement system. The reading is also affected by fumes in the sight path. Because of these difficulties and its low accuracy, handheld radiation pyrometers have replaced optical pyrometers in most applications.

14.5.2 Radiation pyrometers

All alternative forms of radiation pyrometer described below have an optical system similar to that in the optical pyrometer and focus energy emitted from the measured body. However, they differ by omitting the filament and eyepiece and instead having an energy detector in the same focal plane as the eyepiece, as shown in Fig. 14.15. This principle can be used to measure temperature from -100°C to $+3600^{\circ}\text{C}$. The radiation detector is either a thermal detector, which measures the temperature rise in a black body at the focal point of the optical system, or a photon detector.

Thermal detectors respond equally to all wavelengths in the frequency spectrum and consist of thermopiles, RTDs, or thermistors. All of these typically have time constants of several milliseconds because of the time taken for the black body to heat up and the temperature sensor to respond to the temperature change.

Photon detectors respond selectively to a particular band within the full spectrum and are usually of the photoconductive or photovoltaic type. They respond to temperature changes much faster than thermal detectors because they involve atomic processes, and typical measurement time constants are a few microseconds.

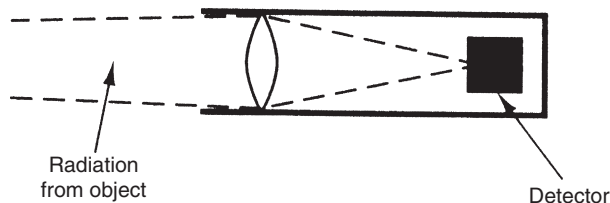


Figure 14.15
Structure of the radiation thermometer.

Fiber-optic technology is frequently used in high-temperature measurement applications to collect incoming radiation and transmit it to a detector and processing electronics that are located remotely. This prevents exposure of the processing electronics to potentially damaging high temperature. Fiber-optic cables are also used to apply radiation pyrometer principles in difficult applications such as measuring the temperature inside jet engines by collecting the radiation from inside the engine and transmitting it outside (see [Section 14.9](#)). The term *fiber-optic pyrometer* is frequently used to describe devices that use fiber optics.

The size of objects measured by a radiation pyrometer is limited by the optical resolution, which is defined as the ratio of target size to distance. 1:300 is regarded as a good ratio, and this would allow temperature measurement of a 1-mm sized object at a range of 300 mm. With large distance/target size ratios, accurate aiming and focusing of the pyrometer at the target is essential. It is common to find through-the-lens viewing provided in pyrometers, using a principle similar to SLR camera technology, because focusing and orientating the instrument for visible light automatically means that it is focused for infrared light. Alternatively, dual laser beams are sometimes used to ensure that the instrument is aimed correctly at the target.

Various forms of electrical output are available from the radiation detector: these are functions of the incident energy on the detector and are therefore functions of the temperature of the measured body. The forms of radiation pyrometer described below differ mainly in the technique used to measure the emitted radiation. They also differ in the range of energy wavelengths, and hence the temperature range, which each is designed to measure. One further difference is the material used to construct the energy-focusing lens. Outside the visible part of the spectrum, glass becomes almost opaque to infrared wavelengths, and other lens materials such as arsenic trisulfide are used.

Broadband (unchopped) radiation pyrometers: The broadband radiation pyrometer finds wide application in industry and has a measurement inaccuracy that varies from $\pm 0.05\%$ of full scale in the best instruments to $\pm 0.5\%$ in the cheapest. However, their accuracy deteriorates significantly over time, and an error of 10°C is common after 1–2 years of operation at high temperatures. As its name implies, the instrument measures radiation across the whole frequency spectrum and so uses a thermal detector. This consists of a blackened platinum disk to which a thermopile⁴ is bonded. The temperature of the detector increases until the heat gain from the incident radiation is balanced by the heat loss due to convection and radiation. For high-temperature measurement, a two-couple thermopile gives acceptable measurement sensitivity and has a fast time constant of about 0.1 s. At lower measured temperatures, at which the level of incident radiation is much less, thermopiles constructed from a greater number of thermocouples must be used to get

⁴ Typically manganin-constantan.

sufficient measurement sensitivity. This increases the measurement time constant to as much as 2 s. Standard instruments of this type are available to measure temperatures between -20°C and $+1800^{\circ}\text{C}$, although in theory much higher temperatures could be measured by this method.

Chopped broadband radiation pyrometers: The construction of this form of pyrometer is broadly similar to that shown in Fig. 14.15, except that a rotary mechanical device is included that periodically interrupts radiation reaching the detector. The voltage output from the thermal detector thus becomes an alternating quantity that switches between two levels. This form of alternating current (AC) output can be amplified much more readily than the DC output coming from an unchopped instrument. This is particularly important when amplification is necessary to achieve an acceptable measurement resolution in situations in which the level of incident radiation from the measured body is low. For this reason, this form of instrument is the more common when measuring body temperatures associated with peak emission in the infrared part of the frequency spectrum. For such chopped systems, the time constant of thermopiles is too long. Instead, thermistors are generally used, giving a time constant of 0.01 s. Standard instruments of this type are available to measure temperatures between $+20^{\circ}\text{C}$ and $+1300^{\circ}\text{C}$. This form of pyrometer suffers accuracy drift similar to that of unchopped forms. Its life is also limited to about 2 years because of motor failures.

Narrow-band radiation pyrometers: Narrow-band radiation pyrometers are highly stable instruments that suffer a drift in accuracy that is typically only 1°C in 10 years. They are also less sensitive to emissivity changes than other forms of radiation pyrometer. They use photodetectors of either the photoconductive or photovoltaic form whose performance is unaffected by carbon dioxide or water vapor in the path between the target object and the instrument. A photoconductive detector exhibits a change in resistance as the incident radiation level changes, whereas a photovoltaic cell exhibits an induced voltage across its terminals that is also a function of the incident radiation level. All photodetectors are preferentially sensitive to a particular narrow band of wavelengths in the range of $0.5\text{--}1.2\ \mu\text{m}$ and all have a form of output that varies in a highly nonlinear fashion with temperature, and thus a microcomputer inside the instrument is highly desirable. Four commonly used materials for photodetectors are cadmium sulfide, lead sulfide, indium antimonide, and lead tin telluride. Each of these is sensitive to a different band of wavelengths, and therefore all find application in measuring the particular temperature ranges corresponding to each of these bands.

The output from the narrow-band radiation pyrometer is normally chopped into an AC signal in the same manner as used in the chopped broadband pyrometer. This simplifies the amplification of the output signal, which is necessary to achieve an acceptable measurement resolution. The typical time constant of a photon detector is only $5\ \mu\text{s}$, which

allows high chopping frequencies up to 20 kHz. This gives such instruments an additional advantage in being able to measure fast transients in temperature as short as 10 μ s.

Two-color pyrometer (ratio pyrometer): As stated earlier, the emitted radiation—temperature relationship for a body depends on its emissivity. This is difficult to calculate, and therefore in practice, all pyrometers have to be calibrated to the particular body they are measuring. The two-color pyrometer (alternatively known as a ratio pyrometer) is a system that largely overcomes this problem by using the arrangement shown in Fig. 14.16. Radiation from the body is split equally into two parts, which are applied to separate narrow-band filters. The outputs from the filters consist of radiation within two narrow bands of wavelength, λ_1 and λ_2 . Detectors sensitive to these frequencies produce output voltages V_1 and V_2 , respectively. The ratio of these outputs (V_1/V_2) can be shown to be a function of temperature and to be independent of the emissivity provided that the two wavelengths, λ_1 and λ_2 , are close together.

The theoretical basis of the two-color pyrometer is that the output is independent of emissivity because the emissivities at the two wavelengths, λ_1 and λ_2 , are equal. This is based on the assumption that λ_1 and λ_2 are very close together. In practice, this assumption does not hold, and therefore the accuracy of the two-color pyrometer tends to be relatively poor. However, the instrument is still greatly useful in conditions where the target is obscured by fumes or dust, which is a common problem in the cement and mineral processing industries. Two-color pyrometers typically cost 50–100% more than other types of pyrometers.

Multiwavelength pyrometer: This is an extension of the two-color pyrometer that uses three or more separate wavelengths. By appropriate mathematical manipulation of the outputs at the detector for each frequency, an accurate temperature measurement is achieved even when the emissivity is unknown, changing, or different at each wavelength.

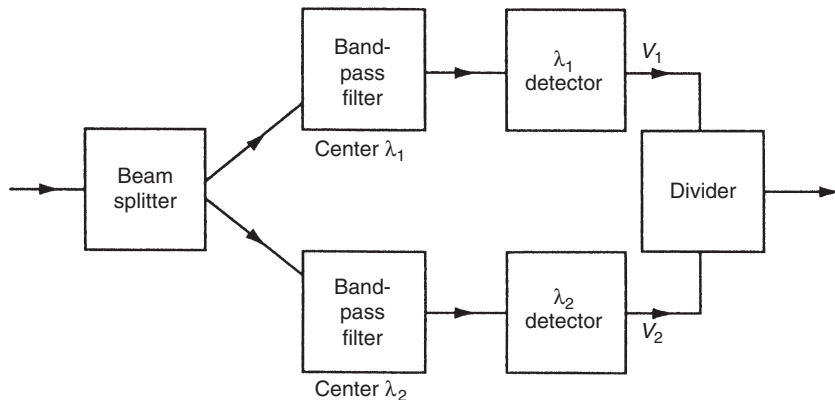


Figure 14.16
Two-color (ratio) pyrometer system.

Selected waveband pyrometer: The selected waveband pyrometer is sensitive to one waveband only (e.g., 5 μm) and is dedicated to particular, special situations in which other forms of pyrometer are inaccurate. One example of such a situation is measuring the temperature of steel billets that are being heated in a furnace. If an ordinary radiation pyrometer is aimed through the furnace door at a hot billet, it receives radiation from the furnace walls (by reflection off the billet) as well as radiation from the billet itself. If the temperature of the furnace walls is measured by a thermocouple, a correction can be made for the reflected radiation, but variations in transmission losses inside the furnace through fumes, and so forth, make this correction inaccurate. However, if a carefully chosen selected-waveband pyrometer is used, this transmission loss can be minimized and the measurement accuracy is thereby greatly improved.

14.6 Thermography (thermal imaging)

Thermography, or thermal imaging, involves scanning an infrared radiation detector across an object. The information gathered is then processed and an output in the form of the temperature distribution across the object is produced. Temperature measurement over the range from -20°C to $+1500^{\circ}\text{C}$ is possible. Elements of the system are shown in Fig. 14.17.

The radiation detector uses the same principles of operation as a radiation pyrometer in inferring the temperature of the point on which the instrument is focused from a measurement of the incoming infrared radiation. However, instead of providing a measurement of the temperature of a single point at the focal point of the instrument, the detector is scanned across a body or scene, and thus provides information about temperature distributions. Because of the scanning mode of operation of the instrument, radiation detectors with a fast response are required and only photoconductive or photovoltaic sensors are suitable. These are sensitive to the portion of the infrared spectrum with wavelengths of 2 to 14 μm .

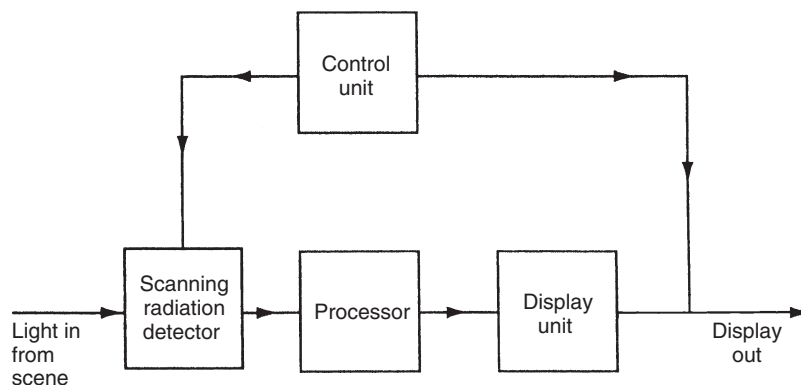


Figure 14.17
Thermography (thermal imaging) system.

Simpler versions of thermal imaging instruments consist of handheld viewers pointed at the object of interest. The output from an array of infrared detectors is directed onto a matrix of red light-emitting diodes assembled behind a glass screen, and the output display thus consists of different intensities of red on a black background, with the different intensities corresponding to different temperatures. Measurement resolution is high, with temperature differences as small as 0.1°C being detectable. Such instruments are used in a wide variety of applications such as monitoring product flows through pipework, detecting insulation faults, and detecting hot spots in furnace linings, electrical transformers, machines, bearings, and so on. The number of applications is extended still further if the instrument is carried in a helicopter or drone, for which uses include scanning electrical transmission lines for faults, searching for lost or injured people, and detecting the source and spread pattern of forest fires.

More complex thermal imaging systems are composed of a tripod-mounted detector connected to a desktop computer and display system. Multicolor displays are commonly used in such systems, where up to 16 different colors represent different bands of temperature across the measured range. The heat distribution across the measured body or scene is thus displayed graphically as a contoured set of colored bands representing the different temperature levels. Such color-thermography systems find many applications such as inspecting electronic circuit boards and monitoring production processes. There are also medical applications in body scanning.

14.7 Thermal expansion methods

Thermal expansion methods make use of the fact that the dimensions of all substances, whether solids, liquids, or gases, change with temperature. Instruments operating on this physical principle include the liquid-in-glass thermometer, the bimetallic thermometer, and the pressure (gas) thermometer.

14.7.1 Liquid-in-glass thermometers

The liquid-in-glass thermometer is a well-known temperature-measuring instrument that is used in a wide range of applications, although there is a growing trend to use a digital thermometer instead. The fluid used in the thermometer is normally either mercury or colored alcohol contained within a bulb and capillary tube, as shown in [Fig. 14.18A](#). As the temperature rises, the fluid expands along the capillary tube and the meniscus level is read against a calibrated scale etched on the tube. Industrial versions of the liquid-in-glass thermometer are normally used to measure temperature in the range between -200°C and $+1000^{\circ}\text{C}$, although instruments are available to special order that can measure temperatures up to 1500°C .

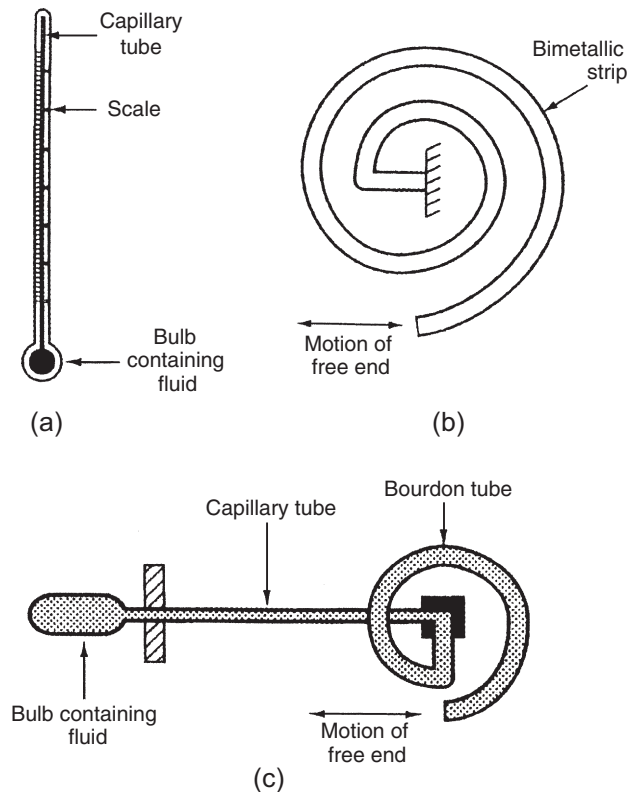


Figure 14.18

Thermal expansion devices: (a) Liquid-in-glass thermometer. (b) Bimetallic thermometer. (c) Pressure thermometer.

Measurement inaccuracy in a mercury-filled thermometer is typically $\pm 1\%$ of a full-scale reading, although inaccuracy of only $\pm 0.15\%$ can be achieved in the best industrial instruments. Alcohol-filled thermometers are less accurate but are preferred in many applications because of the highly toxic nature of mercury if a thermometer is broken. The major source of measurement error arises from the difficulty of correctly estimating the position of the curved meniscus of the fluid against the scale. In the longer term, additional errors are introduced owing to volumetric changes in the glass. Such changes occur because of creep-like processes in the glass, but they occur only over a time scale of years. Annual calibration checks are therefore advisable.

14.7.2 Bimetallic thermometer

The bimetallic principle is probably more commonly known in connection with its use in thermostats. It is based on the fact that if two strips of different metals are bonded, any

temperature change will cause the strip to bend, because this is the only way in which the differing rates of change of length of each metal in the bonded strip can be accommodated. The bimetallic thermostat is used as a switch in control applications. If the magnitude of bending is measured, the bimetallic device becomes a thermometer. For such purposes, the strip is often arranged in a spiral or helical configuration, as shown in Fig. 14.18(b), because this gives a relatively large displacement of the free end for any given temperature change. The measurement sensitivity is increased further by choosing the pair of materials carefully such that the degree of bending is maximized; the combination of invar (a nickel-steel alloy) and brass is commonly used.

The system used to measure the displacement of the strip must be carefully designed. Little resistance must be offered to the end of the strip; otherwise, the spiral or helix will distort and cause a false reading in measurement of the displacement. The device is normally used just as a temperature indicator, in which the end of the strip is made to turn a pointer that moves against a calibrated scale. However, some versions produce an electrical output, using either a linear variable differential transformer (LVDT) or a fiber-optic shutter sensor to transduce the output displacement.

Bimetallic thermometers are used to measure temperatures between -75°C and $+1500^{\circ}\text{C}$. The inaccuracy of the best instruments can be as low as $\pm 0.5\%$, but such devices are quite expensive. Many instrument applications do not require this degree of accuracy in temperature measurements, and in such cases much cheaper bimetallic thermometers with substantially inferior accuracy specifications are used.

All such devices are liable to suffer changes in characteristics owing to contamination of the metal components exposed to the operating environment. Further changes are to be expected that arise from mechanical damage during use, particularly if they are mishandled or dropped. Because the magnitude of these effects varies with their application, the required calibration interval must be determined by practical experimentation.

14.7.3 Pressure thermometers

Pressure thermometers (or *gas thermometers*, as they are sometimes known) have now been superseded by other alternatives in most applications, but they still remain useful in a few applications such as furnace temperature measurement when the level of fumes prevents the use of optical or radiation pyrometers. Examples can still be found of their use as temperature sensors in pneumatic control systems. The sensing element in a pressure thermometer consists of a stainless-steel bulb containing a liquid or gas. If the fluid was not constrained, temperature rises would cause its volume to increase. However, because it is constrained in a bulb and cannot expand, its pressure rises instead. As such, the pressure thermometer does not strictly belong to the thermal expansion class of

instruments but is included because of the relationship between volume and pressure according to Boyle's law: $PV = KT$. The change in pressure of the fluid is measured by a suitable pressure transducer such as the Bourdon tube (see Chapter 15). This transducer is located remotely from the bulb and is connected to it by a capillary tube, as shown in Fig. 14.18(c).

Pressure thermometers can be used to measure temperatures between -250°C and $+2000^{\circ}\text{C}$, and their typical inaccuracy is $\pm 0.5\%$ of a full-scale reading. However, the instrument response has a particularly long time constant.

The need to protect the pressure-measuring instrument from the environment where the temperature is being measured can require the use of capillary tubes up to 5 m long, and the temperature gradient, and hence pressure gradient, along the tube acts as a modifying input that can introduce a significant measurement error. Errors also occur in the short term owing to mechanical damage, and in the longer term owing to small volumetric changes in the glass components. The rate of increase in these errors is mainly related to use, and therefore the required calibration interval must be determined by practical experimentation.

14.8 Fiber-optic temperature sensors

Fiber-optic cables can be used as either intrinsic or extrinsic temperature sensors, as discussed in Chapter 13, although special attention has to be paid to providing a suitable protective coating when high temperatures are measured. Cost varies from \$1000 to \$4000 according to type, and the normal temperature range covered is $250\text{--}3000^{\circ}\text{C}$, although special devices can detect down to 100°C and others can detect up to 3600°C . Their main application is measuring temperatures in hard-to-reach locations, although they are also used when high measurement accuracy is required. Some laboratory versions have an inaccuracy as low as $\pm 0.01\%$, which is better than a Type S thermocouple, although versions used in industry have a more typical inaccuracy of $\pm 1.0\%$.

Although it is often assumed that fiber-optic sensors are intrinsically safe, it has been shown that flammable gas might be ignited by the optical power levels available from some laser diodes. Thus, the power level used with optical fibers must be carefully chosen, and certification of intrinsic safety is necessary if such sensors are to be used in hazardous environments.

One type of intrinsic sensor uses a cable in which the core and cladding have similar refractive indices but different temperature coefficients. Temperature rises cause the refractive indices to become even closer together and losses from the core to increase, thus reducing the quantity of light transmitted. Other types of intrinsic temperature sensor include the cross-talk sensor, phase-modulating sensor, and optical resonator, as described

in Chapter 12. Research into the use of distributed temperature sensing using fiber-optic cable has also been reported. This can be used to measure things such as the temperature distribution along an electricity supply cable. It works by measuring the reflection characteristics of light transmitted down a fiber-optic cable that is bonded to the electrical cable. By analyzing the backscattered radiation, a table of temperature versus distance along the cable can be produced, with a measurement inaccuracy of only $\pm 0.5^\circ\text{C}$.

A common form of extrinsic sensor uses fiber-optic cables to transmit light from a remote targeting lens into a standard radiation pyrometer. This technique can be used with all types of radiation pyrometer, including the two-color version, and a particular advantage is that this method of measurement is intrinsically safe. However, it is impossible to measure very low temperatures, because the small radiation levels that exist at low temperatures are badly attenuated during transmission along the fiber-optic cable. Therefore, the minimum temperature that can be measured is about 50°C , and the light guide for this must not exceed 600 mm in length. At temperatures exceeding 1000°C , lengths of fiber up to 20 m long can be successfully used as a light guide.

Several other types of device that are marketed as extrinsic fiber-optic temperature sensors consist of a conventional temperature sensor (e.g., an RTD) connected to a fiber-optic cable so that the transmission of the signal from the measurement point is free of noise. Such devices must include an electricity supply for the electronic circuit that is needed to convert the sensor output into light variations in the cable. Thus, low-voltage power cables must be routed with the fiber-optic cable, and the device is therefore not intrinsically safe.

14.9 Color indicators

The color of various substances and objects changes as a function of temperature. One use for this is in the optical pyrometer, as discussed earlier. The other main use of color change is in special color indicators that are widely used in industry to determine whether objects placed in furnaces have reached the required temperature. Such color indicators consist of special paints or crayons that are applied to an object before it is placed in a furnace. The color-sensitive component within these is some form of metal salt (usually of chromium, cobalt, or nickel). At a certain temperature, a chemical reaction takes place and a permanent color change occurs in the paint or crayon, although this change does not occur instantaneously but only happens over a period of time.

Hence, the color change mechanism is complicated because the time of exposure as well as the temperature is important. Such crayons or paints usually have a dual rating that specifies the temperature and length of exposure time required for the color change to occur. If the temperature rises above the rated temperature, the color change will occur in less than the rated exposure time. This causes little problem if the rate of temperature rise is slow with

respect to the specified exposure time required for color change to occur. However, if the rate of rise of temperature is high, the object will be significantly above the rated change-temperature of the paint/crayon by the time that the color change happens. Besides wasting energy by leaving the object in the furnace longer than necessary, this can cause difficulty if excess temperature affects the required properties of the heated object.

Paints and crayons are available to indicate temperatures between 50°C and 1250°C. A typical exposure time rating is 30 minutes (i.e., the color change will occur if the paint or crayon is exposed to the rated temperature for this length of time). They have the advantage of low cost, typically a few dollars per application. However, they adhere strongly to the heated object, which can cause difficulty if they have to be cleaned off the object later.

Some liquid crystals also change color at a certain temperature. According to the design of sensors using such liquid crystals, the color change can either occur gradually during a temperature rise of perhaps 50°C or else change abruptly at some specified temperature. The latter kind of sensor is able to resolve temperature changes as small as 0.1°C; according to type, they are used over the temperature range from -20°C to +100°C.

14.10 Pyrometric cones

Pyrometric cones are temperature-indicating devices that change state (i.e., melt) at a particular temperature. They are made from fused oxide and glass material that is formed into a cone shape. The tip of the cone softens and bends over when a particular temperature is reached. They are known by various names such as Orton cone (USA), Staffordshire cone (UK), and Seger cone (Germany) and are commonly used in the ceramics industry. Cones are available that indicate temperatures from 600°C to +2000°C.

14.11 Intelligent temperature-measuring instruments

Intelligent temperature transmitters have been introduced into the catalogs of almost all instrument manufacturers, and they bring about the usual benefits associated with intelligent instruments. Such transmitters are separate boxes designed for use with transducers that have either a DC voltage output in the millivolt range or an output in the form of a resistance change. They are therefore suitable for use in conjunction with thermocouples, thermopiles, RTDs, thermistors, and broadband radiation pyrometers. Transmitters normally have nonvolatile memories in which all constants used in correcting output values for modifying inputs, and so on are stored, thus enabling the instrument to survive power failures without losing such information. Other facilities in intelligent transmitters include adjustable damping, noise rejection, self-adjustment for zero and sensitivity drifts, and expanded measurement range. These features allow an inaccuracy level of $\pm 0.05\%$ of full scale to be specified.

Particular mention must be made of intelligent pyrometers, because some versions of these are able to measure the emissivity of the target body and automatically provide an emissivity-corrected output. This development provides an alternative to the two-color pyrometer when emissivity measurement and calibration for other types of pyrometer are difficult.

Digital thermometers (see [Section 14.2.7](#)) also exist in intelligent versions, in which the inclusion of a microprocessor allows a number of alternative thermocouples and RTDs to be offered as options for the primary sensor.

The cost of intelligent temperature transducers is significantly more than their nonintelligent counterparts, and it is difficult to justify them purely on the grounds of their superior accuracy. However, their expanded measurement range means immediate savings are made in terms of a reduction in the number of different instruments needed to cover a number of measurement ranges. Their capability for self-diagnosis and self-adjustment means that they require attention much less frequently, giving additional savings in maintenance costs. Many transmitters are also largely self-calibrating with respect to their signal processing function, although appropriate calibration routines still have to be applied to each sensor to which the transmitter is connected.

14.12 Microelectromechanical system temperature sensors

The latest development in temperature measurement is the introduction of MEMS-based devices. These are usually noncontact devices based on thermocouples or thermopile sensors (a thermopile is a number of thermocouples connected in series), with polysilicon-gold thermocouples being a common choice. The sensor is powered by a battery that has a long life owing to the very low power consumption of the device.

[Fig. 14.19](#) shows the typical structure of a thermopile-based sensor, in which the hot junctions of a number of thermocouples are placed in the central region of the sensor exposed to the radiated heat from a body, and with the reference junctions situated in the cool outer part of sensor. All of the thermocouples are connected in series to create the thermopile. Such thermal MEMS sensors frequently have integrated signal conditioning within the package. A typical commercially available device contains 32 thermocouples within a 4×4 -mm package, with a measurement range from -40°C to $+125^{\circ}\text{C}$ and a typical measurement inaccuracy of $\pm 1^{\circ}\text{C}$.

14.13 Choice between temperature transducers

The suitability of different temperature-measuring devices in any particular measurement situation depends substantially on whether the medium to be measured is a solid or a fluid.

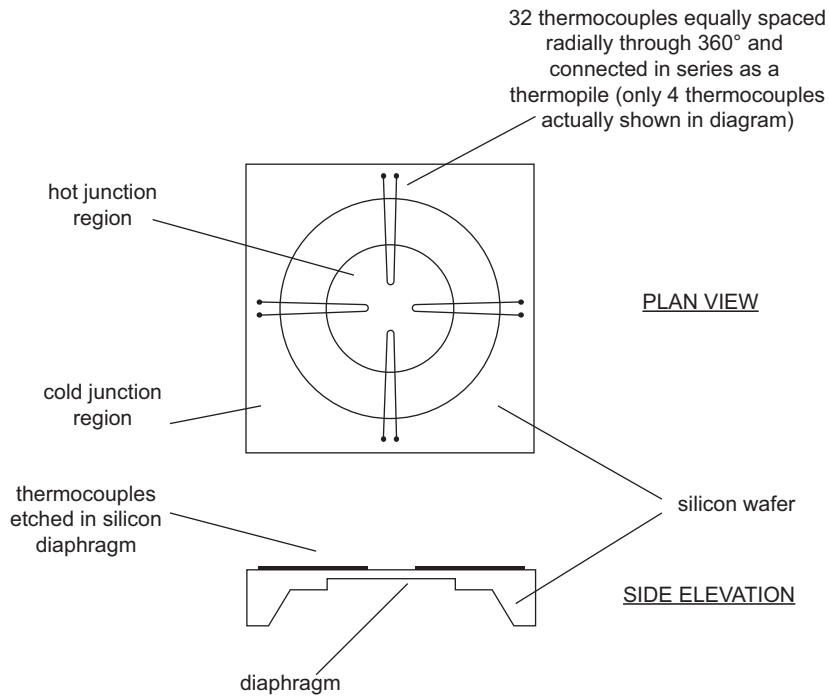


Figure 14.19

Thermopile-based microelectromechanical system temperature sensor.

For measuring the temperature of solids, it is essential for good contact to be made between the body and the transducer unless a radiation thermometer is used. This restricts the range of suitable transducers to thermocouples, thermopiles, RTDs, thermistors, semiconductor devices, color indicators, and pyrometric cones. On the other hand, fluid temperatures can be measured by any of the instruments described in this chapter, with the exception of radiation thermometers.

The most commonly used device in industry to measure temperature is the base metal thermocouple. This is relatively cheap, with prices varying widely from a few dollars upward, according to the thermocouple type and sheath material used. Typical inaccuracy is $\pm 0.5\%$ of full scale over the temperature range of -250°C to $+1200^{\circ}\text{C}$. Noble metal thermocouples are much more expensive but are chemically inert and can measure temperatures up to 2300°C with an inaccuracy of $\pm 0.2\%$ of full scale. However, all types of thermocouple have a low-level output voltage, which makes them prone to noise and therefore unsuitable for measuring small temperature differences.

RTDs are also in common use within the temperature range of -270°C to $+650^{\circ}\text{C}$, with a measurement inaccuracy of $\pm 0.5\%$. Although they have a smaller temperature range than thermocouples, they are more stable and can measure small temperature differences. The

platinum RTD is generally regarded as offering the best ratio of price to performance for measurement in the temperature range of -200°C to $+500^{\circ}\text{C}$, with prices starting from \$15.

Thermistors are another relatively common class of devices. They are small and cheap, with a typical cost of around \$5. They give a fast output response to temperature changes, with good measurement sensitivity, but their measurement range is limited.

Semiconductor devices have better linearity than thermocouples and RTDs and a similar level of accuracy. Thus, they are a viable alternative to these in many applications. Integrated circuit transistor sensors are particularly cheap (from \$10 each), although their accuracy is relatively poor and they have a limited measurement range (-50°C to $+150^{\circ}\text{C}$). Diode sensors are much more accurate and have a wider temperature range (-270°C to $+200^{\circ}\text{C}$), although they are also more expensive (typical costs are anywhere from \$50 to \$500).

A major virtue of radiation thermometers is their noncontact, noninvasive mode of measurement. Costs vary from \$250 up to \$3000 according to type. Although calibration for the emissivity of the measured object often poses difficulties, some instruments provide automatic calibration. Optical pyrometers are used to some extent to monitor temperatures above 600°C in industrial furnaces and such, but their inaccuracy is typically $\pm 5\%$ and they are likely to become obsolete in the near future. Various forms of radiation pyrometer are used over the temperature range between -20°C and $+1800^{\circ}\text{C}$ and they can give measurement inaccuracies as low as $\pm 0.05\%$. One particular merit of narrow-band radiation pyrometers is their ability to measure fast temperature transients of duration as small as $10\ \mu\text{s}$. No other instrument can measure transients anywhere near as fast as this.

The range of instruments working on the thermal expansion principle are mainly used as temperature-indicating devices rather than as components within automatic control schemes. Temperature ranges and costs are: mercury-in-glass thermometers up to $+1000^{\circ}\text{C}$ (cost from a few dollars), bimetallic thermometers up to $+1500^{\circ}\text{C}$ (cost \$50–100) and pressure thermometers up to $+2000^{\circ}\text{C}$ (cost \$100–500). Usual measurement inaccuracy is in the range of $\pm 0.5\%$ to $\pm 1.0\%$. The bimetallic thermometer is more rugged than liquid-in-glass types but less accurate (however, the greater inherent accuracy of liquid-in-glass types can be realized only if the liquid meniscus level is read carefully).

Fiber-optic devices are more expensive than most other forms of temperature sensor (costing up to \$4000) but provide a means of measuring temperature in inaccessible locations. Inaccuracy varies from $\pm 1\%$ to $\pm 0.01\%$ in some laboratory versions. Measurement range also varies with type, but up to $+3600^{\circ}\text{C}$ is possible.

Color indicators are widely used to determine when objects in furnaces have reached the required temperature. These indicators work well if the rate of rise of temperature of the object in the furnace is relatively slow, but because temperature indicators change color only over a period of time, the object will be above the required temperature by the time

the indicator responds if the rate of rise of temperature is large. Cost is low; for example, a crayon typically costs \$3.

The latest devices in temperature measurement are MEMS sensors. These are low-cost devices with a small size and low power consumption. Apart from their use as temperature sensors, they are used for heat detection to turn lights on and off when humans move in and out of an area.

14.14 Calibration of temperature transducers

The fundamental difficulty in establishing an absolute standard for temperature has been mentioned in the introduction to this chapter. The difficulty is that there is no practical way in which a convenient relationship can be established that relates the temperature of a body to another measurable quantity expressible in primary standard units. Instead, it is necessary to use a series of reference calibration points for temperature that are well-defined. These points have been determined by research and international discussion and are published as the *IPTS*. They provide fixed, reproducible reference points for temperature in the form of the freezing points and triple points⁵ of substances in which the transitions among solid, liquid, and gaseous states are sharply defined. The full set of defined points is:

- Triple point of hydrogen: $-259.3467^{\circ}\text{C}$
- Triple point of neon: $-248.5939^{\circ}\text{C}$
- Triple point of oxygen: $-218.7916^{\circ}\text{C}$
- Triple point of argon: $-189.3442^{\circ}\text{C}$
- Triple point of mercury: -38.8344°C
- Triple point of water: $+0.0100^{\circ}\text{C}$
- Melting point of gallium: $+29.7646^{\circ}\text{C}$
- Freezing point of indium: $+156.5985^{\circ}\text{C}$
- Freezing point of tin: $+231.928^{\circ}\text{C}$
- Freezing point of zinc: $+419.527^{\circ}\text{C}$
- Freezing point of aluminum: $+660.323^{\circ}\text{C}$
- Freezing point of silver: $+961.78^{\circ}\text{C}$
- Freezing point of gold: $+1064.18^{\circ}\text{C}$
- Freezing point of copper: $+1084.62^{\circ}\text{C}$

⁵ The triple point of a substance is the temperature and pressure at which the solid, liquid, and gas phases of that substance coexist in thermodynamic equilibrium. For example, in the case of water, the single combination of pressure and temperature at which solid ice, liquid water, and water vapor coexist in a stable equilibrium is a pressure of 611.73 mbars and temperature of 273.16 K (0.01°C).

For calibrating intermediate temperatures, interpolation between the fixed points is carried out by one of the following reference instruments:

- a helium gas thermometer for temperatures below 24.6°K;
- a platinum RTD for temperatures between 13.8°K and 961.8°C; and
- a narrow-band radiation thermometer for temperatures above +961.8°C.

The triple-point method of defining fixed points involves the use of a triple-point cell. The cell consists of a sealed cylindrical glass tube filled with a highly pure version of the reference substance (e.g., mercury). This must be at least 99.9999% pure (such that contamination is less than one part in one million). The cell has a well that allows insertion of the thermometer being calibrated. It also has a valve that allows the cell to be evacuated down to the required triple-point pressure.

The freezing point method of defining fixed points involves the use of an ingot of the reference metal (e.g., tin) that is better than 99.99% pure. This is protected against oxidation inside a graphite crucible with a close-fitting lid. It is heated beyond its melting point and allowed to cool. If its temperature is monitored, an arrest period is observed in its cooling curve at the freezing point of the metal. The melting point method is similar but involves heating the material until it melts (this is used only for materials such as gallium, for which the melting point is more clearly defined than the freezing point). Electric resistance furnaces are available to carry out these procedures. Up to 1100°C, a measurement uncertainty of less than $\pm 0.5^\circ\text{C}$ is achievable.

The accuracy of temperature calibration procedures fundamentally depends on how accurately points on the IPTS can be reproduced. Current limits are:

1K	0.3%	800K	0.001%
10K	0.1%	1500K	0.02%
100K	0.005%	4000K	0.2%
273.15K	0.0001%	10000K	6.7%

14.14.1 Reference instruments and special calibration equipment

The primary reference standard instrument for calibration at the top of the calibration chain is a helium gas thermometer, a platinum RTD, or a narrow-band radiation thermometer according to the temperature range of the instrument being calibrated, as just explained at the end of the last section. However, at lower levels within the calibration chain, almost any instrument from the list of instrument classes given in [Section 14.1](#) might be used for workplace calibration duties under particular circumstances. Where involved in such duties, of course, the instrument used would be one of high accuracy that

was reserved solely for calibration duties. The list of instruments suitable for workplace-level calibration therefore includes mercury-in-glass thermometers, base metal thermocouples (Type K), noble metal thermocouples (Types B, R, and S), platinum RTDs, and radiation pyrometers. However, a subset of this list is commonly preferred for most calibration operations. Up to 950°C, the platinum RTD is often used as a reference standard. Above that temperature up to about 1750°C, a Type S (platinum/rhodium-platinum) thermocouple is usually employed. Type K (chromel-alumel) thermocouples are also used as an alternative reference standard for temperature calibration up to 1000°C.

Although no special types of instrument are needed for temperature calibration, the temperature of the environment within which one instrument is compared with another has to be carefully controlled. This requires purposely designed equipment, which is available commercially from a number of manufacturers.

To calibrate all temperature transducers other than radiation thermometers above a temperature of 20°C, a furnace consisting of an electrically heated ceramic tube is commonly used. The temperature of such a furnace can typically be controlled within limits of $\pm 2^\circ\text{C}$ over the range of 20°C to 1600°C.

Below 20°C, a stirred water bath is used to provide a constant reference temperature, and the same equipment can be used for temperatures up to 100°C. Similar stirred liquid baths containing oil or salts (potassium/sodium nitrate mixtures) can be used to provide reference temperatures up to 600°C.

To calibrate radiation thermometers, a radiation source that approximates as closely as possible the behavior of a black body is required. The actual value of the emissivity of the source must be measured by a surface pyrometer. Some form of optical bench is also required so that instruments being calibrated can be held firmly and aligned accurately.

The simplest form of radiation source is a hot plate heated by an electrical element. The temperature of such devices can be controlled within limits of $\pm 1^\circ\text{C}$ over the range of 0°C to 650°C and the typical emissivity of the plate surface is 0.85. Type R noble metal thermocouples embedded in the plate are normally used as the reference instrument.

A blackbody cavity provides a heat source with much better emissivity. This can be constructed in various alternative forms according to the temperature range of the radiation thermometers to be calibrated, although a common feature is a blackened conical cavity with a cone angle of about 15 degrees.

To calibrate low-temperature radiation pyrometers (measuring temperatures in the range of 20–200°C), the blackbody cavity is maintained at a constant temperature ($\pm 0.5^\circ\text{C}$) by immersing it in a liquid bath. The typical emissivity of a cavity heated in this way is 0.995. Water is suitable for the bath in the temperature range of 20–90°C and a silicone

fluid is suitable for the range of 80–200°C. Within these temperature ranges, a mercury-in-glass thermometer is commonly used as the standard reference calibration instrument, although a platinum RTD is used when better accuracy is required.

Another form of blackbody cavity is one lined with a refractory material and heated by an electrical element. This gives a typical emissivity of 0.998 and is used to calibrate radiation pyrometers at higher temperatures. Within the range of 200–1200°C, temperatures can be controlled within limits of $\pm 0.5^\circ\text{C}$ and a Type R thermocouple is generally used as the reference instrument. At the higher range of 600–1600°C, temperatures can be controlled within limits of $\pm 1^\circ\text{C}$ and a Type B thermocouple (30% rhodium-platinum/6% rhodium-platinum) is normally used as the reference instrument. As an alternative to thermocouples, radiation thermometers can also be used as a standard within $\pm 0.5^\circ\text{C}$ over the temperature range of 400–1250°C.

To provide reference temperatures above 1600°C, a carbon cavity furnace is used. This consists of a graphite tube with a conical radiation cavity at its end. Temperatures up to 2600°C can be maintained with an accuracy of $\pm 5^\circ\text{C}$. Narrow-band radiation thermometers are used as the reference standard instrument.

The equipment mentioned in the paragraphs above merely provides an environment in which radiation thermometers can be calibrated against some other reference standard instrument. To obtain an absolute reference standard of temperature, a fixed-point blackbody furnace is used. This has a radiation cavity consisting of a conical-ended cylinder that contains a crucible of 99.999% pure metal. If the temperature of the metal is monitored as it is heated at a constant rate, an arrest period is observed at the melting point of the metal when the temperature ceases to rise for a short interval. Thus, the melting point, and hence the temperature corresponding to the output reading of the monitoring instrument at that instant, are defined exactly. Measurement uncertainty is on the order of $\pm 0.3^\circ\text{C}$. The list of metals and their melting points was presented earlier at the start of [Section 14.14](#).

In the calibration of radiation thermometers, knowledge of the emissivity of the hot plate or blackbody furnace used as the radiation source is essential. This is measured by special types of surface pyrometer. Such instruments contain a hemispherical, gold-plated surface supported on a telescopic arm that allows it to be put into contact with the hot surface. The radiation emitted from a small hole in the hemisphere is independent of the surface emissivity of the measured body and is equal to that which would be emitted by the body if its emissivity value were 100. This radiation is measured by a thermopile with its cold junction at a controlled temperature. A black hemisphere is also provided with the instrument, which can be inserted to cover the gold surface. This allows the instrument to measure the normal radiation emission from the hot body and so allows the surface emissivity to be calculated by comparing the two radiation measurements.

Within this list of special equipment, mention must also be made of standard tungsten strip lamps, which are used to provide constant known temperatures in the calibration of optical pyrometers. The various versions of these provide a range of standard temperatures between 800°C and 2300°C to an accuracy of $\pm 2^\circ\text{C}$.

14.14.2 Calculating frequency of calibration checks

The manner in which the appropriate frequency for calibration checks is determined for the various temperature-measuring instruments available was discussed in the instrument review presented in [Section 14.1](#). The simplest instruments from a calibration point of view are liquid-in-glass thermometers. The only parameter able to change within these is the volume of the glass used in their construction. This changes only slowly with time, and hence only infrequent (e.g., annual) calibration checks are required.

The required frequency for calibration of all other instruments is either (a) dependent on the type of operating environment and the degree of exposure to it or (b) use-related. In some cases, both of these factors are relevant.

RTDs and thermistors are examples of instruments in which the drift in characteristics depends on the environment in which they are operated and on the degree of protection they have from that environment. Devices such as gas thermometers suffer characteristics drift, which is largely a function of how much they are used (or misused). Any instruments not mentioned so far suffer characteristics drift owing to both environmental and use-related factors. The list of such instruments includes bimetallic thermometers, thermocouples, thermopiles, and radiation thermometers. In the case of thermocouples and thermopiles, it must be remembered that error in the required characteristics is possible even when the instruments are new, and therefore their calibration must be checked before use.

Because factors responsible for characteristics drift vary from application to application, the required frequency of calibration checks can be determined only experimentally. The procedure for doing this is to start by checking the calibration of instruments used in new applications at short intervals and then to lengthen the interval between calibration checks progressively until a significant deterioration in instrument characteristics is observed. The required calibration interval is then defined as the time interval predicted to elapse before the characteristics of the instrument have drifted to limits that are allowable in that particular measurement application.

Working and reference standard instruments and ancillary equipment must also be calibrated periodically. An interval of 2 years is usually recommended between such calibration checks, although monthly checks are advised for the blackbody-cavity furnaces used to provide standard reference temperatures in pyrometer calibration. Standard RTDs

and thermocouples may also need more frequent calibration checks if the conditions (especially of temperature) and frequency of use demand them.

14.14.3 Procedures for calibration

The standard way to calibrate temperature transducers is to put them into a temperature-controlled environment together with a standard instrument or to use a radiant heat source of controlled temperature with high emissivity in the case of radiation thermometers. In either case, the controlled temperature must be measured by a standard instrument whose calibration is traceable to reference standards. This is a suitable method for most instruments in the calibration chain but is not necessarily appropriate or even possible for process instruments at the lower end of the chain.

In the case of many process instruments, their location and mode of fixing make it difficult or sometimes impossible to remove them to a laboratory for calibration checks to be carried out. In this event, it is standard practice to calibrate them in their normal operational position, using a reference instrument that is able to withstand whatever hostile environment may be present. If this practice is followed, it is imperative for the working standard instrument to be checked regularly to ensure it has not been contaminated.

Such *in situ calibration* may also be required when process instruments have characteristics that are sensitive to the environment in which they work, so that they are calibrated under their usual operating conditions and are therefore accurate in normal use. However, the preferred way to deal with this situation is to calibrate them in a laboratory with ambient conditions (of pressure, humidity, etc.) set up to mirror those of the normal operating environment. This alternative avoids having to subject reference calibration instruments to harsh chemical environments commonly associated with manufacturing processes.

For instruments at the lower end of the calibration chain (i.e., those measuring process variables), it is common practice to calibrate them against an instrument that is of the same type but of higher accuracy and reserved only for calibration duties. However, if a large number of different types of instrument have to be calibrated, this practice makes it necessary to keep a large number of different calibration instruments. To avoid this, various reference instruments are available that can be used to calibrate all process instruments within a given temperature-measuring range. Examples are the liquid-in-glass thermometer (0–200°C), platinum RTD (–200°C to +1000°C) and Type S thermocouple (+600 to 1750°C).

To calibrate instruments further up the calibration chain, particular care is needed with regard to both the instruments used and the conditions under which they are used. It is difficult and expensive to meet these conditions, and hence this function is subcontracted

by most companies to specialist laboratories. The reference instruments used are the platinum RTD in the temperature range of -200°C to $+1000^{\circ}\text{C}$, the platinum-platinum/10% rhodium (Type S) thermocouple in the temperature range of $+1000$ – 1750°C , and a narrow-band radiation thermometer at higher temperatures. An exception is *optical pyrometers*, which are calibrated as explained in the final paragraph of this chapter. A particular note of caution must be made when platinum-rhodium thermocouples are used as a standard. These are prone to contamination, and if they need to be handled at all, this should be done with clean hands.

Before ending this chapter, it is appropriate to mention one or two special points concerning the calibration of thermocouples. The mode of construction of thermocouples means that their characteristics can be incorrect even when they are new, because of faults in either the homogeneity of the thermocouple materials or the construction of the device. Therefore, calibration checks should be carried out on all new thermocouples before they are used. The procedure for this is to immerse both junctions of the thermocouple in an ice bath and measure its output with a high-accuracy digital voltmeter ($\pm 5 \mu\text{V}$). Any output greater than $5 \mu\text{V}$ would indicate a fault in the thermocouple material and/or its construction. After this check on thermocouples when they are brand-new, the subsequent rate of change of thermoelectric characteristics with time entirely depends on the operating environment and the degree of exposure to it. Particularly relevant factors in the environment are the type and concentration of trace metal elements and the temperature (the rate of contamination of thermocouple materials with trace elements of metals is a function of temperature). A suitable calibration frequency can therefore be defined only by practical experimentation, and this must be reviewed whenever the operating environment and conditions of use change. A final word of caution when calibrating thermocouples is to ensure that any source of electrical or magnetic fields is excluded, because these will induce erroneous voltages in the sensor.

Special comments are also relevant regarding the calibration of a *radiation thermometer*. In addition to normal accuracy checks, its long-term stability must be verified by testing its output over a period 1 hour longer than the manufacturer's specified warm-up time. This shows up any components within the instrument that are suffering from temperature-induced characteristics drift. It is also necessary to calibrate radiation thermometers according to the emittance characteristic of the body whose temperature is being measured and according to the level of energy losses in the radiation path between the body and measuring instrument. Such emissivity calibration must be carried out for every separate application for which the instrument is used, employing a surface pyrometer.

Finally, it should be noted that the usual calibration procedure for *optical pyrometers* is to sight them on the filament of a tungsten strip lamp in which the current is accurately

measured. This method of calibration can be used at temperatures up to 2500°C. Alternatively, they can be calibrated against a standard radiation pyrometer.

14.15 Summary

Our review at the start of the chapter revealed that there are nine different physical principles that are commonly used as the basis for temperature-measuring devices. During the course of the chapter, we have then looked at how each of these principles is exploited in various classes of temperature-measuring device.

We started by looking at the thermoelectric effect and its use in thermocouples and thermopiles, and also the derived devices of digital thermometers and continuous thermocouples. Thermocouples are the most commonly used device for industrial applications of temperature measurement. However, despite their relatively simple operating concept of generating an output voltage as a function of the temperature to which they are exposed, proper use of thermocouples requires an understanding of two thermoelectric laws. These laws were presented and their application was explained by several examples. We also saw how the output of a thermocouple has to be interpreted by thermocouple tables. We then looked at the different types of thermocouple that are available, ranging from a number of cheap base metal types to expensive ones based on noble metals. We looked at the typical characteristics of these and discussed typical applications. Moving on, we noted that thermocouples are delicate devices that can suffer from both mechanical damage and chemical damage in certain operating environments, and we discussed ways to avoid such problems. We also briefly looked at how thermocouples are made.

Our next subject of study concerned RTDs and thermistors, both of which are devices that convert a change in temperature into a change in the resistance of the device. Both of these are also commonly used measuring devices. We looked at the theoretical principles of each of these and discussed the range of materials used in each class of device. We also looked at the typical device characteristics for each construction material.

Next, we examined semiconductor devices in the form of diodes and transistors and discussed their characteristics and mode of operation. This discussion revealed that although these devices are cheaper and more linear than both thermocouples and RTDs, their typical measurement range is relatively low. This limits their applicability and means that they are not used as widely as they would be if their measurement range were greater.

We then looked at the class of device known as radiation thermometers (alternatively known as radiation pyrometers), which exploit the physical principle that the peak wavelength of radiated energy emission from a body varies with temperature. The instrument is used by pointing it at the body to be measured and analyzing the radiation

emitted from the body. This has the advantage of being a noncontact mode of temperature measurement, which is highly attractive in the food and drug industries and any other application for which contamination of the measured quantity has to be avoided. We also observed that different versions of radiation thermometers are capable of measuring temperatures between -100°C and $+10,000^{\circ}\text{C}$, with measurement inaccuracy as low as $\pm 0.05\%$ in the more expensive versions. Despite these obvious merits, careful calibration of the instrument to the type of body being measured is essential, because the characteristics of radiation thermometers critically depend on the emissivity of the measured body, which varies widely among different materials.

This stage in the chapter marked the end of discussion of the four most commonly used types of temperature-measuring device. The remaining techniques all have niche applications, but none of these are large-volume uses. The first covered of these other techniques was thermography. Also known as thermal imaging, this involves scanning an infrared radiation detector across either a single object or a scene containing several objects. The information gathered is then processed and an output in the form of the temperature distribution across the object is produced. It thus differs from other forms of temperature sensor in providing information on temperature distribution across an object or scene rather than the temperature at a single discrete point. Temperature measurement from -20°C to $+1500^{\circ}\text{C}$ is possible.

Our next subject of study concerned the liquid-in-glass thermometer, the bimetallic thermometer, and the pressure thermometer. These are all usually classed as thermal expansion-based devices, although this is not strictly true in the case of the last one, which is based on the change in pressure of a fluid inside a fixed-volume stainless-steel bulb. The characteristics and typical applications of each of these were discussed.

We then looked at fiber-optic temperature sensors. We saw that their main application is measuring temperatures in hard-to-reach locations, although they are also used when high measurement accuracy is required.

Next, we discussed color indicators, which mainly consist of special paints or crayons that change color at a certain temperature. They are primarily used to determine when the temperature of objects placed in a furnace reach a given temperature. They are relatively cheap, and different paints and crayons are available to indicate temperatures between 50°C and 1250°C . In addition, certain liquid crystals that change color at a certain temperature are also used as color indicators. These have better measurement resolution than paints and crayons, and although some versions can indicate low temperatures down to -20°C , the highest temperature that they can indicate is $+100^{\circ}\text{C}$.

Finally, our discussion of the application of different physical principles in temperature measurement brought us to Seger cones. Also known as pyrometric cones, these have a

conical shape in which the tip melts and bends over at a particular temperature. They are commonly used in the ceramics industry to detect whether a given temperature is reached in a furnace.

The chapter continued with a look at intelligent measuring devices. These are designed for use with various sensors such as thermocouples, thermopiles, RTDs, thermistors, and broadband radiation pyrometers. Intelligence within the device gives them features such as adjustable damping, noise rejection, self-adjustment for zero and sensitivity drifts, self-fault diagnosis, self-calibration, reduced maintenance requirement, and an expanded measurement range. These features reduce typical measurement inaccuracy down to $\pm 0.05\%$ of full scale.

This completion of the discussion on all types of intelligent and nonintelligent device allowed us to consider the mechanisms by which a temperature-measuring device is chosen for a particular application. We reviewed the characteristics of each type of device in turn and looked at circumstances in which each might be used.

Our final subject of study in the chapter was that of calibrating temperature-measuring devices. We noted first that there was a fundamental difficulty in establishing an absolute standard for temperature and that in the absence of such a standard, fixed reference points for temperature were defined in the form of the freezing points and triple points of certain substances. We then looked at the calibration instruments and equipment that are used in workplace calibration. We also established some guidelines about how the frequency of calibration should be set. Finally, we looked in more detail at the appropriate practical procedures for calibrating various types of sensor.

14.16 Problems

- 14.1 Briefly discuss the different physical principles that are used in temperature-measuring instruments and give examples of instruments that use each of these principles.
- 14.2 (a) How are thermocouples manufactured? (b) What are the main differences between base metal and noble metal thermocouples? (c) Give six examples of the materials used to make base metal and noble metal thermocouples. (d) Specify the international code letters used to designate the thermocouples made from each pair of materials that you give in your answer to Part (c).
- 14.3 Explain what each of the following are in relation to thermocouples: (a) extension leads, (b) compensating leads, (c) law of intermediate metals, (d) law of intermediate temperature.
- 14.4 What type of base metal thermocouple would you recommend for each of the following applications?

- (a) measurement of subzero temperatures.
 - (b) measurement in oxidizing atmospheres
 - (c) measurement in reducing atmospheres
 - (d) where high sensitivity measurement is required
- 14.5 Why do thermocouples need protection from some operating environments, and how is this protection given? Discuss any differences between base metal and noble metal thermocouples in the need for protection.
- 14.6 The temperature of a fluid is measured by immersing an iron-constantan thermocouple in it. The reference junction of the thermocouple is maintained at 0°C in an ice bath and an output emf of 5.812 mV is measured. What is the indicated fluid temperature?
- 14.7 The output emf from a chromel-alumel thermocouple (Type K), with its reference junction maintained at 0°C , is 12.207 mV. What is the measured temperature?
- 14.8 The temperature of a fluid is measured by immersing a Type K thermocouple in it. The reference junction of the thermocouple is maintained at 0°C in an ice bath and an output emf of 6.435 mV is measured. What is the indicated fluid temperature?
- 14.9 (a) Draw a graph of the output emf from a nicrosil-nisil thermocouple (Type N) over the temperature range of 0 – 650°C by reading appropriate data from a thermocouple table. Assume that the reference junction of the thermocouple is maintained at 0°C in an ice bath.
- (b) Calculate the measurement sensitivity of the thermocouple over the temperature range of 450 – 650°C .
- 14.10 The output emf from a nicrosil-nisil thermocouple (Type N), with its reference junction maintained at 0°C , is 4.21 mV. What is the measured temperature?
- 14.11 The output emf from a chromel-constantan thermocouple whose hot junction is immersed in a fluid is measured as 18.25 mV. The reference junction of the thermocouple is maintained at 0°C . What is the temperature of the fluid?
- 14.12 A Type S, platinum/10% rhodium-platinum thermocouple is used to measure the temperature of a furnace. The output emf is 17.62 mV with its reference junction maintained at 0°C . What is the temperature of the furnace?
- 14.13 (a) Draw a graph of the output emf from a chromel-alumel thermocouple (Type K) and from a copper-constantan thermocouple (Type T) over the temperature range 0 – 550°C by reading appropriate data from a thermocouple table. Assume that the reference junction of the thermocouple is maintained at 0°C in an ice bath. Draw the graphs for both thermocouples on the same piece of graph paper.
- (b) Calculate the measurement sensitivity of each thermocouple over the temperature range of 250 – 400°C .

- 14.14 A copper-constantan thermocouple is connected to copper-constantan extension wires and the reference junction is exposed to a room temperature of 20°C . If the output voltage measured is 6.537 mV , what is the indicated temperature at the hot junction of the thermocouple?
- 14.15 A nicrosil-nisil thermocouple is connected to nicrosil-nisil extension wires and the reference junction is exposed to a room temperature of 21°C . If the output voltage measured is 10.37 mV , what is the indicated temperature at the hot junction of the thermocouple?
- 14.16 A platinum/10% rhodium-platinum (Type S) thermocouple is used to measure the temperature of a furnace. The output emf, with the reference junction maintained at 50°C , is 5.975 mV . What is the temperature of the furnace?
- 14.17 (a) Draw a graph of the output emf from a chromel-constantan thermocouple (Type E) and from a platinum/10% rhodium-platinum thermocouple (Type S) over the temperature range of $0\text{--}600^{\circ}\text{C}$ by reading appropriate data from a thermocouple table. Assume that the reference junction of the thermocouple is maintained at 0°C in an ice bath. Draw the graphs for both thermocouples on the same piece of graph paper.
- (b) Calculate the measurement sensitivity of each thermocouple over the temperature range of $250\text{--}600^{\circ}\text{C}$.
- 14.18 A chromel-alumel (Type K) thermocouple is used to measure the temperature of a hot fluid. The output emf, with the reference junction maintained at 30°C , is 33.6 mV . What is the temperature of the fluid?
- 14.19 A chromel-constantan (Type E) thermocouple is used to measure the temperature of a furnace. The output emf, with the reference junction maintained at 18°C , is 16.75 mV . What is the temperature of the furnace?
- 14.20 A iron-constantan (Type J) thermocouple is used to measure the temperature of a hot fluid. The output emf, with the reference junction maintained at 16°C , is 24.58 mV . What is the temperature of the fluid?
- 14.21 In a particular industrial situation, a nicrosil-nisil thermocouple with nicrosil-nisil extension wires is used to measure the temperature of a fluid. In connecting this measurement system, the instrumentation engineer responsible has inadvertently interchanged the extension wires from the thermocouple. The ends of the extension wires are held at a reference temperature of 0°C and the output emf measured is 21.0 mV . If the junction between the thermocouple and extension wires is at a temperature of 50°C , what temperature of fluid is indicated and what is the true fluid temperature?
- 14.22 A copper-constantan thermocouple measuring the temperature of a hot fluid is connected by mistake with chromel-constantan extension wires (such that the two constantan wires are connected and the chromel extension wire is connected to the

- copper thermocouple wire. If the actual fluid temperature was 150°C , the junction between the thermocouple and extension wires was at 80°C , and the reference junction was at 0°C , calculate the emf measured at the open ends of the extension wires. What fluid temperature would be deduced from this measured emf (assuming that the error of using the wrong extension wires was not known)? (Hint: apply the law of intermediate metals for the thermocouple-extension lead junction.)
- 14.23 In a particular industrial situation, a chromel-constantan thermocouple with chromel-constantan extension wires is used to measure the temperature of a fluid. In connecting this measurement system, the instrumentation engineer responsible has inadvertently interchanged the extension wires from the thermocouple. The ends of the extension wires are held at a reference temperature of 0°C and the output emf measured is 28.62 mV . If the junction between the thermocouple and extension wires is at a temperature of 40°C , what temperature of fluid is indicated and what is the true fluid temperature?
- 14.24 Suppose an engineer has installed a chromel-constantan thermocouple but has incorrectly used copper-constantan extension leads (such that the two constantan wires were connected and the copper extension wire was connected to the chromel thermocouple wire). If the thermocouple was measuring a hot fluid whose real temperature is 150°C , the junction between the thermocouple and the extension leads was at 80°C , and the reference junction was at 0°C :
- Calculate the emf (voltage) measured at the open ends of the extension wires.
 - What fluid temperature would be deduced from this measured emf, assuming the error in using the incorrect leads was not known?
- 14.25 While installing a chromel-constantan thermocouple to measure the temperature of a fluid, it is connected by mistake with copper-constantan extension leads (such that the two constantan wires are connected and the copper extension wire is connected to the chromel thermocouple wire). If the fluid temperature was actually 250°C and the junction between the thermocouple and extension wires was at 80°C , what emf would be measured at the open ends of the extension wires if the reference junction is maintained at 0°C ? What fluid temperature would be deduced from this (assuming that the connection mistake was not known)?
- 14.26 In connecting extension leads to a chromel-alumel thermocouple that is measuring the temperature of a fluid, a technician connects the leads the wrong way round (such that the chromel extension lead is connected to the alumel thermocouple lead and vice versa). The junction between the thermocouple and extension leads is at a temperature of 100°C and the reference junction is maintained at 0°C in an ice bath. The technician measures an output emf of 12.212 mV at the open ends of the extension leads.
- What fluid temperature would be deduced from this measured emf?
 - What is the true fluid temperature?

- 14.27 A chromel-constantan thermocouple measuring the temperature of a fluid is connected by mistake with copper-constantan extension leads (such that the two constantan wires are connected and the copper extension lead wire is connected to the chromel thermocouple wire). If the fluid temperature was actually 250°C and the junction between the thermocouple and extension leads was at 90°C , what emf would be measured at the open ends of the extension leads if the reference junction is maintained at 0°C ? What fluid temperature would be deduced from this (assuming that the connection error was not known)?
- 14.28 The extension leads used to measure the output emf of an iron-constantan thermocouple measuring the temperature of a fluid are connected the wrong way round by mistake (such that the iron extension lead is connected to the constantan thermocouple wire and vice versa). The junction between the thermocouple and extension leads is at a temperature of 120°C and the reference junction is at a room temperature of 21°C . The output emf measured at the open ends of the extension leads is 27.390 mV .
- (a) What fluid temperature would be deduced from this measured emf assuming that the mistake of connecting the extension leads the wrong way round was not known?
- (b) What is the true fluid temperature?
- 14.29 The temperature of a hot fluid is measured with a copper-constantan thermocouple, but by mistake, this is connected to chromel-constantan extension wires (such that the two constantan wires are connected and the chromel extension wire is connected to the copper thermocouple wire). If the actual fluid temperature was 200°C , the junction between the thermocouple and extension wires was at 50°C , and the reference junction was at 0°C , calculate the emf measured at the open ends of the extension wires. What fluid temperature would be deduced from this measured emf (assuming that the error of using the wrong extension wires was not known)?
- 14.30 In a particular industrial situation, a chromel-alumel thermocouple with chromel-alumel extension wires is used to measure the temperature of a fluid. In connecting this measurement system, the instrumentation engineer responsible has inadvertently interchanged the extension wires from the thermocouple (such that the chromel thermocouple wire is connected to the alumel extension lead wire, etc.). The open ends of the extension leads are held at a reference temperature of 0°C and are connected to a voltmeter that measures an emf of 18.75 mV . If the junction between the thermocouple and extension wires is at a temperature of 38°C ,
- (a) What temperature of fluid is indicated?
- (b) What is the true fluid temperature?
- 14.31 A copper-constantan thermocouple measuring the temperature of a hot fluid is connected by mistake with iron-constantan extension wires (such that the two

- constantan wires are connected and the iron extension wire is connected to the copper thermocouple wire. If the actual fluid temperature was 200°C , the junction between the thermocouple and extension wires was at 160°C , and the reference junction was at 0°C , calculate the emf measured at the open ends of the extension wires. What fluid temperature would be deduced from this measured emf (assuming that the error of using the wrong extension wires was not known)?
- 14.32 In a particular industrial situation, a nicrosil-nisil thermocouple with nicrosil-nisil extension wires is used to measure the temperature of a fluid. In connecting this measurement system, the instrumentation engineer responsible has inadvertently interchanged the extension wires from the thermocouple (such that the nicrosil thermocouple wire is connected to the nisil extension lead wire, etc.). The open ends of the extension leads are held at a reference temperature of 0°C and are connected to a voltmeter that measures an emf of 17.51 mV. If the junction between the thermocouple and extension wires is at a temperature of 140°C ,
- What temperature of fluid is indicated?
 - What is the true fluid temperature?
- 14.33 Explain what the following are: thermocouple, continuous thermocouple, thermopile, digital thermometer.
- 14.34 What is the IPTS? Why is it necessary in temperature sensor calibration and how is it used?
- 14.35 Resistance temperature devices and thermistors are both temperature-measuring devices that convert the measured temperature into a resistance change. What are the main differences between these two types of device with respect to the materials used in their constructions, their cost, and their operating characteristics?
- 14.36 Discuss the main types of radiation thermometer that are available. How do they work and what are their main applications?
- 14.37 Name three kinds of temperature-measuring device that work on the principle of thermal expansion. Explain how each works and what their typical characteristics are.
- 14.38 Explain how fiber-optic cables can be used as temperature sensors.
- 14.39 Discuss the calibration of temperature sensors, mentioning what reference instruments are typically used.
- 14.40 What are semiconductor temperature sensors? What forms of output are available? Describe the characteristics of these kinds of sensors.
- 14.41 What is thermography? Explain how different versions of thermography system work, using a simple sketch to aid your explanation. Describe some typical applications.
- 14.42 What are color indicators and pyrometric cones in the context of temperature measurement? How are they made? How do they work? What are they used for?

- 14.43 What types of intelligent temperature sensor exist? What are their main characteristics?
- 14.44 Carry out an Internet search for MEMS-based temperature sensors. Document the different structures used, their characteristics, and some current applications.
- 14.45 What instruments and special equipment are used in the calibration of temperature sensors at each stage of the calibration chain?
- 14.46 What procedures should be followed in the calibration of temperature sensors?

Pressure measurement

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15.1 Introduction

We are covering pressure measurement next in this chapter because it is very commonly required in most industrial process control systems and is the next-most measured process parameter after temperature. We shall see that many different types of pressure-sensing and pressure-measurement systems are available to satisfy this requirement. However, before considering these in detail, it is important for us to understand that pressure can be quantified in three alternative ways in terms of either absolute pressure, gauge pressure, or differential pressure. The formal definitions of these are as follows:

Absolute pressure: This is the difference between the pressure of a fluid and the absolute zero of pressure.

Gauge pressure: This describes the difference between the pressure of a fluid and atmospheric pressure. Absolute and gauge pressure are therefore related by the expression:

$$\text{Absolute pressure} = \text{Gauge pressure} + \text{Atmospheric pressure.}$$

A typical value of atmospheric pressure is 1.013 bar. However, since atmospheric pressure varies with altitude as well as weather conditions, it is not a fixed quantity. Therefore, since gauge pressure is related to atmospheric pressure, it also is not a fixed quantity.

Differential pressure: This term is used to describe the difference between two absolute pressure values, such as the pressures at two different points within the same fluid (often between the two sides of a flow restrictor in a system measuring volume flow rate).

Pressure is a quantity that is derived from the fundamental quantities of force and area, and is usually measured in terms of the force acting on a known area. The SI unit of pressure is the Pascal, which can alternatively be expressed as Newtons per square meter. The bar, which is equal to 10,000 Pa, is a related metric unit that is more suitable for measuring the most typically met pressure values. The unit of pounds per square inch (psi) is not an SI unit, but it is still in widespread use, especially in the United States and Canada. Pressures are also sometimes expressed as inches of mercury or inches of water, particularly when measuring blood pressure or pressures in gas pipelines. These two measurement units derive from the height of the liquid column in manometers, which was a very common method of pressure measurement in the past. The torr is another unit of measurement that is used, particularly to quantify low pressures (1 torr = 133.3 Pa).

To avoid ambiguity in pressure measurements, it is usual to append one or more letters in parenthesis after the pressure value to indicate whether it is an absolute, gauge, or differential pressure. (a) or (abs) indicates absolute pressure, (g) indicates gauge pressure, and (d) specifies differential pressure. Thus, 2.57 bar (g) means that the pressure is 2.57 bar measured as gauge pressure. In the case of the pounds per square inch unit of

pressure measurement, which is still in widespread use, it is usual to express absolute, gauge, and differential pressure as psia, psig, and psid, respectively.

Absolute pressure measurements are made for such purposes as aircraft altitude measurement (in instruments known as altimeters) and when quantifying atmospheric pressure. Very low pressures are also normally measured as absolute pressure values. Gauge pressure measurements are made by instruments such as those measuring the pressure in vehicle tires and those measuring pressure at various points in industrial processes. Differential pressure is measured for some purposes in industrial processes, especially as part of some fluid flow rate measuring devices.

In most applications, the typical values of pressure measured range from 1.013 bar (the mean atmospheric pressure) up to 7000 bar. This is considered to be the “normal” pressure range, and a large number of pressure sensors are available that can measure pressures in this range. Measurement requirements outside this range are much less common. While some of the pressure sensors developed for the “normal” range can also measure pressures that are either lower or higher than this, it is preferable to use special instruments that have been specially designed to satisfy such low and high pressure measurement requirements. In the case of low pressures, such special instruments are commonly known as vacuum gauges.

Our following discussion summarizes the main types of pressure sensors that are in use. This discussion is primarily concerned only with the measurement of static pressure, because the measurement of dynamic pressure is a very specialized area that is not of general interest. In general, dynamic pressure measurement requires special instruments, although modified versions of diaphragm-type sensors can also be used if they contain a suitable displacement sensor (usually either a piezoelectric crystal or a capacitive element).

15.2 Diaphragms

The diaphragm, shown schematically in [Fig. 15.1](#), is one of three types of elastic-element pressure transducer. Applied pressure causes displacement of the diaphragm and this movement is measured by a displacement transducer. Different versions of diaphragm sensors can measure both absolute pressure (up to 50 bar) and gauge pressure (up to 2000 bar) according to whether the space on one side of the diaphragm is, respectively, evacuated or is open to the atmosphere. A diaphragm can also be used to measure differential pressure (up to 2.5 bar) by applying the two pressures to the two sides of the diaphragm. The diaphragm can be either plastic, metal alloy, stainless steel, or ceramic. A silicon diaphragm is also used in microelectrical mechanical systems (MEMS) sensors. Plastic diaphragms are cheapest, but metal diaphragms give better accuracy. Stainless steel

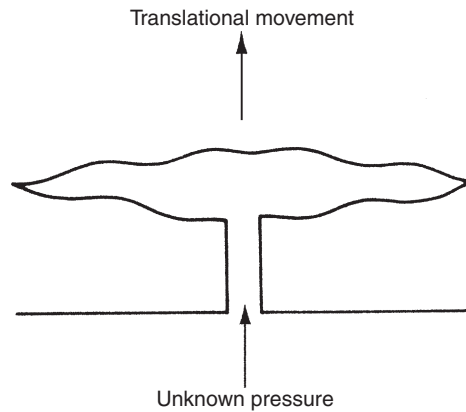


Figure 15.1

Schematic representation of diaphragm pressure sensor.

is normally used in high-temperature or corrosive environments. Ceramic diaphragms are resistant even to strong acids and alkalis, and are used when the operating environment is particularly harsh. The name *aneroid gauge* is sometimes used to describe this type of gauge when the diaphragm is metallic.

The typical magnitude of diaphragm displacement is 0.1 mm, which is well suited to a strain-gauge type of displacement-measuring transducer, although other forms of displacement measurement are also used in some kinds of diaphragm-based sensors. If the displacement is measured with strain gauges, it is normal to use four strain gauges arranged in a bridge circuit configuration. The output voltage from the bridge is a function of the resistance change due to the strain in the diaphragm. This arrangement automatically provides compensation for environmental temperature changes. Older pressure transducers of this type used metallic strain gauges bonded to a diaphragm typically made of stainless steel. However, apart from manufacturing difficulties arising from the problem of bonding the gauges, metallic strain gauges have a low gauge factor, which means that the low output from the strain gauge bridge has to be amplified by an expensive d.c. amplifier. They also have a nonlinear strain/output relationship.

The problem of strain-gauge bonding was solved with the emergence of monolithic piezoresistive pressure transducers. These have a typical measurement uncertainty of $\pm 0.5\%$ and are now the most commonly used type of diaphragm pressure transducer. The monolithic cell consists of a diaphragm made of a silicon sheet into which resistors are diffused during the manufacturing process. Such pressure transducers can be made to be very small and are often known as *MEMS sensors* or *microsensors* (see [Section 15.10](#) for more details). Besides avoiding the difficulty with bonding, monolithic silicon measuring cells have the advantage of being very cheap to manufacture in large quantities. Although

the inconvenience of a nonlinear characteristic remains, this is normally overcome by processing the output signal with an active linearization circuit or incorporating the cell into a microprocessor-based intelligent measuring transducer. The latter usually provide analog-to-digital conversion and interrupt facilities within a single chip and give a digital output that is readily integrated into computer control schemes. Such instruments can also offer automatic temperature compensation, built-in diagnostics, and simple calibration procedures. These features allow measurement inaccuracy to be reduced down to a value as low as $\pm 0.1\%$ of full-scale reading.

15.3 Capacitive pressure sensor

A capacitive pressure sensor is simply a diaphragm-type device in which the diaphragm displacement is determined by measuring the capacitance change between the diaphragm and a metal plate that is close to it. Such devices are in common use and are sometimes known as *Baratron gauges*. It is also possible to fabricate capacitive elements in a silicon chip and thus form very small *microsensors*. These have a typical measurement uncertainty of $\pm 0.2\%$.

15.4 Fiber-optic pressure sensors

Fiber-optic sensors, also known as optical pressure sensors, provide an alternative method of measuring displacements in diaphragm and Bourdon tube pressure sensors by optoelectronic means, and enable the resulting sensors to have lower mass and size compared with sensors in which the displacement is measured by other methods. The Fotic sensor, shown in Fig. 15.2, is one form of fiber-optic displacement sensor in which light travels from a light source, down an optical fiber, is reflected back from a diaphragm, and then travels back along a second fiber to a photodetector. There is a characteristic relationship between the light reflected and the distance from the fiber ends to the

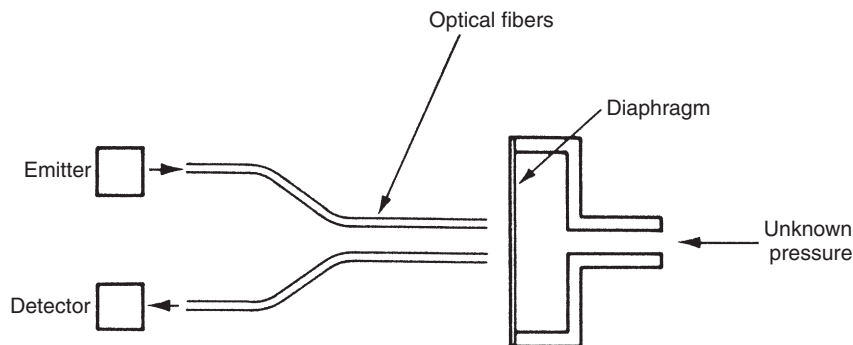


Figure 15.2
Fotic sensor.

diaphragm, thus making the amount of reflected light dependent upon the diaphragm displacement and hence the measured pressure.

Apart from the mass and size advantages of fiber-optic displacement sensors, the output signal is immune to electromagnetic noise. However, the measurement accuracy is usually inferior to that provided by alternative displacement sensors, and choice of such sensors also incurs a cost penalty. Thus, sensors using fiber optics to measure diaphragm or Bourdon tube displacement tend to be limited to applications where their small size, low mass, and immunity to electromagnetic noise are particularly advantageous.

Apart from the limited use above within diaphragm and Bourdon tube sensors, fiber-optic cables are also used in several other ways to measure pressure. A form of fiber-optic pressure sensor known as a *microbend sensor* is sketched in Fig. 13.8(a). In this, the refractive index of the fiber (and hence of the intensity of light transmitted) varies according to the mechanical deformation of the fiber caused by pressure. The sensitivity of pressure measurement can be optimized by applying the pressure via a roller chain such that the bending is applied periodically (see Fig. 13.8(b)). The optimal pitch for the chain varies according to the radius, refractive index, and type of cable involved. Microbend sensors are typically used to measure the small pressure changes generated in Vortex shedding flowmeters. When fiber-optic sensors are used in this flow-measurement role, the alternative arrangement shown in Fig. 15.3 can be used, where a fiber-optic cable is merely stretched across the pipe. This often simplifies the detection of vortices.

Phase-modulating fiber-optic pressure sensors also exist. The mode of operation of these was discussed in Chapter 13.

15.5 Bellows

The bellows, schematically illustrated in Fig. 15.4, is another elastic-element type of pressure sensor that operates on very similar principles to the diaphragm pressure sensor. Pressure changes within the bellows, which is typically fabricated as a seamless tube of either metal or metal alloy, produce translational motion of the end of the bellows that can be measured by capacitive, inductive (LVDT), or potentiometric transducers. Different versions can measure either absolute pressure (up to 2.5 bar) or gauge pressure (up to 150 bar). Double-bellows versions also exist that are designed to measure differential pressures of up to 30 bar.

Bellows have a typical measurement uncertainty of only $\pm 0.5\%$, but they have a relatively high manufacturing cost and are prone to failure. Their principal attribute in the past has been their greater measurement sensitivity compared with diaphragm sensors. However, advances in electronics mean that the high-sensitivity requirement can usually be satisfied now by diaphragm-type devices, and usage of bellows is therefore falling.

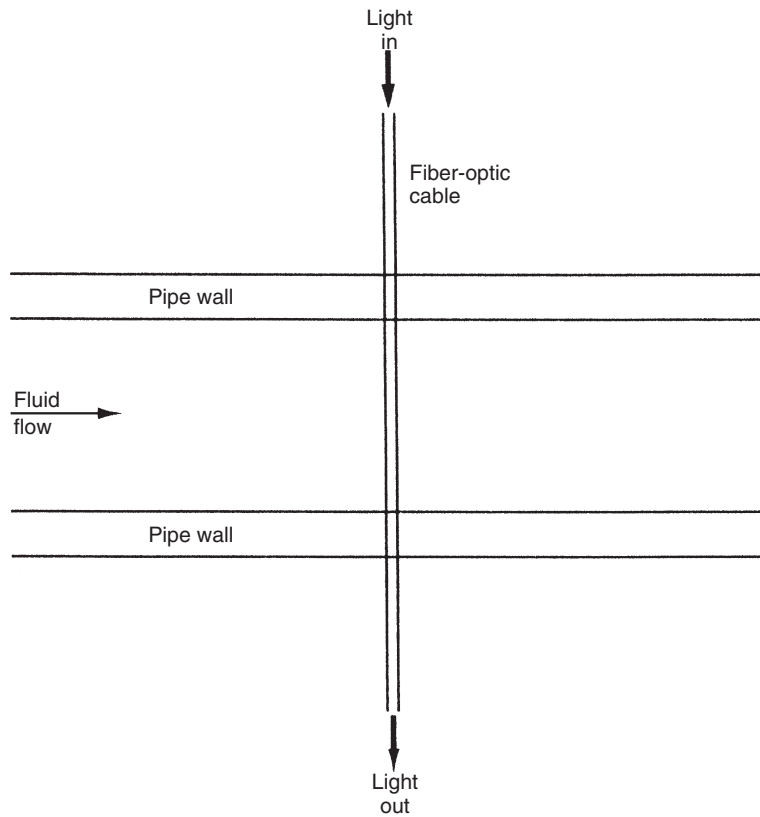


Figure 15.3

Use of fiber-optic pressure sensor in vortex-shedding flowmeter.

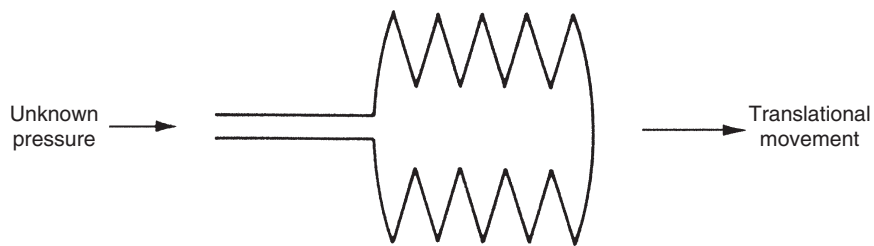


Figure 15.4

Bellows.

15.6 Bourdon tube

The Bourdon tube is also an elastic element type of pressure transducer. It is relatively cheap and is commonly used for measuring the gauge pressure of both gaseous and liquid fluids. It consists of a specially shaped piece of oval-section, flexible, metal tube that is

fixed at one end and free to move at the other end. When pressure is applied at the open, fixed end of the tube, the oval cross section becomes more circular. In consequence, there is a displacement of the free end of the tube. This displacement is measured by some form of displacement transducer, which is commonly a potentiometer or LVDT. Capacitive and optical sensors are also sometimes used to measure the displacement.

The three common shapes of Bourdon tube are shown in Fig. 15.5. The maximum possible deflection of the free end of the tube is proportional to the angle subtended by the arc through which the tube is bent. For a C-type tube, the maximum value for this arc is somewhat less than 360° . Where greater measurement sensitivity and resolution are required, spiral and helical tubes are used. These both give a much greater deflection at the free end for a given applied pressure. However, this increased measurement performance is only gained at the expense of a substantial increase in manufacturing difficulty and cost compared with C-type tubes, and is also associated with a large decrease in the maximum pressure that can be measured. Spiral and helical types are sometimes provided with a rotating pointer that moves against a scale to give a visual indication of the measured pressure.

C-type tubes are available for measuring pressures up to 6000 bar. A typical C-type tube of 25-mm radius has a maximum displacement travel of 4 mm, giving a moderate level of

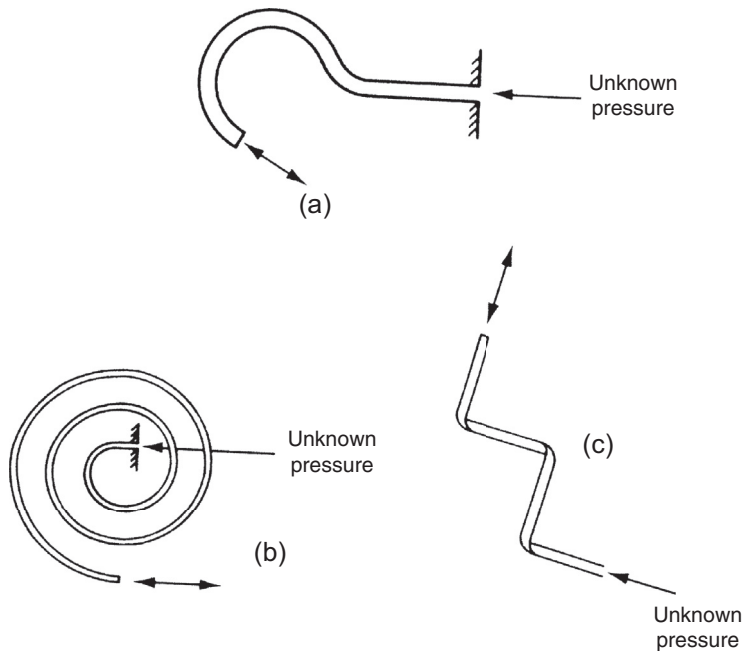


Figure 15.5

Three forms of Bourdon tube. (a) C-type; (b) spiral type; (c) helical type.

measurement resolution. Measurement inaccuracy is typically quoted at $\pm 1\%$ of full-scale deflection. Similar accuracy is available from helical and spiral types, but while the measurement resolution is higher, the maximum pressure measurable is only 700 bar.

The existence of one potentially major source of error in Bourdon tube pressure measurement has not been widely documented, and few manufacturers of Bourdon tubes make any attempt to warn users of their products appropriately. The problem is concerned with the relationship between the fluid being measured and the fluid used for calibration. The pointer of Bourdon tubes is normally set at zero during manufacture, using air as the calibration medium. However, if a different fluid, especially a liquid, is subsequently used with a Bourdon tube, the fluid in the tube will cause a nonzero deflection according to its weight compared with air, resulting in a reading error of up to 6%. This can be avoided by calibrating the Bourdon tube with the fluid to be measured instead of with air, assuming of course that the user is aware of the problem. Alternatively, correction can be made according to the calculated weight of the fluid in the tube. Unfortunately, difficulties arise with both of these solutions if air is trapped in the tube, since this will prevent the tube from being filled completely by the fluid. Then, the amount of fluid actually in the tube, and its weight, will be unknown.

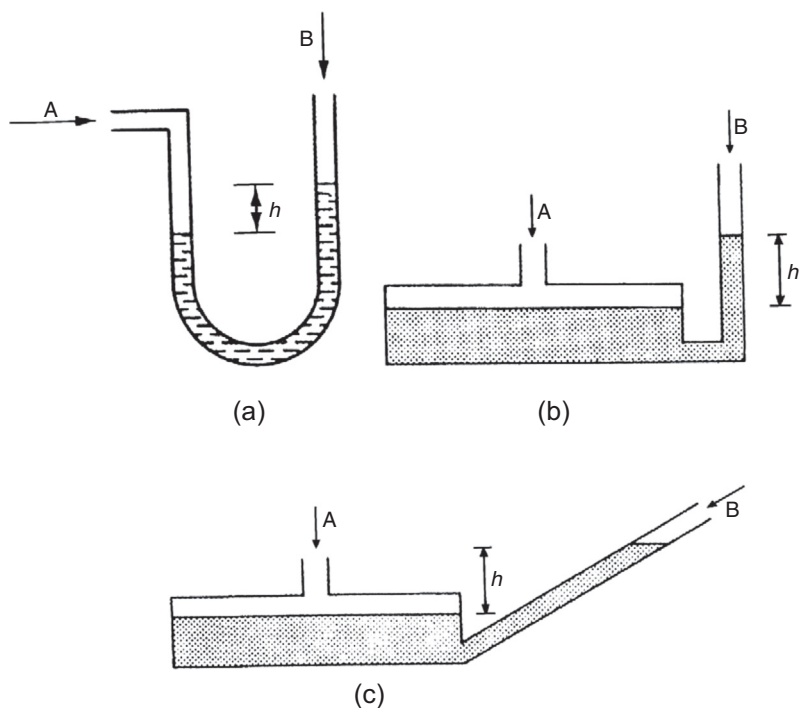
In conclusion, therefore, Bourdon tubes only have guaranteed accuracy limits when measuring gaseous pressures. Their use for accurate measurement of liquid pressures poses great difficulty unless the gauge can be totally filled with liquid during both calibration and measurement, a condition that is very difficult to fulfill practically.

15.7 Manometers

Manometers are passive instruments that give a visual indication of pressure values. Various types exist.

U-tube manometer: The U-tube manometer, shown in Fig. 15.6(a), is the most common form of manometer. Applied pressure causes a displacement of liquid inside the U-shaped glass tube, and the output pressure reading P is made by observing the difference h between the level of liquid in the two halves of the tube A and B , according to the equation $P = h\rho g$, where ρ is the specific gravity of the fluid. If an unknown pressure is applied to side A , and side B is open to the atmosphere, the output reading is gauge pressure. Alternatively, if side B of the tube is sealed and evacuated, the output reading is absolute pressure. The U-tube manometer also measures the differential pressure ($p_1 - p_2$), according to the expression $(p_1 - p_2) = h\rho g$, if two unknown pressures p_1 and p_2 are applied, respectively, to sides A and B of the tube.

Output readings from U-tube manometers are subject to error, principally because it is very difficult to judge exactly where the meniscus levels of the liquid are in the two halves

**Figure 15.6**

Three forms of manometer: (a) U-tube; (b) well-type; (c) inclined-type.

of the tube. In absolute pressure measurement, an addition error occurs because it is impossible to totally evacuate the closed end of the tube.

U-tube manometers are typically used to measure gauge and differential pressures up to about 2 bar. The type of liquid used in the instrument depends on the pressure and characteristics of the fluid being measured. Water is a cheap and convenient choice, but it evaporates easily and is difficult to see. Nevertheless, it is used extensively, with the major obstacles to its use being overcome by using colored water and by regularly topping up the tube to counteract evaporation. However, water is definitely not used when measuring the pressure of fluids that react with or dissolve in water. Water is also unsuitable when high-pressure measurements are required. In such circumstances, liquids such as aniline, carbon tetrachloride, bromoform, mercury, or transformer oil are used instead.

Well-type manometer (cistern manometer): The well-type or cistern manometer, shown in Fig. 15.6(b), is similar to a U-tube manometer but one-half of the tube is made very large so that it forms a well. The change in the level of the well as the measured pressure varies is negligible. Therefore, the liquid level in only one tube has to be measured, which makes the instrument much easier to use than the U-tube manometer. If an unknown

pressure p_1 is applied to port A , and port B is open to the atmosphere, the gauge pressure is given by: $p_1 = h\rho$. It might appear that the instrument would give a better measurement accuracy than the U-tube manometer because the need to subtract two liquid level measurements in order to arrive at the pressure value is avoided. However, this benefit is swamped by errors that arise due to the typical cross-sectional area variations in the glass used to make the tube. Such variations do not affect the accuracy of the U-tube manometer to the same extent.

Inclined manometer (draft gauge): The inclined manometer or draft gauge shown in Fig. 15.6(c) is a variation on the well-type manometer in which one leg of the tube is inclined to increase measurement sensitivity. However, similar comments to those above apply about accuracy.

15.8 Resonant-wire devices

A typical resonant-wire device is shown schematically in Fig. 15.7. Wire is stretched across a chamber containing fluid at unknown pressure subjected to a magnetic field. The wire resonates at its natural frequency according to its tension, which varies with pressure. Thus pressure is calculated by measuring the frequency of vibration of the wire. Such frequency measurement is normally carried out by electronics integrated into the

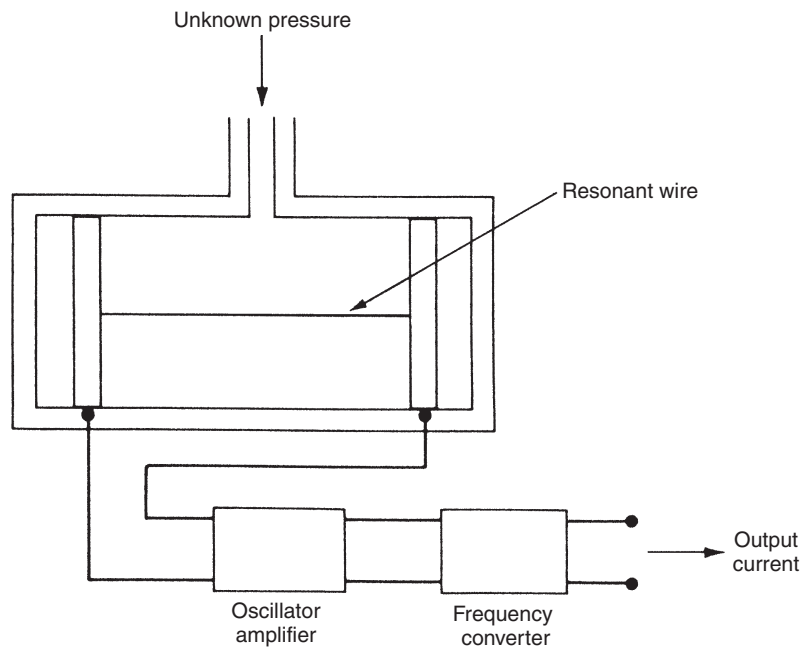


Figure 15.7
Resonant-wire device.

cell. Such devices are highly accurate, with a typical inaccuracy figure being $\pm 0.2\%$ full-scale reading. They are also particularly insensitive to ambient condition changes and can measure pressures between 5 mbar and 2 bar.

15.9 Digital pressure gauges

This section is included since many instrument manufacturer's catalogs have a section entitled "digital pressure gauges" or "electronic pressure gauges." However, in reality, such pressure gauges are merely special forms of the pressure gauges described earlier, in which electronic techniques are applied to improve performance, and a digital output display is included. All of the following commonly appear in instrument catalogs under the heading "digital pressure gauges" or "electronic pressure gauges":

15.9.1 Piezoresistive digital pressure gauge

This is a diaphragm-type sensor that uses piezoresistive strain gauges to measure diaphragm displacement.

15.9.2 Piezoelectric digital pressure gauge

This is a diaphragm-type sensor that uses a piezoelectric crystal to measure diaphragm displacement.

15.9.3 Magnetic digital pressure gauge

This describes a class of diaphragm-type devices that measure diaphragm displacement magnetically using either inductive, variable reluctance, or eddy current sensors.

15.9.4 Capacitive digital pressure gauge

This is a diaphragm-type sensor that measures the variation in capacitance between the diaphragm and a fixed metal plate close to it.

15.9.5 Fiber-optic digital pressure sensor

Known alternatively as an *optical pressure sensor*, this uses a fiber-optic sensor to measure the displacement of either a diaphragm or Bourdon tube pressure sensor.

15.9.6 Potentiometric digital pressure sensor

This is a device where the translational motion of a bellows-type pressure sensor is connected to the sliding element of an electrical potentiometer.

15.9.7 Resonant-wire digital pressure transducer

This is a form of resonant-wire pressure-measuring device in which the pressure-induced frequency change is measured by electronics that are integrated into the device.

15.10 MEMS pressure sensors

MEMS pressure sensors are used to measure the pressure of both air and various other gases. Their basic structure is shown in Fig. 15.8. It consists of a thin silicon diaphragm of typical thickness 100 μm on a silicon substrate. The diaphragm bends under the action of the measured pressure applied to it. There are two main ways in which this deflection is measured:

- (i) By mounting piezoresistors on the diaphragm surface.
- (ii) By measuring the capacitance change between the diaphragm and a silicon beam above the diaphragm.

The piezoresistor method produces a device with a linear input-output relationship, but one that is significantly affected by temperature changes. Devices based on the capacitive method are much less affected by temperature changes, but they are physically larger and have a nonlinear input-output relationship. Capacitance-operation devices are also cheaper than piezoresistive ones. Choice between these two alternatives therefore depends on the requirements of the intended application.

Both gauge and differential pressure can be measured by different versions of MEMS pressure sensors, with a typical inaccuracy of $\pm 3\%$ of full-scale reading. They are now used quite widely in a variety of automotive, medical, and industrial applications. Automotive applications include measuring the intake pressure in vehicle engines and sensing the pressure within car tires. A common medical application is in disposable blood

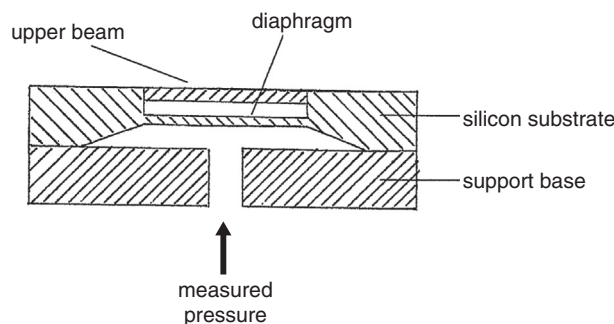


Figure 15.8
Structure of MEMS pressure sensor.

pressure sensors. The variation of air pressure with altitude is exploited in several devices that measure altitude and is used in devices like smartphones and tablet computers to enhance GPS for indoor and outdoor navigation. Devices are also used as a barometer in weather stations and sports watches.

15.11 Special measurement devices for low-pressures

The term *vacuum gauge* is commonly applied to describe any pressure sensor that is designed to measure pressures in the vacuum range (pressures less than atmospheric pressure, i.e., below 1.013 bar). Many special versions of the types of pressure transducer described earlier have been developed for measurement in the vacuum gauge. The typical minimum pressure measurable by these special forms of “normal” pressure-measuring instruments are 10 mbar (Bourdon tubes), 0.1 mbar (manometers and bellows-type instruments), and 0.001 mbar (diaphragms). However, in addition to these special versions of normal instruments, a number of other devices have been specifically developed for measurement of pressures below atmospheric pressure. These special devices include the thermocouple gauge, the Pirani gauge, the thermistor gauge, the McLeod gauge, and the ionization gauge, and they are covered in more detail next. Unfortunately, all of these specialized instruments are quite expensive.

Thermocouple gauge: The thermocouple gauge is one of a group of gauges working on the thermal conductivity principle. At low pressure, the kinematic theory of gases predicts a linear relationship between pressure and thermal conductivity. Thus measurement of thermal conductivity gives an indication of pressure. Fig. 15.9 shows a sketch of a

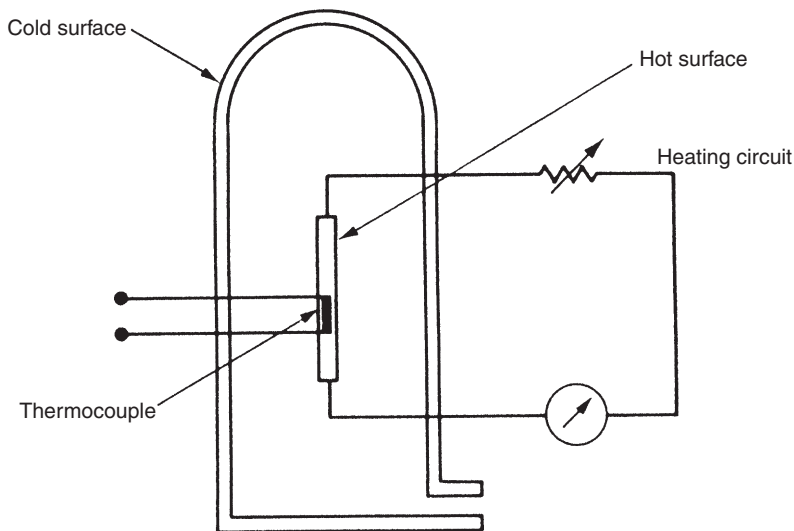


Figure 15.9
Thermocouple gauge.

thermocouple gauge. Operation of the gauge depends on the thermal conduction of heat between a thin, hot metal strip in the center and the cold outer surface of a glass tube (that is normally at room temperature). The metal strip is heated by passing a current through it and its temperature is measured by a thermocouple. The temperature measured depends on the thermal conductivity of the gas in the tube and hence on its pressure. A source of error in this instrument is the fact that heat is also transferred by radiation as well as conduction. This error is of a constant magnitude, independent of pressure. Hence, it can be measured, and thus correction can be made for it. However, it is usually more convenient to design for low radiation loss by choosing a heated element with low emissivity. Thermocouple gauges are typically used to measure pressures in the range 10^{-4} mbar up to 1 mbar.

Thermistor gauge: This is identical in its mode of operation to a thermocouple gauge except that a thermistor is used to measure the temperature of the metal strip rather than a thermocouple. It is commonly marketed under the name *electronic vacuum gauge* in a form that includes a digital LED display and switchable output ranges. It is used over the pressure range from 10^{-4} mbar up to 1 mbar.

Pirani gauge: A typical form of Pirani gauge is shown in Fig. 15.10(a). This is similar to a thermocouple gauge but has a heated element that consists of four coiled tungsten wires connected in parallel. Two identical tubes are normally used, connected in a bridge circuit as shown in Fig. 15.10(b), with one containing the gas at unknown pressure and the other evacuated to a very low pressure. Current is passed through the tungsten element, which attains a certain temperature according to the thermal conductivity of the gas. The resistance of the element changes with temperature and causes an imbalance of the

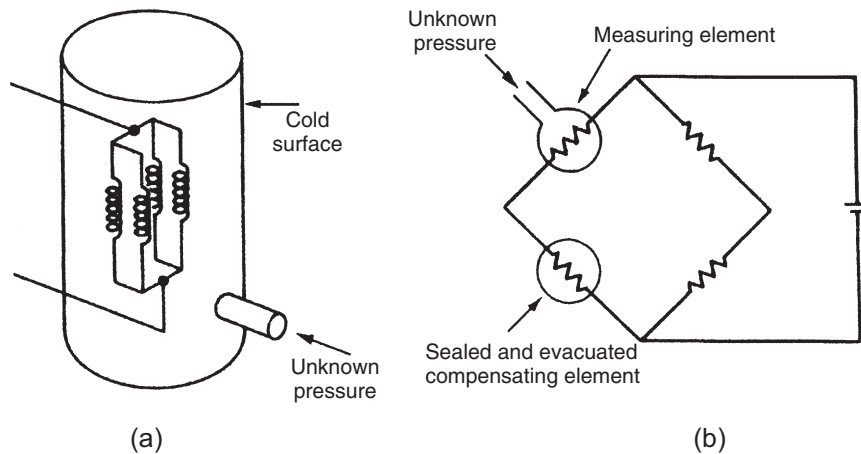


Figure 15.10

(a) Pirani gauge; (b) Wheatstone bridge circuit to measure output.

measurement bridge. Thus, the Pirani gauge avoids the use of a thermocouple to measure temperature (as in the thermocouple gauge) by effectively using a resistance thermometer as the heated element. Such gauges cover the pressure range 10^{-5} mbar to 1 mbar.

McLeod gauge: Fig. 15.11(a) shows the general form of a McLeod gauge, in which low-pressure fluid is compressed to a higher pressure that is then read by manometer techniques. In essence, the gauge can be visualized as a U-tube manometer that is sealed at one end, and where the bottom of the U can be blocked at will. To operate the gauge, the piston is first withdrawn. This causes the level of mercury in the lower part of the gauge to fall below the level of the junction J between the two tubes marked Y and Z in the gauge. Fluid at unknown pressure P_u is then introduced via the tube marked Z, from where it also flows into the tube of cross-sectional area A marked Y. Next, the piston is pushed in, moving the mercury level up to block the junction J. At the stage where J is just blocked, the fluid in tube Y is at pressure P_u and is contained in a known volume V_u . Further movement of the piston compresses the fluid in tube Y, and this process continues until the mercury level in tube Z reaches a zero mark. Measurement of the height (h) above the mercury column in tube Y then allows calculation of the compressed volume of the fluid V_c as: $V_c = hA$.

Then, by Boyle's law: $P_u V_u = P_c V_c$, where P_c is the pressure of the compressed fluid in tube Y.

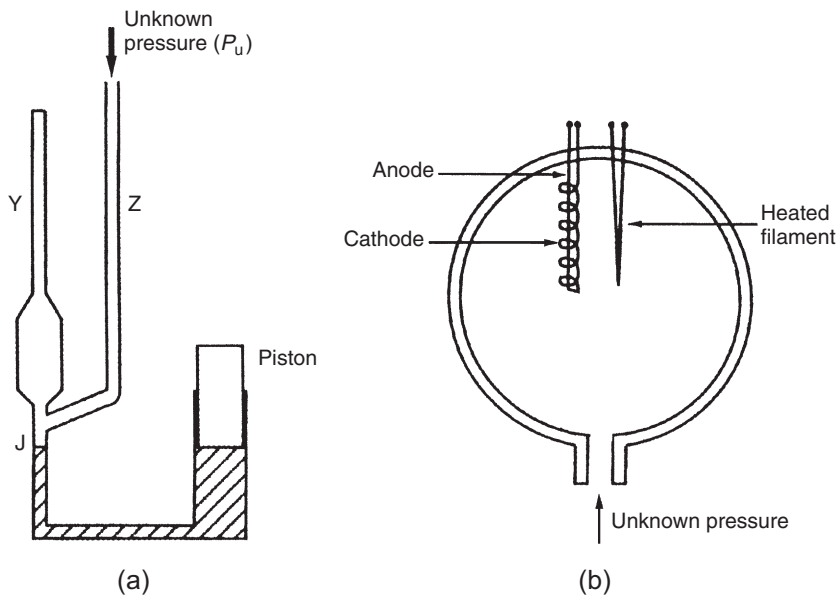


Figure 15.11

Other low-pressure gauges: (a) McLeod gauge, (b) Ionization gauge.

Also, applying the normal manometer equation: $P_c = P_u + h\rho g$

where ρ is the mass density of mercury, the pressure P_u can be calculated as:

$$P_u = \frac{Ah^2\rho g}{V_u - Ah} \quad (15.1)$$

The compressed volume V_c is often very much smaller than the original volume, in which case Eq. (15.1) approximates to:

$$P_u = \frac{Ah^2\rho g}{V_u} \text{ for } Ah \ll V_u \quad (15.2)$$

Although the smallest inaccuracy achievable with McLeod gauges is $\pm 1\%$, this is still better than that which is achievable with most other gauges that are available for measuring pressures in this range. Therefore, the McLeod gauge is often used as a standard against which other gauges are calibrated. The minimum pressure normally measurable is 10^{-1} mbar, although lower pressures can be measured if pressure-dividing techniques are applied.

Ionization gauge: The ionization gauge is a special type of instrument used for measuring very low pressures in the range 10^{-10} mbar to 1 mbar. Normally, they are only used in laboratory conditions because their calibration is very sensitive to the composition of the gases in which they operate, and use of a mass spectrometer is often necessary to determine the gas composition around them. They exist in two forms, known as hot cathode and cold cathode. The hot cathode form is shown schematically in Fig. 15.11(b). In this, gas of unknown pressure is introduced into a glass vessel containing free electrons discharged from a heated filament, as shown in Fig. 15.11(b). Gas pressure is determined by measuring the current flowing between an anode and cathode within the vessel. This current is proportional to the number of ions per unit volume, which in turn is proportional to the gas pressure. Cold cathode ionization gauges operate in a similar fashion except that the stream of electrons is produced by a high-voltage electrical discharge.

15.12 High-pressure measurement (greater than 7000 bar)

Measurement of pressures above 7000 bar is normally carried out electrically by monitoring the change of resistance of wires of special materials. Manganin is now the preferred material since its resistance-pressure characteristics are more linear and sensitive than most other materials. Devices made from *gold-chromium wire* were also used at one time, but manganin wire devices are now predominant. A *manganin pressure sensor* consists of a coil of manganin wire enclosed in a sealed, kerosene-filled, flexible bellows, as shown in Fig. 15.12. The unknown pressure is applied to one end of the bellows, which transmit the pressure to the coil. The magnitude of the applied pressure is then determined by measuring the coil resistance. Pressures up to 30,000 bar can be measured by a manganin-wire pressure sensor, with a typical inaccuracy of $\pm 0.5\%$.

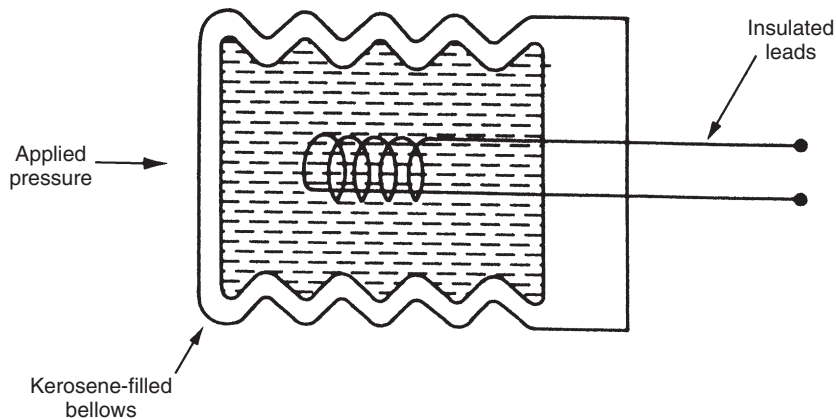


Figure 15.12
High-pressure measurement — wire coil in bellows.

15.13 Intelligent pressure transducers

Adding microprocessor power to pressure transducers brings about substantial improvements in their characteristics. Measurement sensitivity improvement, extended measurement range, compensation for hysteresis and other nonlinearities, and correction for ambient temperature and pressure changes are just some of the facilities offered by intelligent pressure transducers. For example, inaccuracy values as low as $\pm 0.1\%$ can be achieved with silicon piezoresistive-bridge devices.

Inclusion of microprocessors has also enabled the use of novel techniques of displacement measurement, for example, the optical method of displacement measurement shown in Fig. 15.13. In this, the motion is transmitted to a vane that progressively shades one of two monolithic photodiodes that are exposed to infrared radiation. The second photodiode acts as a reference, enabling the microprocessor to compute a ratio signal that is linearized and is available as either an analog or digital measurement of pressure. The typical measurement inaccuracy is $\pm 0.1\%$. Versions of both diaphragms and Bourdon tubes that use this technique are available.

15.14 Differential pressure measuring devices

Differential pressure measuring devices have two input ports. One unknown pressure is applied to each port and the instrument output is the difference between the two pressures. The alternative way to measure differential pressure would be to measure each pressure with a separate instrument and then subtract one reading from the other. However, this would produce a far less accurate measurement of the differential pressure because of the well-known problem that the process of subtracting measurements amplifies the inherent

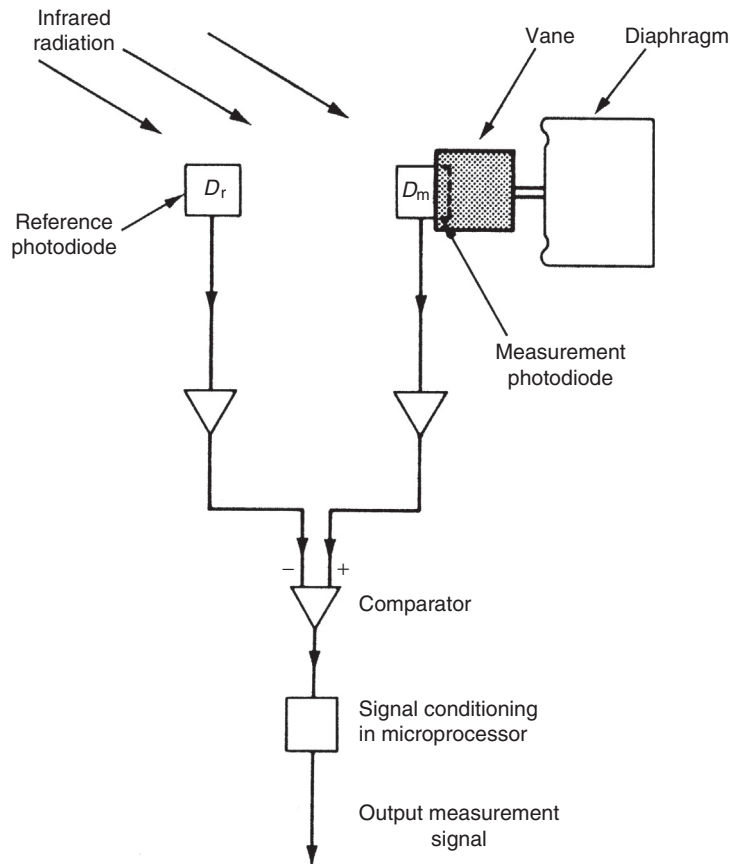


Figure 15.13

Example of intelligent pressure measuring instrument.

inaccuracy in each individual measurement. This is a particular problem when measuring differential pressures of low magnitude.

Differential pressure can be measured by special forms of many of the pressure-measuring devices described earlier. Diaphragm pressure sensors, and their MEMS, piezoresistive, piezoelectric, magnetic, capacitive, and fiber-optic named variants, are all commonly available in a differential-pressure-measuring form in which the two pressures to be subtracted are applied to either side of the diaphragm. Double-bellows pressure transducers (including devices known as potentiometric pressure transducers) are also used, but are much less common than diaphragm-based sensors. A special form of U-tube manometer is also sometimes used when a visual indication of differential pressure values is required. This has the advantage of being a passive instrument that does not require a power supply and it is commonly used in liquid flow-rate indicators.

15.15 Selection of pressure sensors

Choice between the various types of instrument available for measuring midrange pressures (1.013–7000 bar) is usually strongly influenced by the intended application. Manometers are commonly used when just a visual indication of pressure level is required, and dead-weight gauges, because of their superior accuracy, are used in calibration procedures of other pressure-measuring devices. When an electrical form of output is required, the choice is usually either one out of the several types of diaphragm sensor (strain gauge, piezoresistive, piezoelectric, magnetic, capacitive, fiber-optic, or MEMS-scale device) or, less commonly, a Bourdon tube. Bellows-type instruments are also sometimes used for this purpose, but much less frequently. If very high measurement accuracy is required, the resonant-wire device is a popular choice.

In the case of pressure measurement in the vacuum range (less than atmospheric pressure, i.e., below 1.013 bar), adaptations of most of the types of pressure transducer described earlier can be used. Special forms of Bourdon tubes measure pressures down to 10 mbar, manometers and bellows-type instruments measure pressures down to 0.1 mbar, and diaphragms can be designed to measure pressures down to 0.001 mbar. However, a number of more specialized instruments have also been developed to measure vacuum pressures, as discussed in [Section 15.10](#). These generally give better measurement accuracy and sensitivity compared with instruments that are primarily designed for measuring midrange pressures. This improved accuracy is particularly evident at low pressures. Therefore, only the special instruments described in [Section 15.10](#) (Pirani, McLeod, and ionization gauges) are used to measure pressures lower than 10^{-4} mbar.

At high pressures (>7000 bar), the only devices in common use are the manganin-wire sensor and similar devices based on alternative alloys to manganin.

For differential pressure measurement, diaphragm-type sensors are the preferred option, with double-bellows sensors being used occasionally. Manometers are also sometimes used to give visual indication of differential pressure values (especially in liquid flow-rate indicators). These are passive instruments that have the advantage of not needing a power supply.

15.16 Calibration of pressure sensors

Different types of reference instrument are used according to the range of the pressure-measuring instrument being calibrated. In the midrange of pressures from 0.1 mbar to 20 bar, U-tube manometers, dead-weight gauges, and barometers can all be used as reference instruments for calibration purposes. The vibrating cylinder gauge also provides a very accurate reference standard over part of this range. At high pressures above 20 bar,

a gold-chrome alloy resistance reference instrument is normally used. For low pressures in the range of 10^{-1} to 10^{-3} mbar, both the McLeod gauge and various forms of micromanometer are used as a pressure-measuring standard. At even lower pressures below 10^{-3} mbar, a pressure-dividing technique is used to establish calibration. This involves setting up a series of orifices of accurately known pressure ratio and measuring the upstream pressure with a McLeod gauge or micromanometer.

The limits of accuracy with which pressure can be measured by presently known techniques are as follows:

10^{-7} mbar	$\pm 4\%$
10^{-5} mbar	$\pm 2\%$
10^{-3} mbar	$\pm 1\%$
10^{-1} mbar	$\pm 0.1\%$
1 bar	$\pm 0.001\%$
10^4 bar	$\pm 0.1\%$

15.16.1 Reference calibration instruments

Dead-weight gauge (pressure balance)

The dead-weight gauge, also known by the alternative names of *piston gauge* and *pressure balance*, is shown in schematic form in Fig. 15.14. It is a null-reading type of measuring instrument in which weights are added to the piston platform until the piston is adjacent to a fixed reference mark, at which time the downward force of the weights on top of the piston is balanced by the pressure exerted by the fluid beneath the piston. The fluid pressure is therefore calculated in terms of the weight added to the platform and the known area of the piston. The instrument offers the ability to measure pressures to a high degree of accuracy, and it is widely used as a reference instrument against which other pressure-measuring instruments are calibrated in the midrange of pressures. Unfortunately, its mode of measurement makes it inconvenient to use and it is therefore rarely used except for calibration duties.

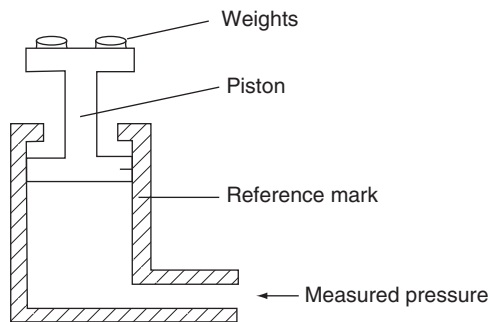


Figure 15.14

Schematic representation of dead-weight gauge.

Special precautions are necessary in the manufacture and use of dead-weight gauges. Friction between the piston and cylinder must be reduced to a very low level, otherwise a significant measurement error would result. Friction reduction is accomplished by designing for a small clearance gap between the piston and cylinder, by machining the cylinder to a slightly greater diameter than the piston. The piston and cylinder are also designed so that they can be turned relative to one another, which reduces friction still further. Unfortunately, as a result of the small gap between the piston and cylinder, there is a finite flow of fluid past the seals. This produces a viscous shear force that partly balances the dead weight on the platform. A theoretical formula exists for calculating the magnitude of this shear force, suggesting that exact correction can be made for it. In practice, however, the piston deforms under pressure and alters the piston/cylinder gap, and so the shear force calculation and correction can only be approximate.

In spite of these difficulties, the instrument gives a typical measurement inaccuracy of only $\pm 0.01\%$. It is normally used for calibrating pressures in the range of 20 mbar up to 20 bar. However, special versions can measure pressures down to 0.1 mbar or up to 7000 bar.

U-tube manometer

Besides its use for normal process measurements, the U-tube manometer is also used as a reference instrument for calibrating instruments measuring midrange pressures. Although it is a deflection rather than null type of instrument, it manages to achieve similar degrees of measurement accuracy to the dead-weight gauge, because of the error sources noted in the latter. The major source of error in U-tube manometers arises out of the difficulty in estimating the meniscus level of the liquid column accurately. There is also a tendency for the liquid level to creep up the tube by capillary action, which creates an additional source of error.

U-tubes for measuring high pressures become unwieldy because of the long lengths of liquid column and tube required. Consequently, U-tube manometers are normally used only for calibrating pressures at the lower end of the midpressure range.

Barometers

The most commonly used type of barometer for calibration duties is the Fortin barometer. This is a highly accurate instrument that provides measurement inaccuracy levels of between $\pm 0.03\%$ of full-scale reading and $\pm 0.001\%$ of full-scale reading depending on the measurement range. To achieve such levels of accuracy, the instrument has to be used under very carefully controlled conditions of lighting, temperature, and vertical alignment. It must also be manufactured to exacting standards and is therefore very expensive to buy. Corrections have to be made to the output reading according to the ambient temperature,

local value of gravity, and atmospheric pressure. Because of its expense and the difficulties in using it, the barometer is not normally used for calibration other than as a primary reference standard at the top of the calibration chain.

Vibrating cylinder gauge

The vibrating cylinder gauge, shown in Fig. 15.15, acts as a reference standard instrument for calibrating pressure measurements up to 3.5 bar. It consists of a cylinder in which vibrations at the resonant frequency are excited by a current-carrying coil. The pressure-dependent oscillation frequency is monitored by a pickup coil, and this frequency measurement is converted to a voltage signal by a microprocessor and signal-conditioning circuitry contained within the package. By evacuating the space on the outer side of the cylinder, the instrument is able to measure the absolute pressure of the fluid inside the cylinder. Measurement errors are less than 0.005% over the absolute pressure range up to 3.5 bar.

Gold-chrome alloy resistance instruments

For measuring pressures above 7000 bar, an instrument based on measuring the resistance change of a metal coil as the pressure varies is used, and the same type of instrument is also used for calibration purposes. Such instruments commonly use manganin or gold-chrome alloys for the coil. Gold chrome has a significantly lower temperature coefficient (i.e., its pressure/resistance characteristic is less affected by temperature changes), and is therefore the normal choice for calibration instruments in spite of its higher cost. An inaccuracy of only $\pm 0.1\%$ is achievable in such devices.

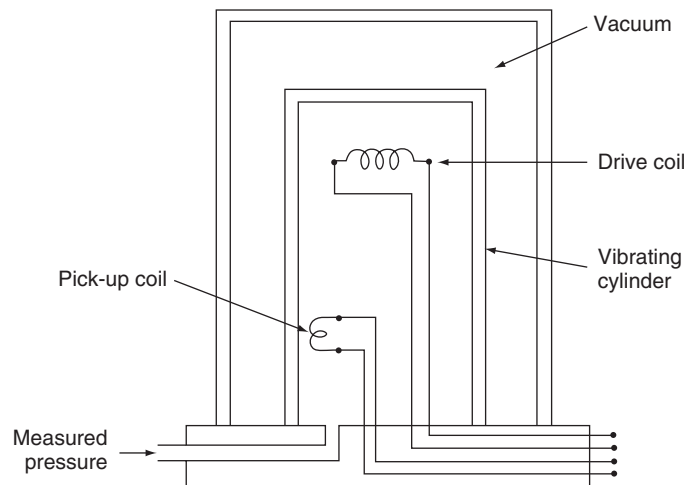


Figure 15.15
Vibrating cylinder gauge.

McLeod gauge

The McLeod gauge, which was discussed earlier in [Section 15.11](#), can be used for the calibration of instruments that are designed to measure low pressures between 10^{-4} mbar and 0.1 mbar (10^{-7} to 10^{-4} bar).

Ionization gauge

The ionization gauge is used to calibrate instruments measuring very low pressures in the range 10^{-13} to 10^{-3} bar. It has the advantage of having a straight-line relationship between output reading and pressure. Unfortunately, its inherent accuracy is relatively poor and specific points on its output characteristic have to be calibrated against a McLeod gauge.

Micromanometers

Micromanometers are instruments that work on the manometer principle but are specially designed to minimize capillary effects and meniscus reading errors. The centrifugal form of micromanometer, shown schematically in [Fig. 15.16](#), is the most accurate type for use as a calibration standard down to pressures of 10^{-3} mbar. In this, a rotating disc serves to amplify a reference pressure, with the speed of rotation being adjusted until the amplified pressure just balances the unknown pressure. This null position is detected by observing when oil droplets sprayed into a glass chamber cease to move. Measurement inaccuracy is $\pm 1\%$.

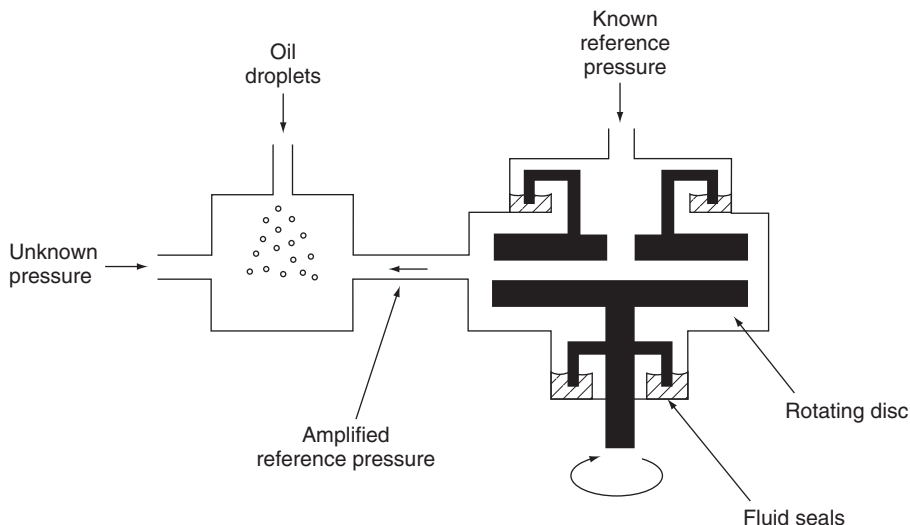


Figure 15.16
Centrifugal micromanometer.

Other types of micromanometer also exist, which give similar levels of accuracy, but only at somewhat higher pressure levels. These can be used as calibration standards at pressures up to 50 mbar.

15.16.2 Calculating frequency of calibration checks

Some pressure-measuring instruments are very stable and unlikely to suffer drift in characteristics with time. Devices in this class include resonant-wire devices, ionization gauges, and high-pressure instruments (those working on the principle of resistance change with pressure). All forms of manometers are similarly stable, although in the longer term small errors can develop in these through volumetric changes in the glass. Therefore, for all these instruments, only annual calibration checks are recommended, unless of course something happens to the instrument that puts its calibration into question.

However, most instruments used to measure pressure consist of an elastic element and a displacement transducer that measures its movement. Both of these component parts are mechanical in nature. Devices of this type include diaphragms, bellows, and Bourdon tubes. Such instruments can suffer changes in characteristics for a number of reasons. One factor is the characteristics of the operating environment and the degree to which the instrument is exposed to it. Another reason is the amount of mishandling they receive. These parameters are entirely dependent upon the particular application the instrument is used in, and the frequency with which it is used and exposed to the operating environment. A suitable calibration frequency can therefore only be determined on an experimental basis.

A third class of instrument from the calibration-requirements viewpoint is the range of devices working on the thermal conductivity principle. This range includes the thermocouple gauge, Pirani gauge, and thermistor gauge. Such instruments have characteristics that vary with the nature of the gas being measured and must therefore be calibrated each time that they are used.

15.16.3 Procedures for calibration

Pressure calibration requires the output reading of the instrument being calibrated to be compared with the output reading of a reference-standard instrument when the same pressure is applied to both. This necessitates designing a suitable leakproof seal to connect the pressure-measuring chambers of the two instruments.

The calibration of pressure transducers used for process measurements often has to be carried out in situ in order to avoid serious production delays. Such devices are often

remote from the nearest calibration laboratory and to transport them there for calibration would take an unacceptably long time. Because of this, portable reference instruments have been developed for calibration at this level in the calibration chain. These use a standard air supply connected to an accurate pressure regulator to provide a range of reference pressures. An inaccuracy of $\pm 0.025\%$ is achieved when calibrating midrange pressures in this manner. Calibration at higher levels in the calibration chain must of course be carried out in a proper calibration laboratory maintained in the correct manner. However, irrespective of where calibration is carried out, several special precautions are necessary when calibrating certain types of instruments, as described in the following paragraphs.

U-tube manometers must have their vertical alignment carefully set up before use. Particular care must also be taken to ensure that there are no temperature gradients across the two halves of the tube. Such temperature differences would cause local variations in the specific weight of the manometer fluid, resulting in measurement errors. Correction must also be made for the local value of g (acceleration due to gravity). These comments apply similarly to the use of other types of manometer and micromanometer.

The existence of one potentially major source of error in *Bourdon tube* pressure measurement has not been widely documented, and few manufacturers of Bourdon tubes make any attempt to warn users of their products appropriately. This problem is concerned with the relationship between the fluid being measured and the fluid used for calibration. The pointer of Bourdon tubes is normally set at zero during manufacture, using air as the calibration medium. However, if a different fluid, especially a liquid, is subsequently used with a Bourdon tube, the fluid in the tube will cause a nonzero deflection according to its weight compared with air, resulting in a reading error of up to 6% of full-scale deflection.

This can be avoided by calibrating the Bourdon tube with the fluid to be measured instead of with air. Alternatively, correction can be made according to the calculated weight of the fluid in the tube. Unfortunately, difficulties arise with both of these solutions if air is trapped in the tube, since this will prevent the tube being filled completely by the fluid. Then, the amount of fluid actually in the tube, and its weight, will be unknown. To avoid this problem, at least one manufacturer now provides a bleed facility in the tube, allowing measurement uncertainties of less than 0.1% to be achieved.

When using a *McLeod gauge*, care must be taken to ensure that the measured gas does not contain any vapor. This would be condensed during the compression process, causing a measurement error. A further recommendation is the insertion of a liquid–air cold trap between the gauge and the instrument being calibrated to prevent the passage of mercury vapor into the latter.

15.17 Summary

We began this chapter by looking at the formal definitions of the three ways in which pressure is quantified in terms of absolute, gauge, and differential pressure. We then went on to look at the devices used for measuring pressure in three distinct ranges, normal, midrange pressures between 1.013 bar (the mean atmospheric pressure) and 7000 bar, low or vacuum pressures below 1.013 bar, and high pressures above 7000 bar.

We saw that a large number of devices are available for measurements in the “normal” range. Of these, sensors containing a diaphragm are the most commonly used. We looked at the type of material used for the diaphragm in diaphragm-based sensors and also examined the different ways in which diaphragm movement can be measured. These different ways of measuring diaphragm displacement give rise to a number of different names for diaphragm-based sensors, such as capacitive and fiber-optic (optical) sensors.

Moving on, we examined various other devices that are used to measure midrange pressures. These included bellows sensors, Bourdon tubes, and several types of manometer and resonant-wire sensors. We also looked at the range of devices commonly called digital (or electronic) pressure gauges. Many of these are diaphragm-based sensors that use an electronic means of measuring diaphragm displacement and include a digital output display, with names like piezoresistive pressure sensor, piezoelectric pressure sensor, magnetic pressure sensor, and potentiometric pressure sensor. This led to an overview of the rapidly growing area of diaphragm-based MEMS devices.

We then went on to study the measurement of low pressures. To start with, we observed that special forms of instruments commonly used to measure midrange pressures can also measure pressures below atmospheric pressure (Bourdon tubes down to 10 mbar, bellows-type instruments down to 0.1 mbar, manometers down to 0.1 mbar, and diaphragm-based sensors down to 0.001 mbar). As well as these special versions of Bourdon tube, etc., several other instruments have been specially developed to measure in the low-pressure range. These include thermocouple and thermistor gauges (measure between 10^{-4} and 1 mbar), the Pirani gauge (measures between 10^{-5} and 1 mbar), the McLeod gauge (measures down to 10^{-1} mbar, or even lower pressures if it is used in conjunction with pressure-dividing techniques), and the ionization gauge (measures from 10^{-10} mbar to 1 mbar).

When we looked at measurement of high pressures, we found that our choice of instrument was much more limited. All currently available instruments for this pressure range involve monitoring the change of resistance in a coil of wire made from special materials. The two usual devices of this type are the manganin-wire pressure sensor (more common) and gold-chromium-wire pressure sensor (less common).

The following three sections were devoted to intelligent pressure-measuring devices, instruments measuring differential pressure, and some guidance about which type of device to use in particular circumstances.

Our final subject of study in the chapter was the means of calibrating pressure-measuring devices. We looked at various instruments that are used for calibration, including the dead-weight gauge, special forms of the U-tube manometer, barometers, the vibrating cylinder gauge, gold-chrome alloy resistance instruments, the McLeod gauge, the ionization gauge, and micromanometers. We then considered how the frequency of recalibration should be determined for various kinds of pressure-measuring device. Finally, we looked in more detail at the appropriate practical procedures and precautions that should be taken for calibrating different types of instruments.

15.18 Problems

- 15.1 Explain the difference between absolute pressure, gauge pressure, and differential pressure. When pressure readings are being written down, what is the mechanism for defining whether the value is a gauge, absolute, or differential pressure?
- 15.2 Give examples of situations where pressure measurements are normally given as (a) absolute pressure, (b) gauge pressure, and (c) differential pressure.
- 15.3 Summarize the main classes of device used for measuring absolute pressure.
- 15.4 Summarize the main classes of device used for measuring gauge pressure.
- 15.5 Summarize the main classes of device used for measuring differential pressure.
- 15.6 Explain what a diaphragm pressure sensor is. What are the different materials used in the construction of a diaphragm pressure sensor and what are their relative merits?
- 15.7 Strain gauges are commonly used to measure the displacement in a diaphragm pressure sensor. What are the difficulties in bonding a standard strain gauge to the diaphragm and how are these difficulties usually solved?
- 15.8 What are the advantages in using a monolithic piezoresistive displacement transducer with diaphragm pressure sensors?
- 15.9 What other types of device apart from strain gauges are used to measure the displacement in a diaphragm strain gauge? Summarize the main features of each of these alternative types of displacement sensor.
- 15.10 Discuss the mode of operation of fiber-optic pressure sensors. What are their principal advantages over other forms of pressure sensor?
- 15.11 What are bellows pressure sensors? How do they work? Describe some typical applications.
- 15.12 How does a Bourdon tube work? What are the three common shapes of Bourdon tubes and what is the typical measurement range of each type?

- 15.13 Describe the three types of manometers available. What is the typical measurement range of each type?
- 15.14 What is a resonant-wire pressure-measuring device and what is it typically used for?
- 15.15 What is an electronic pressure gauge? Discuss the different types of electronic gauges that exist.
- 15.16 Describe the structure of MEMS pressure sensors (including a simple sketch). What are their main characteristics?
- 15.17 What are the relevant criteria in choosing a pressure sensor for a particular application?
- 15.18 Discuss the range of instruments that are available for measuring very low pressures (pressures below atmospheric pressure).
- 15.19 How are high pressures (>7000 bar) normally measured?
- 15.20 What advantages do intelligent pressure transducers have over their nonintelligent counterparts?
- 15.21 A differential pressure can be measured by subtracting the readings from two separate pressure transducers. What is the problem with this? Suggest a better way of measuring differential pressures.
- 15.22 How are pressure transducers calibrated? How is a suitable frequency of calibration determined?
- 15.23 Which instruments are used as a reference standard in the calibration of pressure sensors that are designed to measure midrange pressures (0.1 mbar–20 bar)? Sketch their structure and discuss their characteristics.
- 15.24 Carry out an internet search for MEMS pressure sensors, documenting the structure and characteristics of alternative types and noting current applications.
- 15.25 Which instruments are used as a reference standard in the calibration of pressure sensors that are designed to measure (a) high pressures (>20 bar) and (b) low pressures (<0.1 mbar)? Sketch their structure and discuss their characteristics.

Flow measurement

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16.1 Introduction

We now move on to look at flow measurement in this chapter. Flow measurement is concerned with quantifying the rate of flow of materials. Such measurement is quite a common requirement in the process industries. The material measured may be in either a solid, liquid, or gaseous state. When the material is in a solid state, the flow can only be quantified as the mass flow rate, this being the mass of material that flows in one unit of time. When the material is in a liquid or gaseous state, the flow can be quantified either as the mass flow rate or the volume flow rate, the latter being the volume of material that flows in one unit of time. Of the two, a flow measurement in terms of the mass flow rate is preferred if very accurate measurement is required. The greater accuracy of mass flow measurement arises from the fact that mass is invariant whereas volume is a variable quantity.

A particular complication in the measurement of flow rate of liquids and gases that are flowing in pipes is the need to consider whether the flow is laminar or turbulent. Laminar flow is characterized by a motion of the fluid being in a direction parallel to the sides of the pipe, and it occurs in straight lengths of pipe when the fluid is flowing at a low velocity. However, it should be noted that even laminar flow is not uniform across the cross section of the pipe, with the velocity being greatest at the center of the pipe and decreasing to zero immediately next to the wall of the pipe. By contrast, turbulent flow involves a complex pattern of flow that is not in a uniform direction. Turbulent flow occurs in nonstraight sections of pipe and also occurs in straight sections when the fluid velocity exceeds a critical value. Because of the difficulty in measuring turbulent flow, the usual practice is to restrict flow measurement to places where the flow is laminar, or at least approximately laminar. This can be achieved by measuring the flow in the center of a long, straight length of pipe if the flow velocity is below the critical velocity for turbulent flow. In the case of high mean fluid velocity, it is often possible to find somewhere within the flow path where a larger diameter pipe exists and therefore the flow velocity is lower.

16.2 Mass flow rate

The method used to measure mass flow rate is determined by whether the measured quantity is in a solid, liquid, or gaseous state, since different techniques are appropriate for each. The main techniques available for measuring mass flow rate are summarized below.

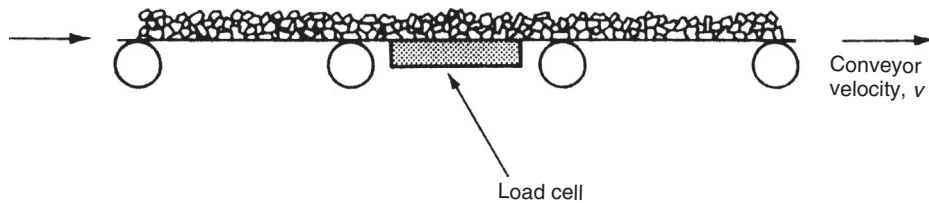


Figure 16.1

Conveyor-based mass flow rate measurement.

16.2.1 Conveyor-based methods

Conveyor-based methods are appropriate for measuring the flow of solids that are in the form of powders or small granular particles. Such powders and particles are commonly produced by crushing or grinding procedures in process industries, and a conveyor is a very suitable means of transporting materials in this form. Transporting materials on a conveyor allows the mass flow rate to be calculated in terms of the mass of material on a given length of conveyor multiplied by the speed of the conveyor. Fig. 16.1 shows a typical measurement system. A load cell measures the mass M of material distributed over a length L of the conveyor. If the conveyor velocity is v , the mass flow rate Q is given by:

$$Q = Mv/L$$

As an alternative to weighing the flowing material, a *nuclear mass-flow sensor* can be used, in which a gamma-ray source is directed at the material being transported along the conveyor. The material absorbs some radiation, and the amount of radiation received by a detector on the other side of the material indicates the amount of material on the conveyor. This technique has obvious safety concerns, and is therefore subject to licensing and strict regulation.

16.2.2 Coriolis flowmeter

As well as sometimes being known by the alternative name of *inertial flowmeter*, the Coriolis flowmeter is often referred to simply as a *mass flowmeter* because of its dominance in the mass flowmeter market. However, this assumption that a mass flowmeter always refers to a Coriolis meter is wrong since several other types of device are available to measure mass flow, although it is true to say that they are much less common than Coriolis meters.

Coriolis meters are primarily used to measure the mass flow rate of liquids, although they have also been successfully used in some gas-flow measurement applications. The flowmeter consists either of a pair of parallel vibrating tubes or else as a single vibrating tube that is formed into a configuration that has two parallel sections. The two vibrating

tubes (or the two parallel sections of a single tube) deflect according to the mass flow rate of the measured fluid that is flowing inside. Tubes are made of various materials, of which stainless steel is the most common. They are also manufactured in different shapes such as B-shaped, D-shaped, U-shaped, triangular-shaped, helix-shaped, and straight. These alternative shapes are sketched in Fig. 16.2A, and a U-shaped tube is shown in more detail in Fig. 16.2B. The tubes are anchored at two points. An electromechanical drive unit, positioned midway between the two anchors, excites vibrations in each tube at the tube resonant frequency. The vibrations in the two tubes, or the two parallel sections of a single tube, are 180 degrees out of phase. The vibratory motion of each tube causes forces on the particles in the flowing fluid. These forces induce motion of the fluid particles in a direction that is orthogonal to the direction of flow, and this produces a Coriolis force. This Coriolis force causes a deflection of the tubes that is superimposed on top of the vibratory motion. The net deflection of one tube relative to the other is given by $d = kfR$,

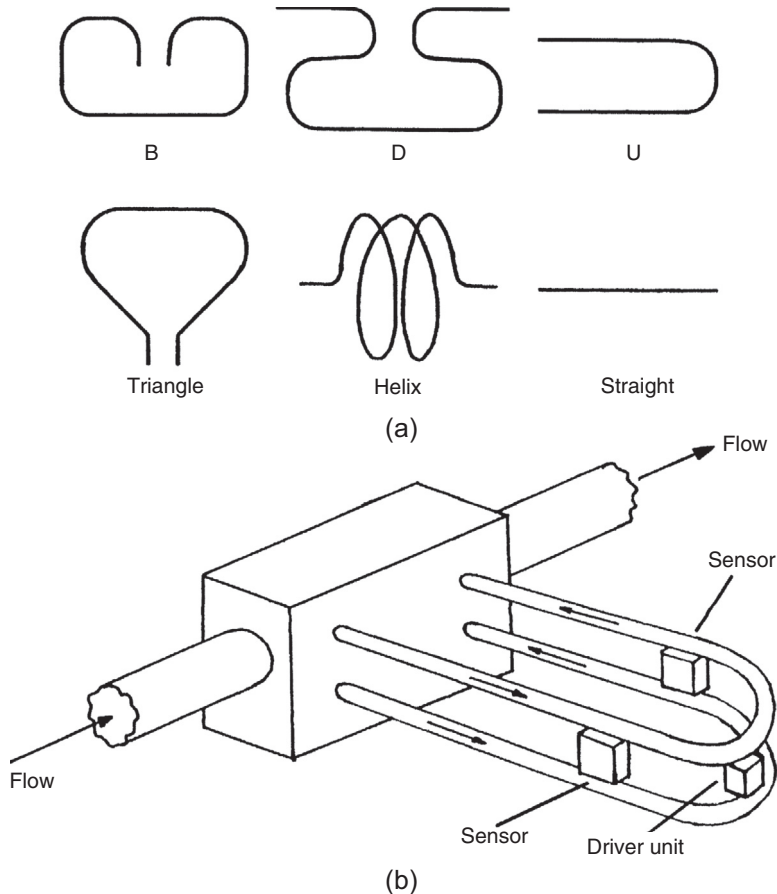


Figure 16.2

(A) Coriolis flowmeter shapes; (B) Detail of U-shaped Coriolis flowmeter.

where k is a constant, f is the frequency of the tube vibration, and R is the mass flow rate of the fluid inside the tube. This deflection is measured by a suitable sensor.

Coriolis meters give excellent accuracy, with measurement uncertainties of $\pm 0.2\%$ being typical. They also have low maintenance requirements. However, apart from being expensive (typical cost is \$6000), they suffer from a number of operational problems. Failure may occur after a period of use because of mechanical fatigue in the tubes. Tubes are also subject to both corrosion caused by chemical interaction with the measured fluid and abrasion caused by particles within the fluid. Diversion of the flowing fluid around the flowmeter causes it to suffer a significant pressure drop, though this is much less evident in straight tube designs.

16.2.3 Thermal mass flow measurement

Thermal mass flowmeters are primarily used to measure the flow rate of gases. The structure of the device is shown in Fig. 16.3. The principle of operation is to direct the flowing material past a heated element and measure the temperature of the flowing material before and after the heater. The temperatures either side of the heater are commonly measured by a platinum resistance thermometer inside a protective sheath. The mass flow rate is inferred in one of two ways: (1) by measuring the temperature rise in the flowing material or (2) by measuring the heater power required to achieve a constant set temperature difference in the flowing material. In both cases, the specific heat and density of the flowing fluid must be known. Typical measurement uncertainty is $\pm 2\%$. Standard instruments require the measured gas to be clean and noncorrosive. However, versions made from special alloys can cope with more aggressive gases.

MEMS thermal mass flowmeters are also now available. These have the same basic structure as shown in Fig. 16.3, but of course the heater and temperature sensors are both microelectrical mechanical systems (MEMS) scale and contained within a tiny package. The temperature sensors used are thermopile devices of the form shown in Fig. 14.9.

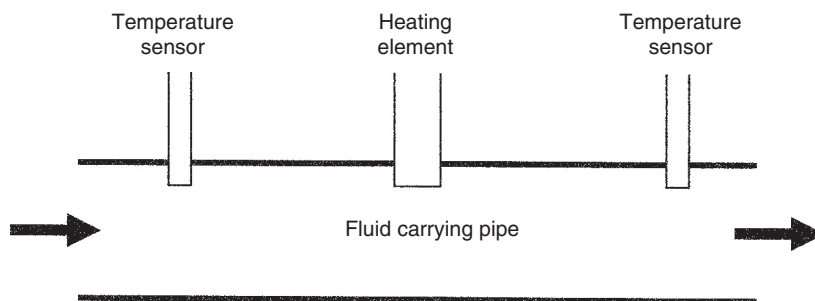


Figure 16.3
Structure of thermal mass flowmeter.

MEMS thermal mass flowmeters have a number of advantages over other forms of diaphragm-based flowmeter, including small size, fast response time, high accuracy, low pressure drop across the device, very low power consumption, and high reliability due to the absence of moving parts. Flow rates up to 100 L per minute can typically be measured. However, since this form of sensor also has excellent sensitivity to very small flow rates, devices are available that can measure flow rates in the range of microliters (10^{-6} L) per minute or nanoliters (10^{-9} L) per minute.

16.2.4 Joint measurement of volume flow rate and fluid density

Before the advent of the Coriolis meter, the usual way of measuring mass flow rate was to compute this from separate, simultaneous measurements of the volume flow rate and the fluid density. In many circumstances, this is still the cheapest option, although measurement accuracy is substantially inferior to that provided by a Coriolis meter.

16.3 Volume flow rate

Volume flow rate is an appropriate way of quantifying the flow of all materials that are in a gaseous, liquid, or semiliquid slurry form (where solid particles are suspended in a liquid host), although measurement accuracy is inferior to mass flow measurement as noted earlier. Materials in these forms are usually carried in pipes, and various instruments can be used to measure the volume flow rate as described below. As noted in the introduction, these all assume laminar flow. In addition, flowing liquids are sometimes carried in an open channel, in which case the volume flow rate can be measured by an open channel flowmeter.

16.3.1 Differential pressure (obstruction-type) meters

Differential pressure meters involve the insertion of some device into a fluid-carrying pipe that causes an obstruction and creates a pressure difference on either side of the device. Such meters are sometimes known as obstruction-type meters or flow-restriction meters. Devices used to obstruct the flow include the *orifice plate*, the *Venturi tube*, the *flow nozzle*, and the *Dall flow tube*, as illustrated in Fig. 16.4. When such a restriction is placed in a pipe, the velocity of the fluid through the restriction increases and the pressure decreases. The volume flow rate is then proportional to the square root of the pressure difference across the obstruction. The manner in which this pressure difference is measured is important. Measuring the two pressures with different instruments and calculating the difference between the two measurements is not satisfactory because of the large measurement error that can arise when the pressure difference is small, as explained in Chapter 4. Therefore, the normal procedure is to use a differential pressure transducer, which is commonly a diaphragm-type device.

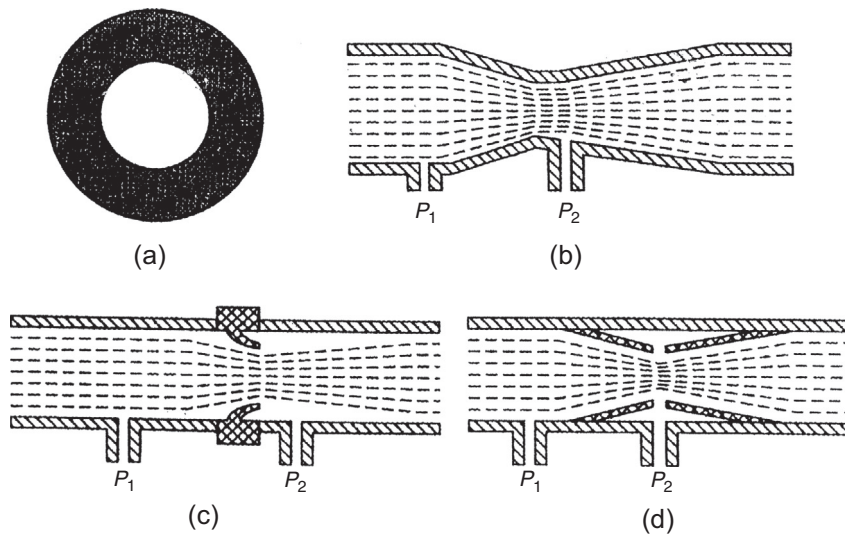


Figure 16.4

Obstruction devices: (A) Orifice plate; (B) Venturi; (C) Flow nozzle; (D) Dall flow tube.

The *Pitot static tube* is a further device that measures flow by creating a pressure difference within a fluid-carrying pipe. However, in this case, there is negligible obstruction of flow in the pipe. The Pitot tube is a very thin tube that obstructs only a small part of the flowing fluid and thus measures flow at a single point across the cross section of the pipe. This measurement only equates to average flow velocity in the pipe for the case of uniform flow. The *Annubar* is a type of multiport Pitot tube that does measure the average flow across the cross section of the pipe by forming the mean value of several local flow measurements across the cross section of the pipe.

All applications of this method of flow measurement assume laminar flow by ensuring that the flow conditions upstream of the obstruction device are in steady state, and a certain minimum length of straight run of pipe ahead of the flow measurement point is specified to achieve this. The minimum lengths required for various pipe diameters are specified in standards tables. However, a useful rule of thumb widely used in the process industries is to specify a length of 10 times the pipe diameter. If physical restrictions make this impossible to achieve, special flow smoothing vanes can be inserted immediately ahead of the measurement point.

Flow-restriction-type instruments are popular because they have no moving parts and are therefore robust, reliable, and easy to maintain. However, one significant disadvantage of this method is that the obstruction causes a permanent loss of pressure in the flowing fluid. The magnitude and hence importance of this loss depends on the type of obstruction element used, but where the pressure loss is large, it is sometimes necessary to recover the

lost pressure by an auxiliary pump further down the flow line. This class of device is not normally suitable for measuring the flow of slurries, as the tappings into the pipe to measure the differential pressure are prone to blockage, although the Venturi tube can be used to measure the flow of dilute slurries.

Fig. 16.5 illustrates approximately the way in which the flow pattern is interrupted when an orifice plate is inserted into a pipe. The other obstruction devices also have a similar effect to this although the magnitude of pressure loss is smaller. Of particular interest is the fact that the minimum cross-sectional area of flow occurs not within the obstruction but at a point downstream of there. Knowledge of the pattern of pressure variation along the pipe, as shown in Fig. 16.6, is also of importance in using this technique of volume-flow-rate measurement. This shows that the point of minimum pressure coincides with the point of minimum cross-section flow, a little way downstream of the obstruction. Fig. 16.6 also shows that there is a small rise in pressure immediately before the obstruction. It is therefore important not only to position the instrument measuring P_2 exactly at the point of minimum pressure but also to measure the pressure P_1 at a point upstream of the point where the pressure starts to rise before the obstruction.

In the absence of any heat transfer mechanisms, and assuming frictionless flow of an incompressible fluid through the pipe, the theoretical volume flow rate of the fluid, Q , is given by:

$$Q = \left[\frac{A_2}{\sqrt{1 - (A_2/A_1)^2}} \right] \left[\sqrt{\frac{2(P_1 - P_2)}{\rho}} \right] \quad (16.1)$$

where A_1 and P_1 are the cross-sectional area and pressure of the fluid flow before the obstruction, A_2 and P_2 are the cross-sectional area and pressure of the fluid flow at the narrowest point of the flow beyond the obstruction, and ρ is the fluid density.

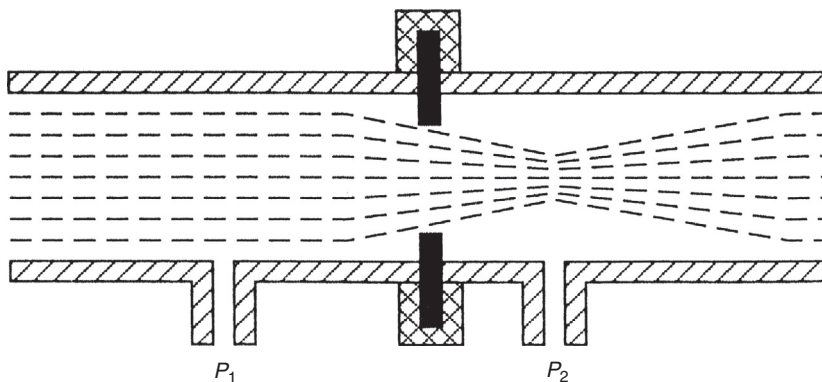


Figure 16.5
Profile of flow across orifice plate.

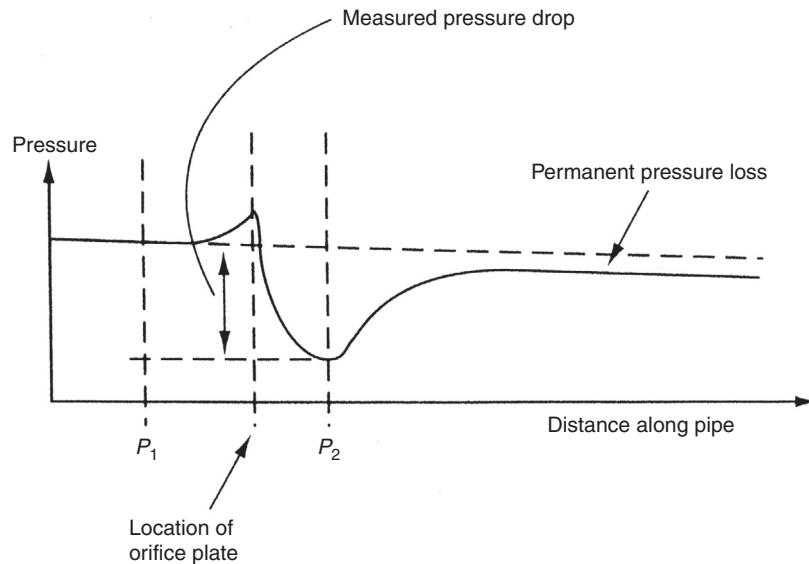


Figure 16.6
Pattern of pressure variation either side of orifice plate.

Eq. (16.1) is never entirely applicable in practice for two main reasons. Firstly, the flow is always impeded by a friction force, which varies according to the type of fluid and its velocity and is quantified by a constant known as the Reynold's number. Secondly, the cross-sectional area of the fluid flow ahead of the obstruction device is less than the diameter of the pipe carrying it, and the minimum cross-sectional area of the fluid after the obstruction is less than the diameter of the obstruction. This latter problem means that neither A_1 nor A_2 can be measured accurately. Fortunately, provided the pipe is smooth and therefore the friction force is small, these two problems can be adequately accounted for by applying a constant called the discharge coefficient. This modifies Eq. (16.1) to the following:

$$Q = \left[\frac{C_D A'_2}{\sqrt{1 - (A'_2/A'_1)^2}} \right] \left[\sqrt{\frac{2(P_1 - P_2)}{\rho}} \right] \quad (16.2)$$

where A'_1 and A'_2 are the actual pipe diameters before and at the obstruction and C_D is the discharge coefficient that corrects for the friction force and the difference between the pipe and flow cross-section diameters.

Before Eq. (16.2) can be evaluated, the discharge coefficient must be calculated. As this varies between each measurement situation, it would appear at first sight that the discharge coefficient must be determined by practical experimentation in every case. However, provided that certain conditions are met, standard tables can be used to obtain the value of the discharge coefficient appropriate to the pipe diameter and fluid involved.

One particular problem with all flow restriction devices is that the pressure drop ($P_1 - P_2$) varies as the square of the flow rate Q according to Eq. (16.2). The difficulty of measuring small pressure differences accurately has already been noted earlier. In consequence, the technique is only suitable for measuring flow rates that are between 30% and 100% of the maximum flow rate that a given device can handle. This means that alternative flow measurement techniques have to be used in applications where the flow rate can vary over a large range that can drop to below 30% of the maximum rate.

Orifice plate

The orifice plate is a metal disk with a concentric hole in it, which is inserted into the pipe carrying the flowing fluid. Orifice plates are simple, cheap, and available in a wide range of sizes. In consequence, they account for almost 50% of the instruments used in industry for measuring volume flow rate. One limitation of the orifice plate is that its inaccuracy is typically at least $\pm 2\%$ and may approach $\pm 5\%$. Also, the permanent pressure loss caused in the measured fluid flow is between 50% and 90% of the magnitude of the pressure difference ($P_1 - P_2$). Other problems with the orifice plate are a gradual change in the discharge coefficient over a period of time as the sharp edges of the hole wear away, and a tendency for any particles in the flowing fluid to stick behind the hole and thereby gradually reduce its diameter as the particles build up. The latter problem can be minimized by using an orifice plate with an eccentric hole. If this hole is close to the bottom of the pipe, solids in the flowing fluid tend to be swept through, and buildup of particles behind the plate is minimized. A very similar problem arises if there are any bubbles of vapor or gas in the flowing fluid when liquid flow is involved. These also tend to build up behind an orifice plate and distort the pattern of flow. This difficulty can be avoided by mounting the orifice plate in a vertical run of pipe.

Venturis and similar devices

A number of obstruction devices are available that are specially designed to minimize the pressure loss in the measured fluid. These have various names such as Venturi, flow nozzle, and Dall flow tube. They are all much more expensive than an orifice plate but have better performance. The smooth internal shape means that they are not prone to solid particles or bubbles of gas sticking in the obstruction, as is likely to happen in an orifice plate. The smooth shape also means that they suffer much less wear, and consequently have a longer life than orifice plates. They also require less maintenance and give greater measurement accuracy.

Venturi: The Venturi has a precision-engineered tube of a special shape. This offers measurement uncertainty of only $\pm 1\%$. However, the complex machining required to manufacture it means that it is the most expensive of all the obstruction devices discussed. Permanent pressure loss in the measured system is 10%–15% of the pressure difference ($P_1 - P_2$) across it.

Dall flow tube: The Dall flow tube consists of two conical reducers inserted into the fluid-carrying pipe. It has a very similar internal shape to the Venturi, except that it lacks a throat. This construction is much easier to manufacture and this gives the Dall flow tube an advantage in cost over the Venturi, although the typical measurement inaccuracy is a little higher ($\pm 1.5\%$). Another advantage of the Dall flow tube is its shorter length, which makes the engineering task of inserting it into the flow line easier. The Dall tube has one further operational advantage, in that the permanent pressure loss imposed on the measured system is only about 5% of the measured pressure difference ($P_1 - P_2$).

Flow nozzle: The flow nozzle is of simpler construction still, and is therefore cheaper than either a Venturi or a Dall flow tube, but the pressure loss imposed on the flowing fluid is 30%–50% of the measured pressure difference ($P_1 - P_2$) across the nozzle.

Pitot static tube

The Pitot static tube is mainly used for making temporary measurements of flow, although it is also used in some instances for permanent flow monitoring. It measures the local velocity of flow at a particular point within a pipe rather than the average flow velocity as measured by other types of flowmeter. This may be very useful where there is a requirement to measure local flow rates across the cross section of a pipe in the case of nonuniform flow. Multiple Pitot tubes are normally used to do this.

The instrument depends on the principle that a tube placed with its open end in a stream of fluid, as shown in Fig. 16.7, will bring to rest that part of the fluid that impinges on it, and the loss of kinetic energy will be converted to a measurable increase in pressure inside the tube. This pressure (P_1), as well as the static pressure of the undisturbed free stream of flow (P_2), is measured. The flow velocity can then be calculated from the formula:

$$v = C\sqrt{2g(P_1 - P_2)}$$

The constant C , known as the Pitot tube coefficient, is a factor that corrects for the fact that not all fluid incident on the end of the tube will be brought to rest: a proportion will

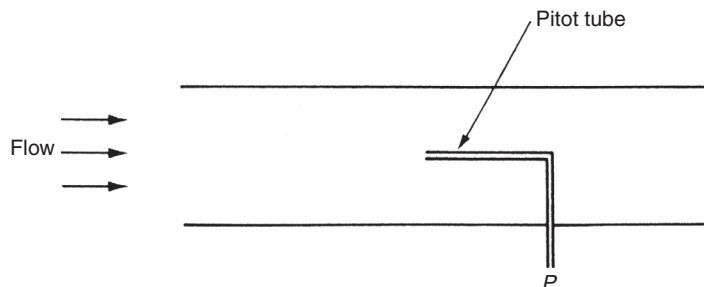


Figure 16.7
Pitot tube.

slip around it according to the design of the tube. Having calculated v , the volume flow rate can then be calculated by multiplying v by the cross-sectional area of the flow pipe, A .

Pitot tubes have the advantage that they cause negligible pressure loss in the flow. They are also cheap, and the installation procedure consists of the very simple process of pushing them down a small hole drilled in the flow-carrying pipe. Their main failing is that the measurement inaccuracy is typically about $\pm 5\%$, although more expensive versions can reduce inaccuracy down to $\pm 1\%$. The *annubar* is a development of the Pitot tube that has multiple sensing ports distributed across the cross section of the pipe and thus provides an approximate measurement of the mean flow rate across the pipe.

16.3.2 Variable area flowmeters (Rotameters)

In the variable area flowmeter (sometimes known as a Rotameter), the differential pressure across a variable aperture is used to adjust the area of the aperture. The aperture area is then a measure of the flow rate. The instrument is reliable and cheap and is used extensively throughout industry, accounting for about 20% of all flowmeters sold. Normally, this type of instrument only gives a visual indication of flow rate, and so it is of no use in automatic control schemes. However, special versions of variable area flowmeters are now available that incorporate fiber optics. In these, a row of fibers detects the position of the float by sensing the reflection of light from it, and an electrical signal output can be derived from this.

In its simplest form, shown in Fig. 16.8, the instrument consists of a tapered glass tube containing a float that takes up a stable position where its submerged weight is balanced by the upthrust due to the differential pressure across it. The position of the float is a measure of the effective annular area of the flow passage and hence of the flow rate. The inaccuracy of the cheapest instruments is typically $\pm 5\%$, but more expensive versions offer measurement inaccuracies as low as $\pm 0.5\%$.

16.3.3 Positive displacement flowmeters

Positive displacement flowmeters account for nearly 10% of the total number of flowmeters used in industry and are used in large numbers for metering domestic gas and water consumption. The cheapest instruments have a typical inaccuracy of about $\pm 2\%$, but the inaccuracy in more expensive ones can be as low as $\pm 0.5\%$. These higher-quality instruments are used extensively within the oil industry, as such applications can justify the high cost of such instruments.

All positive displacement meters operate by using mechanical divisions to displace discrete volumes of fluid successively. While this principle of operation is common, many different mechanical arrangements exist for putting the principle into practice. However,

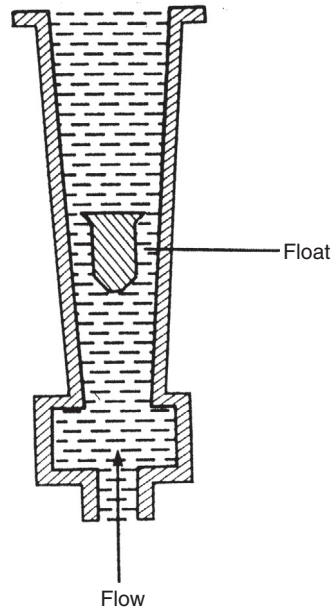


Figure 16.8
Variable area flowmeter.

all versions of positive displacement meter are low friction, low maintenance, and long-life devices, although they do impose a small permanent pressure loss on the flowing fluid. Low friction is especially important when measuring gas flows, and meters with special mechanical arrangements to satisfy this requirement have been developed.

The **rotary piston meter** is a common type of positive displacement meter that is used particularly for the measurement of domestic water supplies. It consists, as shown in Fig. 16.9, of a slotted cylindrical piston moving inside a cylindrical working chamber that has an inlet port and an outlet port. The piston moves round the chamber such that its outer surface maintains contact with the inner surface of the chamber, and, as this happens, the piston slot slides up and down a fixed division plate in the chamber. At the start of each piston motion cycle, liquid is admitted to volume B from the inlet port. The fluid pressure causes the piston to start to rotate around the chamber, and, as this happens, liquid in volume C starts to flow out of the outlet port, and also liquid starts to flow from the inlet port into volume A. As the piston rotates further, volume B becomes shut off from the inlet port, while liquid continues to be admitted into A and pushed out of C. When the piston reaches the endpoint of its motion cycle, the outlet port is opened to volume B, and the liquid that has been transported round inside the piston is expelled. After this, the piston pivots about the contact point between the top of its slot and the division plate, and volume A effectively becomes volume C ready for the start of the next motion cycle. A peg on top of the piston causes a reciprocating motion of a lever attached

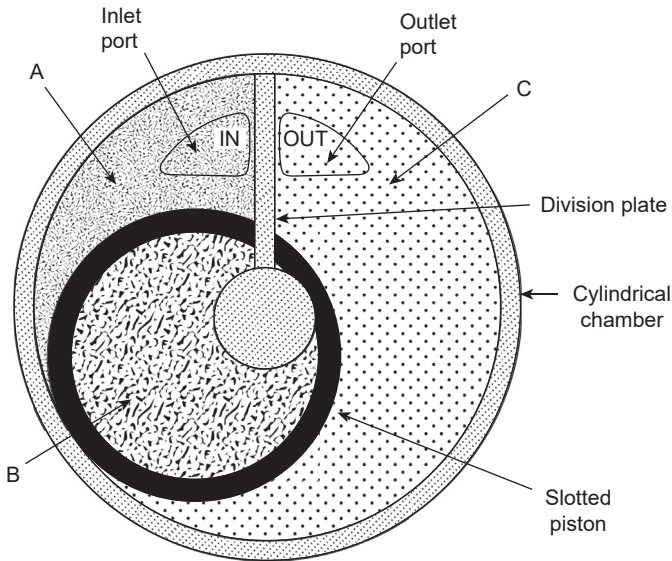


Figure 16.9

Rotary piston form of positive displacement flowmeter.

to it. This is made to operate a counter, and the flow rate is therefore determined from the count in unit time multiplied by the quantity (fixed) of liquid transferred between the inlet and outlet ports for each motion cycle.

The *nutating disk meter* is another form of positive displacement meter in which the active element is a disk inside a precision-machined chamber. Liquid flowing into the chamber causes the disk to nutate (wobble), and these nutations are translated into a rotary motion by a roller cam. Rotations are counted by a pulse transmitter that provides a measurement of the flow rate. This form of meter is noted for its ruggedness and long life. It has a typical measurement accuracy of $\pm 1.0\%$. It is commonly used for water supply measurement.

The *oval gear meter* is yet another form of positive displacement meter that has two oval-shaped gear wheels. It is used particularly for measuring the flow rate of high-viscosity fluids. It can also cope with measuring fluids that have variable viscosity.

16.3.4 Turbine meters

A turbine flowmeter consists of a multibladed wheel mounted in a pipe along an axis parallel to the direction of fluid flow in the pipe, as shown in Fig. 16.10. The flow of fluid past the wheel causes it to rotate at a rate that is proportional to the volume flow rate of the fluid. This rate of rotation has traditionally been measured by constructing the flowmeter such that it behaves as a variable reluctance tachogenerator. This is achieved by

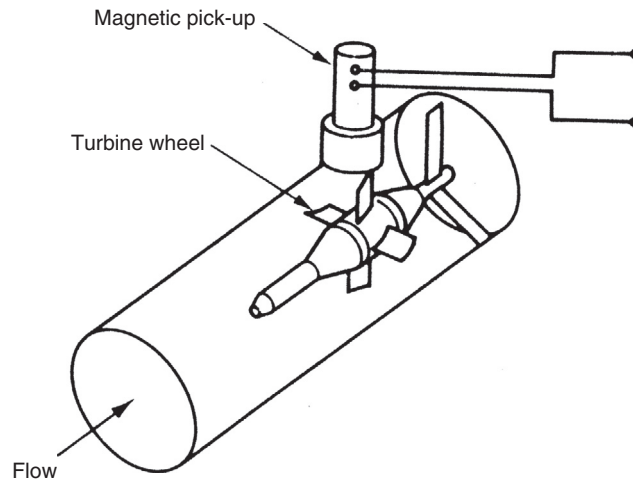


Figure 16.10
Turbine flowmeter.

fabricating the turbine blades from a ferromagnetic material and placing a permanent magnet and coil inside the meter housing. A voltage pulse is induced in the coil as each blade on the turbine wheel moves past it, and if these pulses are measured by a pulse counter, the pulse frequency and hence flow rate can be deduced. In recent instruments, fiber optics are also now sometimes used to count the rotations by detecting reflections off the tip of the turbine blades.

Provided that the turbine wheel is mounted in low-friction bearings, measurement inaccuracy can be as low as $\pm 0.2\%$. However, turbine flowmeters are less rugged and reliable than flow-restriction-type instruments, and are badly affected by any particulate matter in the flowing fluid. Bearing wear is a particular problem and they also impose a permanent pressure loss on the measured system. Turbine meters are particularly prone to large errors when there is any significant second phase in the fluid measured. For instance, using a turbine meter calibrated on pure liquid to measure a liquid containing 5% air produces a 50% measurement error. As an important application of the turbine meter is in the petrochemical industries, where gas/oil mixtures are common, special procedures are being developed to avoid such large measurement errors.

Readers may find reference in manufacturers' catalogs to a *Woltmann meter*. This is a type of turbine meter that has helical blades and is used particularly for measuring high flow rates. It is also sometimes known as a *helix meter*.

Turbine meters have a similar cost and market share to positive displacement meters, and compete for many applications, particularly in the oil industry. Turbine meters are smaller and lighter than the latter and are preferred for low-viscosity, high-flow measurements. However, positive displacement meters are superior in conditions of high viscosity and low flow rate.

16.3.5 Electromagnetic flowmeters

Electromagnetic flowmeters, sometimes known just as *magnetic flowmeters*, are limited to measuring the volume flow rate of electrically conductive fluids. The typical measurement inaccuracy of around $\pm 1\%$ is acceptable in many applications, but the instrument is expensive both in terms of the initial purchase cost and also in running costs, mainly due to its electricity consumption. A further reason for high cost is the need for careful calibration of each instrument individually during manufacture, as there is considerable variation in the properties of the magnetic materials used.

The instrument, shown in Fig. 16.11, consists of a stainless steel cylindrical tube, fitted with an insulating liner, which carries the measured fluid. Typical lining materials used are neoprene, polytetrafluoroethylene (PTFE) and polyurethane. A magnetic field is created in the tube by placing mains-energized field coils either side of it, and the voltage induced in the fluid is measured by two electrodes inserted into opposite sides of the tube. The ends

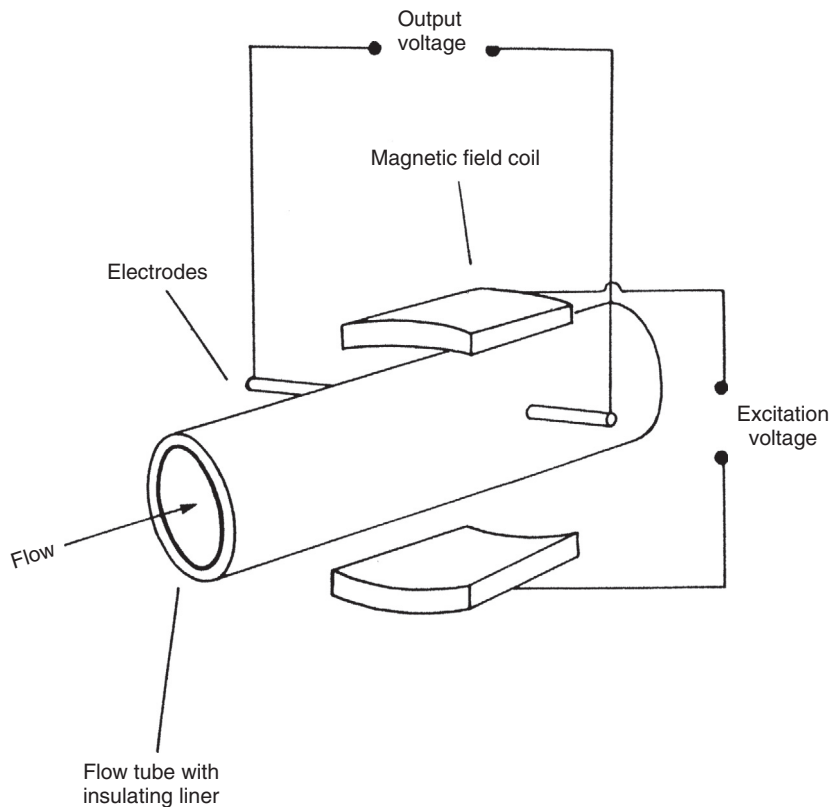


Figure 16.11
Electromagnetic flowmeter.

of these electrodes are usually flush with the inner surface of the cylinder. The electrodes are constructed from a material that is unaffected by most types of flowing fluid, such as stainless steel, platinum-iridium alloys, Hastelloy, titanium, and tantalum. In the case of the rarer metals in this list, the electrodes account for a significant part of the total instrument cost.

By Faraday's law of electromagnetic induction, the voltage, E , induced across a length, L , of the flowing fluid moving at velocity, v , in a magnetic field of flux density, B , is given by:

$$E = BLv \quad (16.3)$$

L is the distance between the electrodes, which is the diameter of the tube, and B is a known constant. Hence, measurement of the voltage E induced across the electrodes allows the flow velocity v to be calculated from Eq. (16.3). Having thus calculated v , it is a simple matter to multiply v by the cross-sectional area of the tube to obtain a value for the volume flow rate. The typical voltage signal measured across the electrodes is 1 mV when the fluid flow rate is 1 m/s.

The internal diameter of electromagnetic flowmeters is normally the same as that of the rest of the flow-carrying pipework in the system. Therefore, there is no obstruction to the fluid flow and consequently no pressure loss associated with measurement. Like other forms of flowmeter, the electromagnetic type requires a minimum length of straight pipework immediately prior to the point of flow measurement in order to guarantee the accuracy of measurement, although a length equal to five pipe diameters is usually sufficient.

While the flowing fluid must be electrically conductive, the method is of use in many applications and is particularly useful for measuring the flow of slurries in which the liquid phase is electrically conductive. Corrosive fluids can be handled providing a suitable lining material is used. At the present time, electromagnetic flowmeters account for about 15% of the new flowmeters sold, and this total is slowly growing. One operational problem is that the insulating lining is subject to damage when abrasive fluids are being handled, and this can give the instrument a limited life.

New developments in electromagnetic flowmeters are producing instruments that are physically smaller than before. Also, by employing better coil designs, electricity consumption is reduced. This means that battery-powered versions are now commercially available. Also, whereas conventional electromagnetic flowmeters require a minimum fluid conductivity of $10 \mu\text{mho}/\text{cm}^3$, new versions can cope with fluid conductivities as low as $1 \mu\text{mho}/\text{cm}^3$.

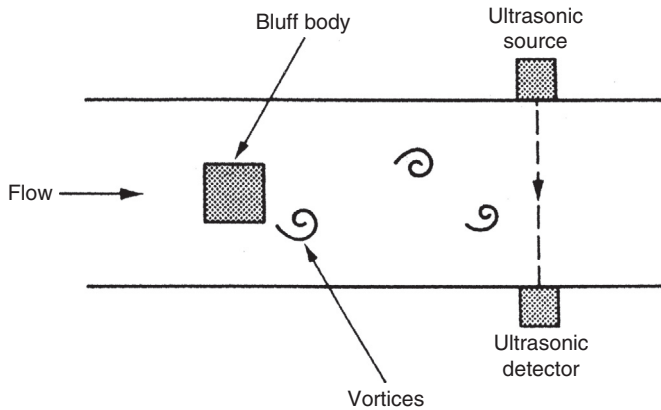


Figure 16.12
Vortex-shedding flowmeter.

16.3.6 Vortex-shedding flowmeters

The vortex-shedding flowmeter is used as an alternative to traditional differential pressure meters in many applications. The operating principle of the instrument is based on the natural phenomenon of vortex shedding, created by placing an unstreamlined obstacle (known as a bluff body) in a fluid-carrying pipe, as indicated in Fig. 16.12. When fluid flows past the obstacle, boundary layers of viscous, slow-moving fluid are formed along the outer surface. Because the obstacle is not streamlined, the flow cannot follow the contours of the body on the downstream side, and the separate layers become detached and roll into eddies or vortices in the low-pressure region behind the obstacle. The shedding frequency of these alternately shed vortices is proportional to the fluid velocity past the body. Various thermal, magnetic, ultrasonic, and capacitive vortex detection techniques are employed in different instruments.

Such instruments have no moving parts, operate over a wide flow range, have a small power consumption, require little maintenance, and have a similar cost to measurement using an orifice plate. They can measure both liquid and gas flows and a common inaccuracy value quoted is $\pm 1\%$ of full scale reading, though this can be seriously downgraded in the presence of flow disturbances upstream of the measurement point, and a straight run of pipe before the measurement point of 50 pipe diameters is recommended. Another problem with the instrument is its susceptibility to pipe vibrations, although new designs are becoming available that have a better immunity to such vibrations.

16.3.7 Ultrasonic flowmeters

The ultrasonic technique of volume flow rate measurement is, like the magnetic flowmeter, a noninvasive method. It is not restricted to conductive fluids, however, and is particularly

useful for measuring the flow of corrosive fluids and slurries. Besides its high reliability and low maintenance requirements, a further advantage of an ultrasonic flowmeter over an electromagnetic flowmeter is that the instrument can be clamped externally onto existing pipework rather than being inserted as an integral part of the flow line. As the procedure of breaking into a pipeline to insert a flowmeter can be as expensive as the cost of the flowmeter itself, the ultrasonic flowmeter has enormous cost advantages. Its clamp-on mode of operation also has significant safety advantages in avoiding the possibility of personnel installing flowmeters coming into contact with hazardous fluids such as poisonous, radioactive, flammable, or explosive ones. Also, any contamination of the fluid being measured (e.g., food substances and drugs) is avoided. Ultrasonic meters are still less common than differential pressure or electromagnetic flowmeters, though usage continues to expand year by year.

Two different types of ultrasonic flowmeter exist that employ distinct technologies, one based on Doppler shift and the other on transit time. In the past, the existence of these alternative technologies has not always been readily understood, and has resulted in ultrasonic technology being rejected entirely when one of these two forms has been found to be unsatisfactory in a particular application. This is unfortunate because the two technologies have distinct characteristics and areas of application, and many situations exist where one form is very suitable and the other not suitable. To reject both, having only tried out one, is therefore a serious mistake. Recently, ultrasonic flowmeters have become available that combine both Doppler shift and transit time technologies.

Particular care has to be taken to ensure a stable flow profile in ultrasonic flowmeter applications. It is usual to increase the normal specification of the minimum length of straight pipe run prior to the point of measurement, expressed as a number of pipe diameters, from a value of 10 up to 20 or in some cases even 50 diameters. Analysis of the reasons for poor performance in many instances of ultrasonic flowmeter application has shown failure to meet this stable flow-profile requirement to be a significant factor.

Doppler shift ultrasonic flowmeter

The principle of operation of the Doppler shift flowmeter is shown in [Fig. 16.13](#).

A fundamental requirement of these instruments is the presence of scattering elements within the flowing fluid, which deflect the ultrasonic energy output from the transmitter such that it enters the receiver. These can be provided by either solid particles, gas bubbles, or eddies in the flowing fluid. The scattering elements cause a frequency shift between the transmitted and reflected ultrasonic energy, and measurement of this shift enables the fluid velocity to be inferred.

The instrument consists essentially of an ultrasonic transmitter-receiver pair clamped onto the outside wall of a fluid-carrying vessel. Ultrasonic energy consists of a train of short

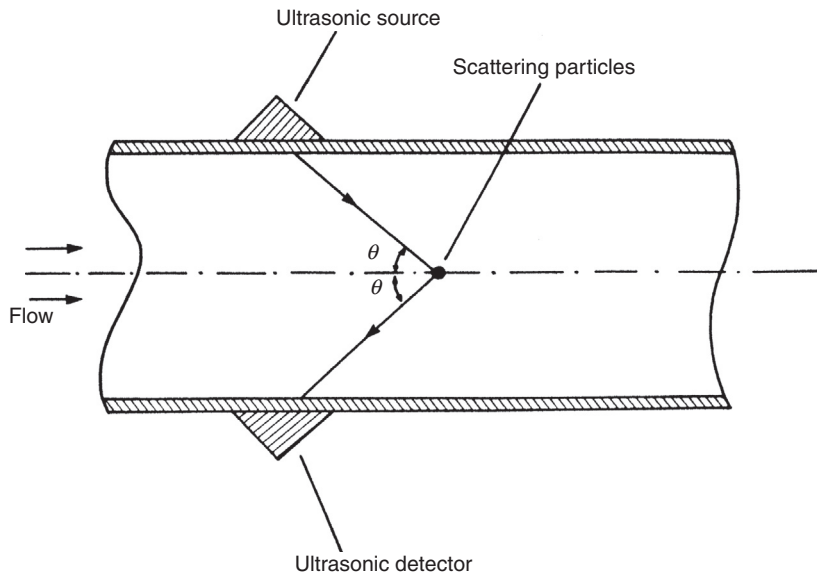


Figure 16.13
Doppler shift ultrasonic flowmeter.

bursts of sinusoidal waveforms at a frequency between 0.5 and 20 MHz. This frequency range is described as ultrasonic because it is outside the range of human hearing. The flow velocity, v , is given by:

$$v = \frac{c(f_t - f_r)}{2f_t \cos(\theta)} \quad (16.4)$$

where f_t and f_r are the frequencies of the transmitted and received ultrasonic waves, respectively, c is the velocity of sound in the fluid being measured, and θ is the angle that the incident and reflected energy waves make with the axis of flow in the pipe. Volume flow rate is then readily calculated by multiplying the measured flow velocity by the cross-sectional area of the fluid-carrying pipe.

The electronics involved in Doppler-shift flowmeters is relatively simple and therefore cheap. Ultrasonic transmitters and receivers are also relatively inexpensive, being based on piezoelectric oscillator technology. Therefore, as all of its components are cheap, the Doppler shift flowmeter itself is inexpensive. The measurement accuracy obtained depends on many factors such as the flow profile, the constancy of pipe-wall thickness, the number, size, and spatial distribution of scatterers, and the accuracy with which the speed of sound in the fluid is known. Consequently, accurate measurement can only be achieved by the tedious procedure of carefully calibrating the instrument in each particular flow measurement application. Otherwise, measurement errors can approach $\pm 10\%$ of the

reading, and for this reason Doppler-shift flowmeters are often used merely as flow indicators rather than for accurate quantification of the volume flow rate.

Versions are now available that are being fitted inside the flow pipe, flush with its inner surface. This overcomes the problem of variable pipe thickness, and an inaccuracy level as small as $\pm 0.5\%$ is claimed for such devices. Other recent developments are the use of multiple-path ultrasonic flowmeters that use an array of ultrasonic elements to obtain an average velocity measurement. This substantially reduces the error due to nonuniform flow profiles, but there is a substantial cost penalty involved in such devices.

Transit-time ultrasonic flowmeter

The transit-time ultrasonic flowmeter is an instrument that is designed for measuring the volume flow rate in clean liquids or gases. It consists of a pair of ultrasonic transducers mounted along an axis aligned at an angle θ with respect to the fluid-flow axis, as shown in Fig. 16.14.

Each transducer consists of a transmitter–receiver pair, with the transmitter emitting ultrasonic energy that travels across to the receiver on the opposite side of the pipe. These ultrasonic elements are normally piezoelectric oscillators of the same type as used in Doppler-shift flowmeters. Fluid flowing in the pipe causes a time difference between the transit times of the beams traveling upstream and downstream, and measurement of this difference allows the flow velocity to be calculated. The typical magnitude of this time difference is 100 ns in a total transit time of 100 μs , and high-precision electronics are therefore needed to measure the difference. There are three distinct ways of measuring the

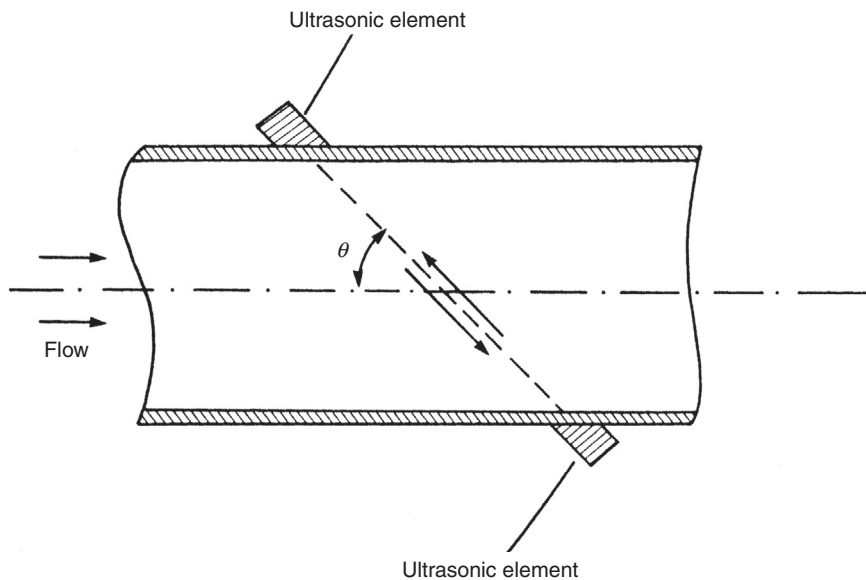


Figure 16.14

Transit-time ultrasonic flowmeter.

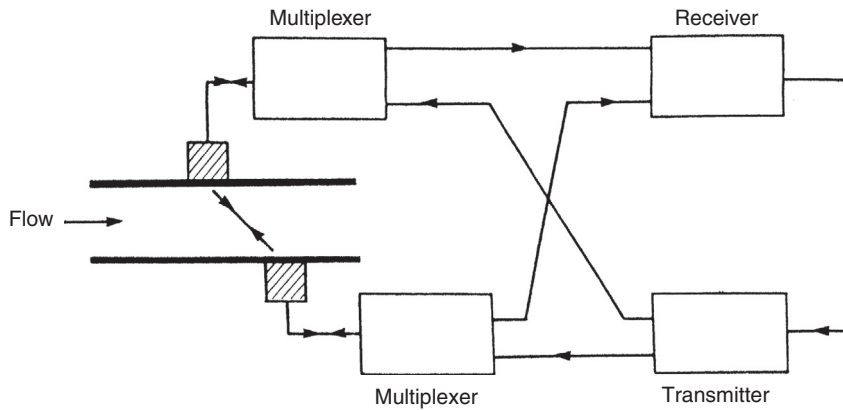


Figure 16.15
Transit-time measurement system.

time shift. These are direct measurement, conversion to a phase change, and conversion to a frequency change. The third of these options is particularly attractive, as it obviates the need to measure the speed of sound in the measured fluid as required by the first two methods. A scheme applying this third option is shown in Fig. 16.15. This also multiplexes the transmitting and receiving functions, so that only one ultrasonic element is needed in each transducer. The forward and backward transit times across the pipe, T_f and T_b , are given by:

$$T_f = \frac{L}{c + v \cos(\theta)} \quad ; \quad T_b = \frac{L}{c - v \cos(\theta)}$$

where c is the velocity of sound in the fluid, v is the flow velocity, L is the distance between the ultrasonic transmitter and receiver, and θ is the angle of the ultrasonic beam with respect to the fluid flow axis.

The time difference δT is given by:

$$\delta T = T_b - T_f = \frac{2vL \cos(\theta)}{c^2 - v^2 \cos^2(\theta)}$$

This requires knowledge of c before it can be solved. However, a solution can be found much more simply if the receipt of a pulse is used to trigger the transmission of the next ultrasonic energy pulse. Then, the frequencies of the forward and backward pulse trains are given by:

$$F_f = \frac{1}{T_f} = \frac{c - v \cos(\theta)}{L} \quad ; \quad F_b = \frac{1}{T_b} = \frac{c + v \cos(\theta)}{L}$$

If the two frequency signals are now multiplied together, the resulting beat frequency is given by:

$$\delta F = F_b - F_f = \frac{2v \cos(\theta)}{L}$$

c has now been eliminated and v can be calculated from a measurement of δF as:

$$v = \frac{L\delta F}{2 \cos(\theta)}$$

This is often known as the *sing-around flowmeter*.

Transit-time flowmeters are of more general use than Doppler-shift flowmeters, particularly where the pipe diameter involved is large and hence the transit time is consequently sufficiently large to be measured with reasonable accuracy. It is possible then to reduce the inaccuracy value down to $\pm 0.5\%$. However, the instrument costs more than a Doppler-shift flowmeter because of the greater complexity of the electronics needed to make accurate transit-time measurements.

Combined Doppler-shift/transit time flowmeters

Recently, some manufacturers have developed ultrasonic flowmeters that use a combination of Doppler-shift and transit time. The exact mechanism by which these work is rarely, if ever, disclosed since manufacturers wish to protect the details from competitors. However, details of various forms of combined Doppler-shift/transit time measurement techniques are filed in patent offices.

16.3.8 Other types of flowmeter for measuring volume flow rate

Vane meter: This consists of a spring-loaded, hinged flap mounted at right angles to the direction of fluid flow in the fluid-carrying pipe. The original form of this was known as a *gate meter*, in which the movement of the flap was measured by connecting it to a pointer outside the pipe that moved against a graduated scale. The major difficulty with the gate meter was in preventing leaks at the hinge point. The vane meter avoids this difficulty by measuring the deflection of the flap by a potentiometer mounted inside the fluid-carrying pipe. It is typically used to measure airflow within automotive fuel-injection systems.

Another similar device is the *target meter*. This consists of a circular-disk-shaped flap in the pipe. Fluid flow rate is inferred from the force exerted on the disk measured by strain gauges bonded to it. This meter is very useful for measuring the flow of dilute slurries, but it does not find wide application elsewhere as it has a relatively high cost. Measurement uncertainty in all of these types of meters varies between 1% and 5% according to cost and design of each instrument.

Jet meter: These come in two forms, the single jet meter and the multiple jet meter. In the first, the flow is diverted into a single jet that impinges on the radial vanes of an impeller. The multiple jet form diverts the flow into multiple jets that are arranged at equal angles around an impeller that is mounted on a horizontal axis.

Paddle wheel meter: This is a variation of the single jet meter in which the impeller only projects partially into the flowing fluid.

Pelton wheel flowmeter: This uses a similar mechanical arrangement to the old-fashioned water wheels that were used for power generation at the time of the industrial revolution. The flowing fluid is directed onto the blades of the flowmeter wheel by a jet, and the flow rate is determined from the rate of rotation of the wheel. This type of flowmeter is used to measure the flow rate of a diverse range of materials including acids, aggressive chemicals, and hot fats at both low and high flow rates. Special versions can measure very small flow rates down to 3 mL/min.

Laser Doppler flowmeter: This instrument gives direct measurements of flow velocity for liquids containing suspended particles flowing in a pipe. Light from a laser is focused by an optical system to a point in the flow, with fiber-optic cables being commonly used to transmit the light. The movement of particles causes a Doppler shift of the scattered light and produces a signal in a photodetector that is related to the fluid velocity. A very wide range of flow velocities between 10 $\mu\text{m/s}$ and 105 m/s can be measured by this technique.

Sufficient particles for satisfactory operation are normally present naturally in most liquid and gaseous fluids, and the introduction of artificial particles is rarely needed. The technique is advantageous in measuring flow velocity directly rather than inferring it from a pressure difference. It also causes no interruption in the flow, and, as the instrument can be made very small, it can measure velocity in confined areas. One limitation is that it measures local flow velocity in the vicinity of the focal point of the light beam, which can lead to large errors in the estimation of mean volume flow rate if the flow profile is not uniform. However, this limitation can be used constructively in applications of the instrument where the flow profile across the cross section of a pipe is determined by measuring the velocity at a succession of points.

The final comment on this instrument has to be that, although it could potentially be used in many applications, it has competition from many other types of instruments that offer similar performance at lower cost. Its main application at the present time is in measuring blood flow in medical applications.

Thermal anemometers: Thermal anemometry was first used in the *hot-wire anemometer* to measure the volume flow rate of gases flowing in pipes. The hot-wire anemometer consists of a piece of thin (typical diameter 5 microns), electrically heated wire (usually tungsten, platinum or a platinum-iridium alloy) that is inserted into the gas flow. The

flowing gas has a cooling effect on the wire that reduces its resistance. Measurement of the resistance change (usually by a bridge circuit) allows the volume flow rate of the gas to be calculated. Unfortunately, the device is not robust because of the very small diameter of the wire used in its construction. However, it has a very fast speed of response, which makes it an ideal measurement device in conditions where the flow velocity is changing. It is also insensitive to the direction of gas flow, making it a very useful measuring device in conditions of turbulent flow. Recently, more robust devices have been made by using a thin metal film instead of a wire. In this form the device is known as a **hot-film anemometer**. Typically, the film is platinum and it is deposited on a quartz probe of typical diameter 0.05 mm. The increased robustness means that the hot-film anemometer is also used to measure the flow rate of liquids such as water.

Coriolis meter: While the Coriolis meter is primarily intended to be a mass-flow measuring instrument, it can also be used to measure volume flow rate when high measurement accuracy is required. However, its high cost means that alternative instruments are normally used for measuring volume flow rate.

16.3.9 Open channel flowmeters

Open channel flowmeters measure the flow of liquids in open channels and are particularly relevant to measuring the flow of water in rivers as part of environmental management schemes. The normal procedure is to build a weir or flume of constant width across the flow and measure the velocity of flow and the height of liquid immediately before the weir or flume with an ultrasonic or radar level sensor, as shown in Fig. 16.16. The volume flow rate can then be calculated from this measured height.

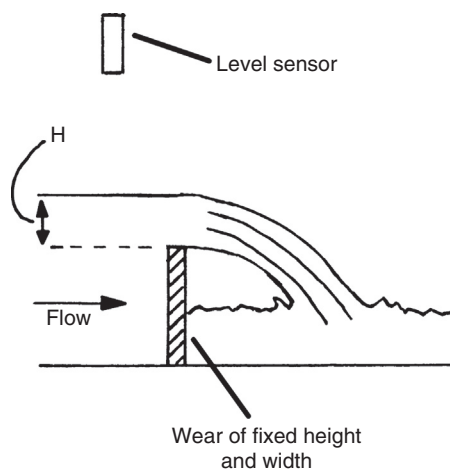


Figure 16.16
Open channel flowmeter.

As an alternative to building a weir or flume, electromagnetic flowmeters up to 180 mm wide are available that can be placed across the channel to measure the flow velocity, providing the flowing liquid is conductive. If the channel is wider than 180 mm, two or more electromagnetic meters can be placed side by side. Apart from measuring the flow velocity in this way, the height of the flowing liquid must also be measured, and the width of the channel must also be known in order to calculate the volume flow rate.

As a third alternative, ultrasonic flowmeters are also used to measure flow velocity in conjunction with a device to measure the liquid depth.

16.4 Intelligent flowmeters

All the usual benefits associated with intelligent instruments are applicable to most types of flowmeter. Indeed, all types of mass flowmeters routinely have intelligence as an integral part of the instrument. For volume flow rate measurement, intelligent differential pressure measuring instruments can be used to good effect in conjunction with obstruction-type flow transducers. One immediate benefit of this in the case of the commonest flow restriction device, the orifice plate, is to extend the lowest flow measurable with acceptable accuracy down to 20% of the maximum flow value. In positive displacement meters, intelligence allows compensation for thermal expansion of meter components and temperature-induced viscosity changes. Correction for variations in flow pressure is also provided for. Intelligent electromagnetic flowmeters are also available, and these have a self-diagnosis and self-adjustment capability. The useable instrument range is typically from 3% to 100% of the full-scale reading, and the quoted maximum inaccuracy is $\pm 0.5\%$. It is also normal to include a nonvolatile memory to protect constants used for correcting for modifying inputs, etc., against power supply failures. Intelligent turbine meters are able to detect their own bearing wear and also report deviations from initial calibration due to blade damage, etc. Some versions also have self-adjustment capability.

The ability to carry out digital signal processing has also led to the emergence of the ***cross-correlation ultrasonic flowmeter***. This is a variant of the transit-time form of ultrasonic flowmeter in which a series of ultrasonic signals are injected into the flowing liquid. The ultrasonic receiver stores the echo pattern from each input signal, and then cross-correlation techniques are used to produce a map of the profile of the water flow in different layers. Thus, the instrument provides information on the profile of the flow rate across the cross section of the pipe rather than just giving a measurement of the mean flow rate in the pipe.

The trend is now moving toward total flow computers that can process inputs from almost any type of transducer. Such devices allow user input of parameters like specific gravity, fluid density, viscosity, pipe diameters, thermal expansion coefficients, discharge

coefficients, etc. Auxiliary inputs from temperature transducers are also catered for. After processing the raw flow transducer output with this additional data, flow computers are able to produce measurements of flow to a very high degree of accuracy.

16.5 Choice between flowmeters for particular applications

The first question to answer in specifying a suitable flowmeter for a given application is whether the flowing material is a solid, liquid, or gas. If the material is a solid (powders or granular particles), the only option is to measure the mass flow rate using some form of mass flowmeter. However, if the flowing material is in liquid or gaseous form, the flow can be measured either as the mass flow rate or as the volume flow rate. Of the two, measurement of the flow in terms of the mass flow rate is usually preferred if there is a requirement for high measurement accuracy.

If the flowing material is a solid, the only technique available to measure the flow rate is a conveyor-based method. The standard way to do this is to measure the mass flowing in unit time. However, an alternative nuclear mass flow sensor is now available, as discussed in [Section 16.2.1](#).

If the material flowing is in liquid or gaseous form, there are a large number of factors to be considered when specifying a flowmeter for a particular application. These include the temperature and pressure of the fluid, its density, viscosity, chemical properties, and abrasiveness, whether it contains particles, whether it is a liquid or gas, etc. This narrows the field to a subset of instruments that are physically capable of making the measurement. Next, the required performance factors of accuracy, rangeability, acceptable pressure drop, output signal characteristics, reliability, and service life must be considered. Accuracy requirements vary widely across different applications, with measurement uncertainty of $\pm 5\%$ being acceptable in some and less than $\pm 0.5\%$ being demanded in others. Finally, the economic viability must be assessed and this must take account not only of purchase cost but also of reliability, installation difficulties, maintenance requirements, and service life.

Where only a visual indication of fluid flow rate is needed, the variable-area meter is popular. Where a flow measurement in the form of an electrical signal is required, the choice of available instruments is very large. The orifice plate is used extremely commonly for such purposes and accounts for almost 50% of the instruments currently in use in industry. Other forms of differential pressure meter and electromagnetic flowmeters are used in significant numbers. Currently, there is a trend away from rotating devices such as turbine meters and positive displacement meters. At the same time, usage of ultrasonic and vortex meters is expanding. Where high accuracy flow measurement is needed, mass flow rate devices like the Coriolis meter and thermal mass flowmeter (including MEMS devices) are commonly used.

16.6 Calibration of flowmeters

The first consideration in choosing a suitable way to calibrate flow-measuring instruments is to establish exactly what accuracy level is needed so that the calibration system instituted does not cost more than necessary. In some cases, such as handling valuable fluids or where there are legal requirements as in fuel pumps, high accuracy levels (e.g., error $\leq 0.1\%$) are necessary and the expensive procedures necessary to achieve such accuracy are justified. However, in other situations, such as in measuring additives to the main stream in a process plant, only low levels of accuracy are needed (e.g., error $\approx 5\%$ is acceptable) and relatively cheap calibration procedures are sufficient.

The accuracy of flow measurement is greatly affected by the flow conditions and characteristics of the flowing fluid. Therefore, wherever possible, process flow-measuring instruments are calibrated on-site in their normal measuring position. This ensures that calibration is performed in the actual flow conditions, which are difficult or impossible to reproduce exactly in a laboratory. To ensure the validity of such calibration, it is also normal practice to repeat flow calibration checks until the same reading is obtained in two consecutive tests.

If on-site calibration is not feasible or is not accurate enough, the only alternative is to send the instrument away for calibration using special equipment provided by instrument manufacturers or other specialist calibration companies. However, this is usually an expensive option. Furthermore, the calibration facility does not replicate the normal operating conditions of the meter tested, and appropriate compensation for the differences between calibration conditions and normal use conditions must be applied.

The equipment and procedures used for calibration depend on whether mass, liquid, or gaseous flows are being measured. Therefore, separate sections are devoted to each of these cases. It must also be stressed that all calibration procedures mentioned in the following paragraphs with respect to fluid flow only refer to flows of single-phase fluids (i.e., liquids or gases). Where a second or third phase is present, calibration is much more difficult and specialist advice should be sought from the manufacturer of the instrument used for measurement.

16.6.1 Calibration equipment and procedures for mass flow measuring instruments

Where the conveyor method is used for measuring the mass flow of solids in the form of particles or powders, both mass-measuring and velocity-measuring instruments are involved. Suitable calibration techniques for each of these are discussed in later chapters.

In the case of the Coriolis and thermal mass flowmeters, the usual method of calibrating these while in situ in their normal measurement position is to provide a diversion valve

after the meter. During calibration procedures, the valve is opened for a measured time period to allow some of the fluid to flow into a container that is subsequently weighed. Alternatively, the meter can be removed for calibration using special test rigs that are normally provided by the instrument manufacturer.

16.6.2 Calibration equipment and procedures for instruments measuring the volume flow rate of liquids

Calibrated tank

Probably the simplest piece of equipment available for calibrating instruments measuring liquid flow rates is the calibrated tank. This consists of a cylindrical vessel, as shown in Fig. 16.17, with conical ends that facilitate draining and cleaning of the tank. A *sight tube* with a graduated scale is placed alongside the final, upper, cylindrical part of the tank, which allows the volume of liquid in the tank to be measured accurately. Flow-rate calibration is performed by measuring the time taken, starting from an empty tank, for a given volume of liquid to flow into the vessel.

Because the calibration procedure starts and ends in zero-flow conditions, it is not suitable for calibrating instruments that are affected by flow acceleration and deceleration

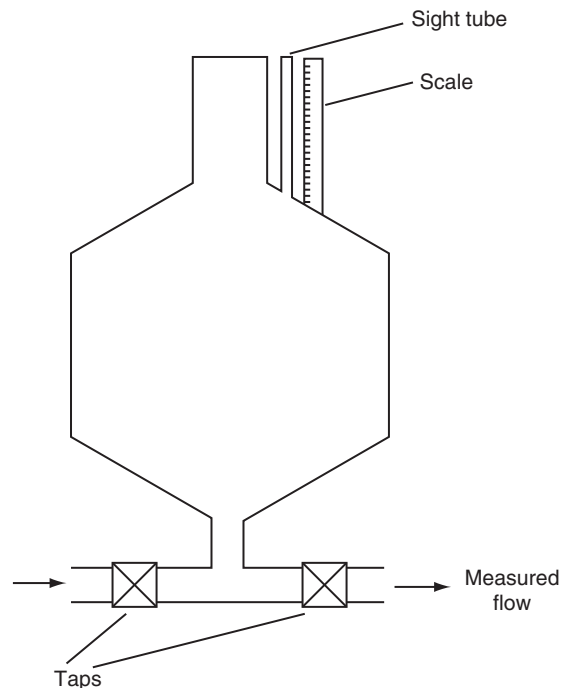


Figure 16.17
Calibrated tank.

characteristics. This therefore excludes instruments like differential pressure meters (orifice plate, flow nozzle, Venturi, Dall flow tube, Pitot tube), turbine flowmeters, and vortex-shedding flowmeters. The technique is further limited to the calibration of low-viscosity liquid flows, although lining the tank with an epoxy coating can allow the system to cope with somewhat higher viscosities. The limiting factor in this case is the drainage characteristics of the tank, which must be such that the residue liquid left after draining has an insufficient volume to affect the accuracy of the next calibration.

Gravimetric method

A variation on the principle of measuring the volume of liquid flowing in a given time is to weigh the quantity of fluid flowing in a given time. Apart from its applicability to a wider range of instruments, this technique is not limited to low-viscosity fluids as any residual fluid in the tank before calibration will be detected by the load cells and therefore compensated for. In the simplest implementation of this system, fluid is allowed to flow for a measured length of time into a tank resting on load cells. As before, the stop-start mode of fluid flow makes this method unsuitable for calibrating differential pressure, turbine, and vortex-shedding flowmeters. It is also unsuitable for measuring high flow rates because of the difficulty in bringing the fluid to rest. These restrictions can be overcome by directing the flowing fluid into the tank via diverter valves. In this alternative, it is important that the timing system be carefully synchronized with the operation of the diverter valves.

All versions of gravimetric calibration equipment are less robust than volumetric types and so on-site use is not recommended.

Pipe prover

The commonest form of pipe prover is the bidirectional type, shown in [Fig. 16.18](#), which consists of a U-shaped tube of metal of accurately known cross section. The purpose of the U-bend is to give a long flow path within a compact spatial volume. Alternative versions with more than one U-bend also exist to cater for situations where an even longer flow path is required. Inside the tube is a hollow, inflatable sphere that is filled with water until its diameter is about 2% larger than that of the tube. As such, the sphere forms a seal with the sides of the tube and acts as a piston. The prover is connected into the existing fluid-carrying pipe network via tappings either side of a by-pass valve. A four-way valve at the start of the U-tube allows fluid to be directed in either direction around it. Calibration is performed by diverting flow into the prover and measuring the time taken for the sphere to travel between two detectors in the tube. The detectors are normally of an electromechanical, plunger type.

Unidirectional versions of the above also exist in which fluid only flows in one direction around the tube. A special handling valve has to be provided to return the sphere to the

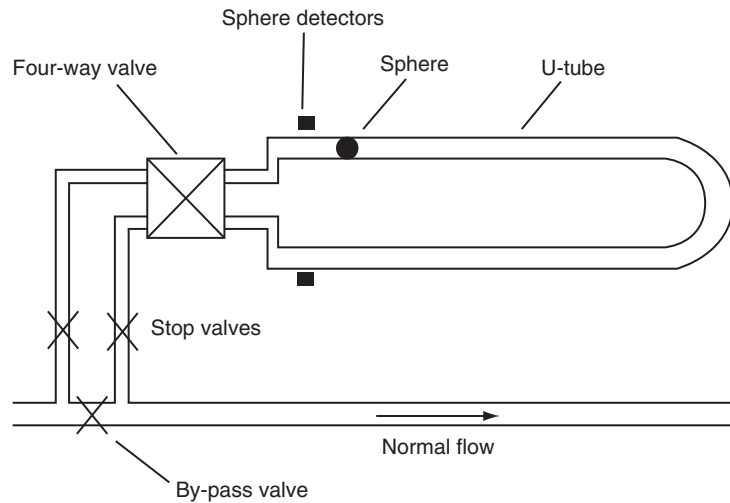


Figure 16.18
Bidirectional pipe prover.

starting point after each calibration, but the absence of a four-way flow control valve makes such devices significantly cheaper than bidirectional types.

Pipe provers are particularly suited to the calibration of pressure-measuring instruments that have a pulse type of output, such as turbine meters. In such cases, the detector switches in the tube can be made to gate the instrument's output pulse counter. This enables not only the basic instrument to be calibrated but also the ancillary electronics within it at the same time. The inaccuracy level of such provers can be as low as $\pm 0.1\%$. This level of accuracy is maintained for high fluid-viscosity levels and also at very high flow rates. Even higher accuracy is provided by an alternative form of prover that consists of a long, straight metal tube containing a metal piston. However, such devices are more expensive than the other types discussed above, and their large space requirements also often cause great difficulties.

Compact prover

The compact prover has an identical operating principle to that of the other pipe provers described above but occupies a much smaller spatial volume. It is therefore used extensively in situations where there is insufficient room to use a larger prover. Many different designs of compact prover exist, operating both in the unidirectional and bidirectional modes; one such design is shown in Fig. 16.19. Common features of compact provers are an accurately machined cylinder containing a metal piston that is driven between two reference marks by the flowing fluid. The instants at which the reference marks are passed are detected by switches, of optical form in the case of the version shown in Fig. 16.19. Provision has to be made within these instruments for returning the

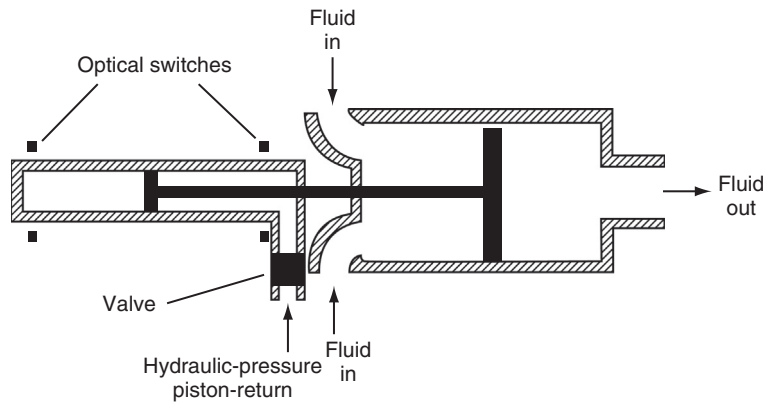


Figure 16.19
Compact prover.

piston back to the starting point after each calibration, and a hydraulic system is commonly used for this. Again, measuring the piston traverse time is made easier if the switches can be made to gate a pulse train, and therefore compact provers are also most suited to instruments having a pulse-type output such as turbine meters. Measurement uncertainty levels down to $\pm 0.1\%$ are possible.

The main technical difficulty in compact provers is measuring the traverse time, which can be as small as 1 second. The pulse count from a turbine meter in this time would typically be only about 100, making the possible measurement error 1%. To overcome this problem, electronic pulse-interpolation techniques have been developed that can count fractions of pulses.

Positive displacement meter

High-quality versions of the positive displacement flowmeter can be used as a reference standard in flowmeter calibration. The general principles of these were explained in Section 16.3.3. Such devices can give measurement inaccuracy levels down to $\pm 0.2\%$.

Orifice plate

A flow line equipped with a certified orifice plate is sometimes used as a reference standard in flow calibration, especially for high flow rates through large bore pipes. While measurement uncertainty is of the order of $\pm 1\%$ at best, this is adequate for calibrating many flow-measuring instruments.

Turbine meter

Turbine meters are also used as a reference standard for testing flowmeters. Their main application, as for orifice plates, is in calibrating high flow rates through large-bore pipes. Measurement uncertainty down to $\pm 0.2\%$ is attainable.

16.6.3 Calibration equipment and procedures for instruments measuring the volume flow rate of gases

Calibration of gaseous flows poses considerable difficulties compared with calibrating liquid flows. These problems include the lower density of gases, their compressibility, and the difficulty in establishing a suitable liquid/air interface as utilized in many liquid flow measurement systems.

In consequence, the main methods of calibrating gaseous flows, as described below, are small in number. Certain other specialized techniques, including the gravimetric method and the pressure–volume–temperature method, are also available. These provide primary reference standards for gaseous flow calibration with measurement uncertainty down to $\pm 0.3\%$. However, the expense of the equipment involved is such that it is usually only available in national standards laboratories.

Bell prover

The bell prover consists of a hollow, inverted, metal cylinder suspended over a bath containing light oil, as shown in Fig. 16.20. The air volume in the cylinder above the oil is connected, via a tube and a valve, to the flowmeter being calibrated. An air flow through the meter is created by allowing the cylinder to fall downwards into the bath, thus displacing the air contained within it. The flow rate, which is measured by timing the rate of fall of the cylinder, can be adjusted by changing the value of counterweights attached via a low-friction pulley system to the cylinder. This is essentially laboratory-only equipment and therefore on-site calibration is not possible.

Positive displacement meter

As for liquid flow calibration, positive displacement flowmeters can be used for the calibration of gaseous flows with inaccuracy levels down to $\pm 0.2\%$.

Compact prover

Compact provers of the type used for calibrating liquid flows can also be applied to the calibration of instruments measuring gaseous flows. However, instruments accurate enough to calibrate gaseous flows tend to be expensive, and so are mainly used in standards laboratories.

16.6.4 Reference standards

Traceability of flow rate calibration to fundamental standards is provided for by reference to primary standards of the separate quantities that flow rate is calculated from. Mass measurements are calibrated by comparison with a standard mass (see Chapter 18) and

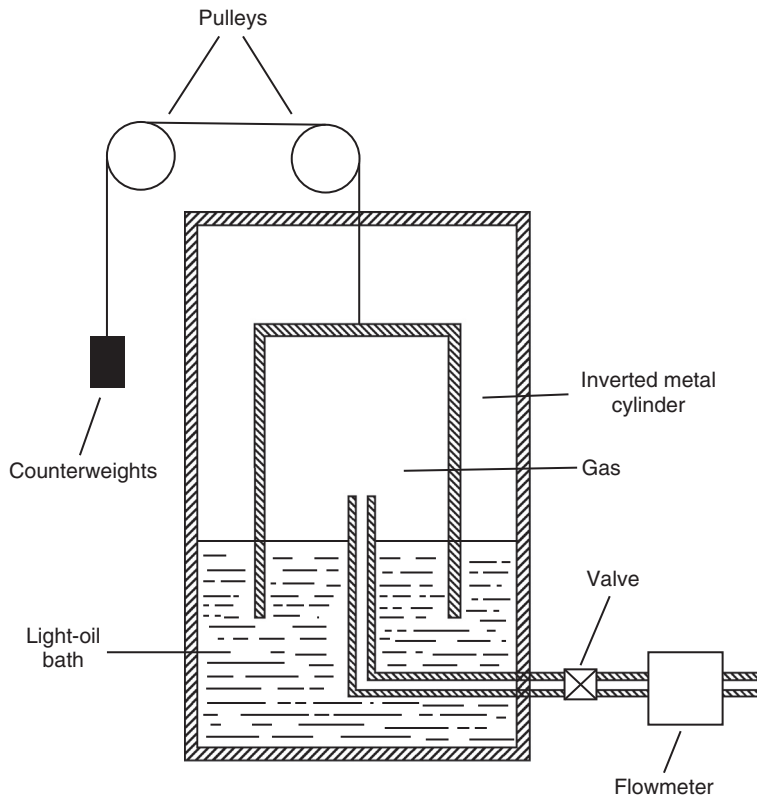


Figure 16.20
Bell prover.

time is calibrated by reference to a cesium resonator standard. Volume measurements are calibrated against standard reference volumes, which are themselves calibrated gravimetrically using a mass measurement system traceable to the standard kilogram.

16.7 Summary

We started this chapter off by observing that flow rate could be measured either as mass flow rate or volume flow rate. We also observed that the material being measured could be in solid, liquid, or gaseous form. In the case of solids, we quickly found that this could only be measured in terms of the mass flow rate. However, in the case of liquids and gases, we found that we have the option of measuring either the mass flow rate or the volume flow rate. Of these two alternatives, we observed that mass flow measurement was the more accurate.

Before proceeding to look at flow measurement in detail, we had a brief look at the differences between laminar flow and turbulent flow. This taught us that the flow rate was

difficult to measure in turbulent flow conditions and even in laminar flow at high velocities. Therefore, as far as possible, the measurement was made at a point in the flow where the flow was at least approximately laminar and the flow velocity was as small as possible.

This allowed us to look at flow-measuring instruments in more detail. We start with mass flow and observed that this could be measured in one of three ways, conveyor-based methods, the Coriolis flowmeter, and the thermal mass flowmeter. We examined the mode of operation of each of these and made some comments about their applicability. In the case of the thermal mass flowmeter, we also looked at MEMS versions of this.

Moving on, we then started to look at volume flow rate measurement and worked progressively through a large number of different instruments that can be used. Firstly, we looked at obstruction devices. These are placed in a fluid-carrying pipe and cause a pressure difference across the obstruction, which is a function of the flow rate of the fluid. Various obstruction devices were discussed, from the commonly used cheap, but less accurate, orifice plate to more expensive, but more accurate, devices like the Venturi tube, the flow nozzle, and the Dall flow tube. We also briefly looked at Pitot tubes and their primary use for temporary flow measurement.

After looking at flow obstruction devices, we looked at a number of other instruments for measuring volume flow rate of fluids flowing in pipes, including the variable area flowmeter (rotameter), positive displacement flowmeter, turbine flowmeter, electromagnetic flowmeter, vortex-shedding flowmeter, and finally ultrasonic flowmeters in both Doppler shift and transit-time forms. We also looked briefly at several other devices including gate-type meters, the laser Doppler flowmeter, and the thermal anemometer. Finally, we also had a brief look at measuring fluid flow in open channels and observed three ways of doing this.

We rounded off our discussion of flow measurement by looking at intelligent devices. We observed that these bring the usual benefits associated with intelligent instruments, including improved measurement accuracy and extended measurement range, with facilities for self-diagnosis and self-adjustment also being common. This led to some discussion about the most appropriate instrument to use in particular flow measurement situations and applications out of all the instruments covered in the chapter.

We then concluded the chapter by considering the subject of flow meter calibration. These calibration methods were considered in three parts. Firstly, we looked at the calibration of instruments measuring mass flow. Secondly, we looked at the calibration of instruments measuring the volume flow rate of liquids. Finally, we looked at the calibration of instruments measuring the volume flow rate of gases.

16.8 Problems

- 16.1 Name and discuss three different kinds of instruments that are used for measuring the mass flow rate of substances (mass flowing in unit time).
- 16.2 Instruments to measure the volume flow rate of fluids (volume flowing in unit time) can be divided into a number of different types. Explain what these different types are and briefly discuss how instruments in each class work, using sketches of instruments as appropriate.
- 16.3 An orifice plate is used to measure the fluid flow rate in a pipe. Sketch a graph showing how the pressure varies either side of the point where the orifice plate is inserted into the pipe. Mark the points in the pipe where sensors should be positioned to measure the minimum and maximum pressure. List the positive and negative features of an orifice plate as a flow rate measuring device. List the alternative obstruction devices that can be used instead of an orifice plate and write down the ways in which these (a) are better than orifice plates, (b) are worse than orifice plates, and (c) have similar problems to orifice plates.
- 16.4 What is a Coriolis meter? What is it used for and how does it work?
- 16.5 Name four different kinds of differential pressure meter. Briefly discuss how each one works and explain the main advantages and disadvantages of each type.
- 16.6 Give a sketch showing the structure of a thermal mass flowmeter. What is this device principally used for? Explain how the device works and describe its main characteristics. What different versions of the device are available?
- 16.7 Explain how each of the following works and give typical applications: rotameter, rotary piston meter.
- 16.8 How does an electromagnetic flowmeter work and what is it typically used for?
- 16.9 Discuss the mode of operation and applications of each of the following: turbine meter, vortex-shedding flowmeter.
- 16.10 What are the two main types of ultrasonic flowmeter? Discuss the mode of operation of each.
- 16.11 How do each of the following work and what are they particularly useful for: gate-type meter, jet meter, Pelton wheel meter, laser Doppler flowmeter, thermal anemometer?
- 16.12 What is an open-channel flowmeter? Draw a sketch of one and explain how it works.
- 16.13 What instruments, special equipment, and procedures are used in the calibration of flowmeters that are used for measuring the flow of liquids?
- 16.14 What instruments, special equipment, and procedures are used in the calibration of flowmeters that are used for measuring the flow of gases?

Level measurement

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17.1 Introduction

Level measurement is required in a wide range of applications and can involve the measurement of solids that are in the form of powders or small particles as well as liquids. While some applications require a level to be measured to a high degree of accuracy, other applications only need an approximate indication of level. A wide variety of instruments are available to meet these differing needs.

Simple devices like as dipsticks or float systems are relatively inexpensive. Although only offering limited measurement accuracy, they are entirely adequate for many applications and find widespread use. A number of higher accuracy devices are also available for applications that require a better level of accuracy. The list of devices in common use that offer good measurement accuracy includes pressure measuring devices, capacitive devices, ultrasonic devices, radar devices, and radiation devices. A number of other devices that are less commonly used are also available. All of these devices are discussed in more detail in the following sections.

17.2 Dipsticks

Dipsticks offer a simple means of measuring the level of liquids approximately. The *ordinary dipstick* is the cheapest device available. This consists of a metal bar on which a scale is etched, as shown in Fig. 17.1. The bar is fixed at a known position in the liquid-containing vessel. A level measurement is made by removing the instrument from the vessel and reading off how far up the scale the liquid has wetted. As a human operator is required to remove and read the dipstick, this method can only be used in relatively small and shallow vessels. One common use is in checking the remaining amount of beer in an ale cask.

17.3 Float systems

Float systems are simple and cheap and provide an alternative way of approximately measuring the level of liquids that is widely used. The system consists of a float on the surface of the liquid whose position is measured by means of a suitable transducer. They have a typical measurement inaccuracy of $\pm 1\%$. The system using a potentiometer, shown earlier in Fig. 2.2, is very common, and is well known for its application to monitoring the level in motor vehicle fuel tanks. An alternative system, which is used in greater numbers, is called the *float and tape gauge* (or *tank gauge*). This has a tape attached to the float that passes round a pulley situated vertically above the float. The other end of the tape is attached to either a counterweight or a negative-rate counter spring. The amount of rotation of the pulley, measured by either a synchro or a potentiometer, is then proportional to the liquid level. These two essentially mechanical systems of measurement are popular in many applications, but their maintenance requirements are always high.

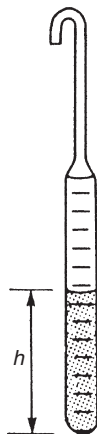


Figure 17.1
Dipstick.

17.4 Pressure-measuring devices (Hydrostatic systems)

Pressure-measuring devices measure liquid level to a better accuracy and use the principle that the hydrostatic pressure due to a liquid is directly proportional to its depth and hence to the level of its surface. Several instruments are available that use this principle, and they are widely used in many industries, particularly in harsh chemical environments. In the case of open-topped vessels (or covered ones that are vented to the atmosphere), the level can be measured by inserting a pressure sensor at the bottom of the vessel, as shown in Fig. 17.2a. The liquid level h is then related to the measured pressure P according to: $h = P/\rho g$, where ρ is the liquid density and g is the acceleration due to gravity. One source of error in this method can be imprecise knowledge of the liquid density. This can be a particular problem in the case of liquid solutions and mixtures (especially hydrocarbons), and in some cases only an estimate of density is available. Even with single liquids, the density is subject to variation with temperature, and therefore temperature measurement may be required if very accurate level measurements are needed.

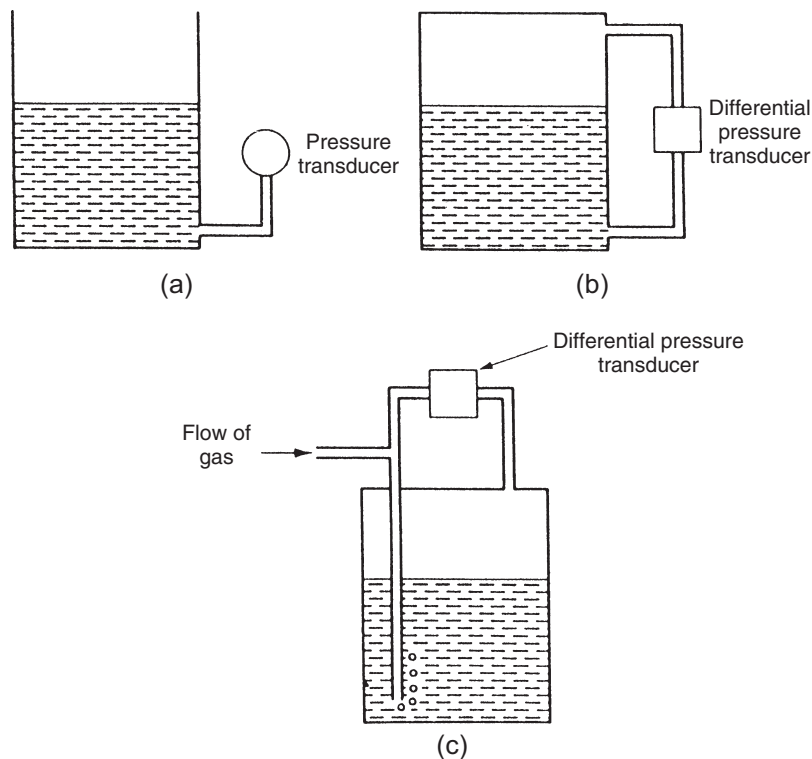


Figure 17.2

Hydrostatic systems: (a) open-topped vessel; (b) sealed vessel; (c) bubbler unit.

Where liquid-containing vessels are totally sealed, the liquid level can be calculated by measuring the differential pressure between the top and bottom of the tank, as shown in Fig. 17.2c. The differential pressure transducer used is normally a standard diaphragm type, although silicon-based microsensors are being used in increasing numbers. The liquid level is related to the differential pressure measured, δP , according to: $h = \delta P / \rho g$. The same comments as for the case of the open vessel apply regarding uncertainty in the value of ρ . An additional problem that can occur is an accumulation of liquid on the side of the differential pressure transducer that is measuring the pressure at the top of the vessel. This can arise because of temperature fluctuations, which allow liquid to alternately vaporize from the liquid surface and then condense in the pressure tapping at the top of the vessel. The effect of this on the accuracy of the differential pressure measurement is severe, but the problem is easily avoided by placing a drain pot in the system.

A final pressure-related system of level measurement is the *bubbler unit* shown in Fig. 17.2c. This uses a dip pipe that reaches to the bottom of the tank and is purged free of liquid by a steady flow of gas through it. The rate of flow is adjusted until gas bubbles are just seen to emerge from the end of the tube. The pressure in the tube, measured by a pressure transducer, is then equal to the liquid pressure at the bottom of the tank. It is important that the gas used is inert with respect to the liquid in the vessel. Nitrogen, or sometimes just air, is suitable in most cases. Gas consumption is low, and a cylinder of nitrogen may typically last for 6 months. The method is suitable for measuring the liquid pressure at the bottom of both open and sealed tanks. It is particularly advantageous in avoiding the large maintenance problem associated with leaks at the bottom of tanks at the site of the pressure tappings required by alternative methods.

Measurement uncertainty varies according to the application and the condition of the measured material. A typical value would be $\pm 0.5\%$ of full-scale reading, although $\pm 0.1\%$ can be achieved in some circumstances.

17.5 Capacitive devices

Capacitive devices are widely used for measuring the level of both liquids and solids in powdered or granular form. They perform well in many applications, but become inaccurate if the measured substance is prone to contamination by agents that change the dielectric constant. Ingress of moisture into powders is one such example of this. They are also suitable for use in extreme conditions measuring liquid metals (high temperatures), liquid gases (low temperatures), corrosive liquids (acids, etc.), and high-pressure processes. Two versions are used according to whether the measured substance is conducting or not. For *nonconducting substances* (less than $0.1 \mu\text{mho}/\text{cm}^3$), two bare-metal capacitor plates in the form of concentric cylinders are immersed in the substance, as shown in Fig. 17.3. The substance behaves as a dielectric between the plates according

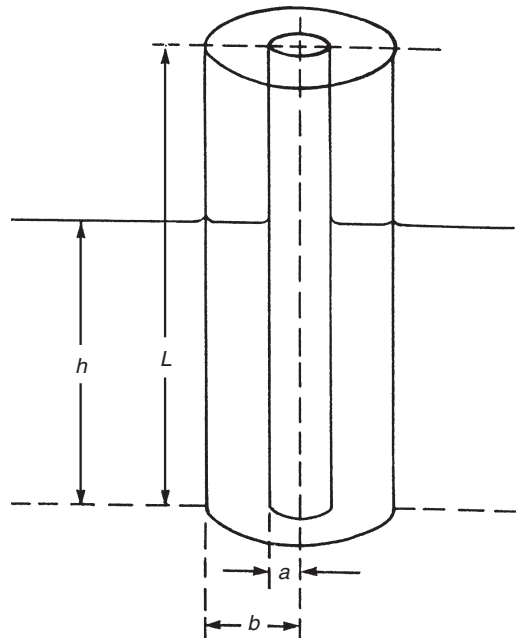


Figure 17.3
Capacitive level sensor.

to the depth of the substance. For concentric cylinder plates of radius a and b ($b > a$), and total height L , the depth of the substance h is related to the measured capacitance C by:

$$h = \frac{C \log_e \left(\frac{b}{a} \right) - 2\pi\epsilon_0}{2\pi\epsilon_0(\epsilon - 1)} \quad (17.1)$$

where ϵ is the relative permittivity of the measured substance and ϵ_0 is the permittivity of free space. In the case of **conducting substances**, exactly the same measurement techniques are applied, but the capacitor plates are encapsulated in an insulating material. The relationship between C and h in Eq. (17.1) then has to be modified to allow for the dielectric effect of the insulator. Measurement uncertainty is typically 1%–2%.

17.6 Ultrasonic level gauge

Ultrasonic level measurement is one of a number of noncontact techniques available. It is primarily used to measure the level of materials that are either in a highly viscous liquid form or in solid (powder or granular) form. The principle of the ultrasonic level gauge is that energy from an ultrasonic source above the material is reflected back from the material surface into an ultrasonic energy detector, as illustrated in Fig. 17.4. Measurement

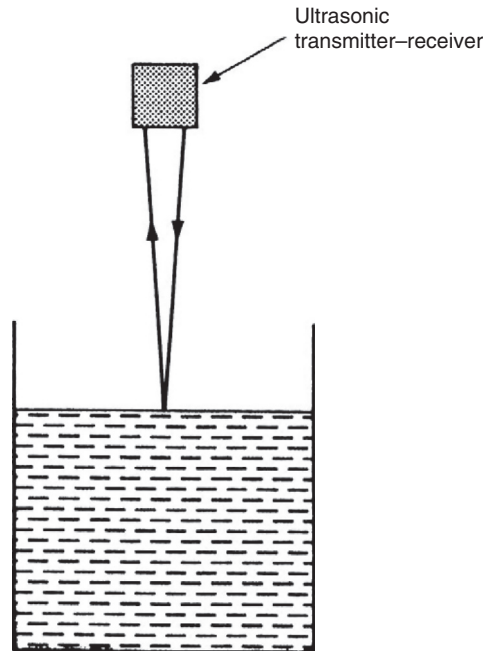


Figure 17.4
Ultrasonic level gauge.

of the time of flight allows the level of the material surface to be inferred. In alternative versions (only valid for liquids), the ultrasonic source is placed at the bottom of the vessel containing the liquid, and the time of flight between emission, reflection off the liquid surface, and detection back at the bottom of the vessel is measured.

Ultrasonic techniques are especially useful in measuring the position of the interface between two immiscible liquids contained in the same vessel, or measuring the sludge or precipitate level at the bottom of a liquid-filled tank. In either case, the method employed is to fix the ultrasonic transmitter-receiver transducer at a known height in the upper liquid, as shown in Fig. 17.5. This establishes the level of the liquid/liquid or liquid/sludge level in absolute terms. When using ultrasonic instruments, it is essential that proper compensation is made for the working temperature if this differs from the calibration temperature, since the speed of ultrasound through air varies with temperature (see Chapter 13). Ultrasound speed also has a small sensitivity to humidity, air pressure, and carbon dioxide concentration, but these factors are usually insignificant. Temperature compensation can be achieved in two ways. Firstly, the operating temperature can be measured and an appropriate correction made. Secondly, and preferably, a comparison method can be used in which the system is calibrated each time it is used by measuring the transit time of ultrasonic energy between two known reference points. This second

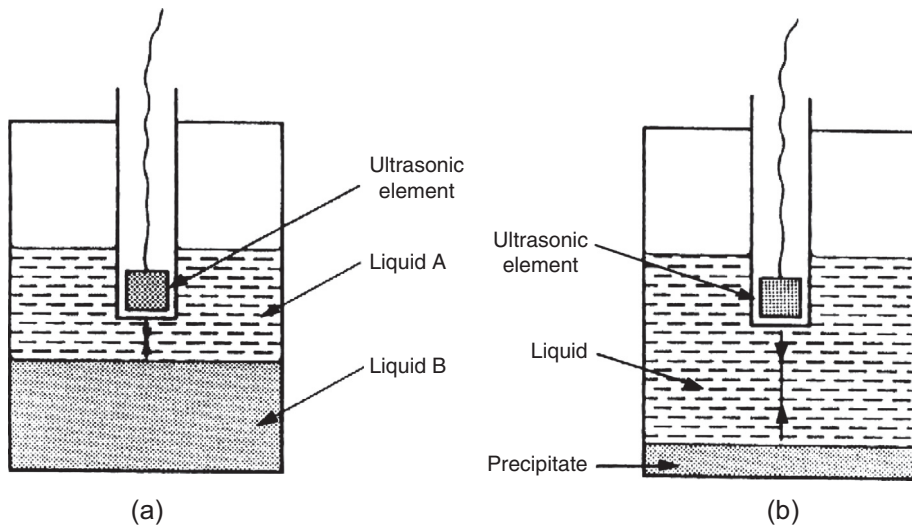


Figure 17.5

Measuring interface positions: (a) liquid/liquid interface; (b) liquid/precipitate interface.

method takes account of humidity, pressure, and carbon dioxide concentration variations as well as providing temperature compensation. With appropriate care, measurement uncertainty can be reduced to about $\pm 1\%$.

17.7 Radar (microwave) sensors

Level-measuring instruments using microwave radar are an alternative technique for noncontact measurement. They are able to provide successful level measurement in applications that are otherwise very difficult, such as measurement in closed tanks, measurement where the liquid is turbulent, and measurement in the presence of obstructions and steam condensate. They can also be used for detecting the surface of solids in powder or particulate form. The technique involves directing a constant-amplitude, frequency-modulated microwave signal at the liquid surface. A receiver measures the phase difference between the reflected signal and the original signal transmitted directly through air to it, as shown in Fig. 17.6. This measured phase difference is linearly proportional to the liquid level. The system is similar in principle to ultrasonic level measurement, but has the important advantage that the transmission time of radar through air is almost totally unaffected by ambient temperature and pressure fluctuations. However, as the microwave frequency is within the band used for radio communications, strict conditions on amplitude levels have to be satisfied, and the appropriate licenses have to be obtained.

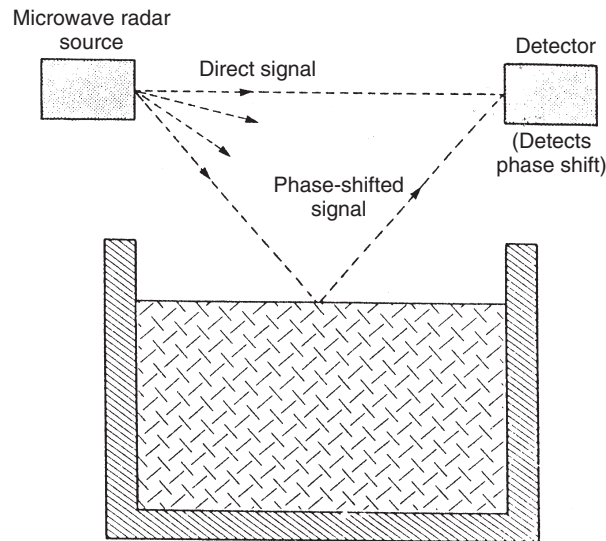


Figure 17.6
Radar level detector.

17.8 Nucleonic (or radiometric) sensors

Nucleonic, sometimes called radiometric, sensors are relatively expensive. They use a radiation source and detector system located outside a tank in the manner shown in Fig. 17.7. The noninvasive nature of this technique in using a source and detector system outside the tank is particularly attractive. The absorption of both beta rays and gamma rays varies with the amount of material between the source and detector, and hence is a function of the level of the material in the tank. The gamma ray source commonly used is either caesium-137 or cobalt-60. The radiation level measured by the detector I is related to the length of material in the path x according to:

$$I = I_o \exp(-\mu\rho x) \quad (17.2)$$

where I_o is the intensity of radiation that would be received by the detector in the absence of any material, μ is the mass absorption coefficient for the measured material, and ρ is the mass density of the measured material.

In the arrangement shown in Fig. 17.7, the radiation follows a diagonal path across the material, and therefore some trigonometrical manipulation has to be carried out to determine the material level h from x . In some applications, the radiation source can be located in the center of the bottom of the tank, with the detector vertically above it. Where this is possible, the relationship between the radiation detected and material level is obtained by directly substituting h in place of x in Eq. (17.2). Apart from use with liquid materials at normal temperatures, this method is commonly used for measuring the level of hot, liquid metals and also for measuring solid materials in a powdered or granular form.

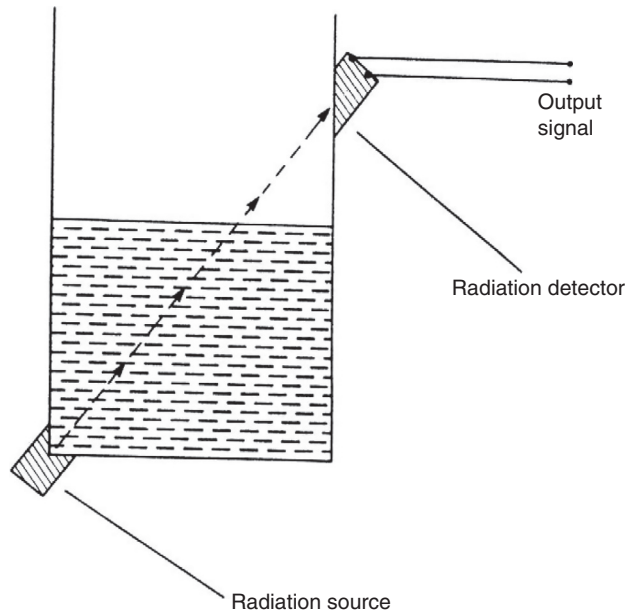


Figure 17.7

Using a radiation source to measure level.

Unfortunately, because of the obvious dangers associated with using radiation sources, very strict safety regulations have to be satisfied when applying this technique. Very low-activity radiation sources are used in some systems to overcome safety problems but the system is then sensitive to background radiation and special precautions have to be taken regarding the provision of adequate shielding. Because of the many difficulties in using this technique, it is only used in special applications.

17.9 Vibrating level sensor

The principle of the vibrating level sensor is illustrated in Fig. 17.8. The instrument consists of two piezoelectric oscillators fixed to the inside of a hollow probe that generate flexural vibrations in the probe at its resonant frequency. The resonant frequency of the probe varies according to the depth of its immersion in the liquid. A phase-locked loop circuit is used to track these changes in resonant frequency and adjust the excitation frequency applied to the probe by the piezoelectric oscillators. Liquid level measurement is therefore obtained in terms of the output frequency of the oscillator when the probe is resonating. The sensor operates reliably and is easy to clean. Its operation is also little affected by any buildup of material deposits on the probe.

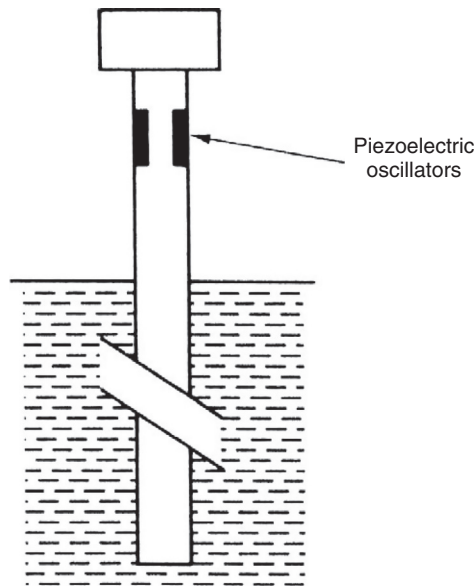


Figure 17.8
Vibrating level sensor.

A variation of this technique is also used in the *vibrating level switch*. This can be used with both liquids and powder/granular solids. When the probe comes in contact with the material being monitored, there is a step change in the vibration frequency and the sensor outputs a switching command.

17.10 Intelligent level-measuring instruments

Most types of level gauges are now available in intelligent form, usually with a digital output display. The pressure-measuring devices (Section 17.3) are obvious candidates for inclusion within intelligent level-measuring instruments, and versions claiming $\pm 0.05\%$ accuracy are now on the market. Such instruments can also carry out additional functions, such as providing automatic compensation for liquid density variations. Microprocessors are also used to simplify installation and setup procedures.

17.11 Choice between different level sensors

The first consideration in choosing a level sensor is whether it is a liquid or a solid that is being measured. The second consideration is the degree of measurement accuracy required.

If it is liquids that are being measured and a relatively low level of accuracy is acceptable, dipsticks and float systems would often be used. Of these, dipsticks require a human operator whereas float systems provide an electrical output that can be recorded or output to an electronic display as required.

Where greater measurement accuracy is required in the measurement of liquid level, a number of different devices can be used. These can be divided into two distinct classes according to whether the instrument does, or does not, make contact with the material whose level is being measured. The advantage of noncontact devices is that they have a higher reliability than contact devices for a number of reasons. All pressure-measuring devices (hydrostatic systems) fall into the class of device that does make contact with the measured liquid and are used quite frequently. However, if there is a particular need for high reliability, noncontact devices such as capacitive, ultrasonic, or radiation devices are preferred. Of these, capacitive sensors are most commonly used but are unsuitable for applications where the liquid may become contaminated, since this changes its dielectric constant and hence the capacitance value. Ultrasonic sensors are less affected by contamination of the measured fluid but only work well with highly viscous fluids. Radar (microwave) and radiation sensors have the best immunity to changes in the temperature, composition, moisture content, and density of the measured material and so are preferred in many applications. However, both of these are relatively expensive.

In the case of measuring the level of solids (which must be in powdered or particle form), the choice of instrument is limited to the options of capacitive, ultrasonic, radar (microwave), and radiation sensors. As for measuring the level of liquids, radar and radiation sensors have the best immunity to changes in the temperature, composition, moisture content, and density of the measured material and so are preferred in many applications. However, they both have a high cost. Either capacitive and ultrasonic devices provide a cheaper solution. Capacitive devices generally perform better but become inaccurate if the measured material is contaminated, in which case ultrasonic sensors are preferred out of these two cheaper solutions.

17.12 Calibration of level sensors

The sophistication of calibration procedures for level sensors depends on the degree of accuracy required. If the accuracy demands are not too high and a tank is relatively shallow, a simple dipstick inserted into a tank will suffice to verify the output reading of any other form of level sensor that is being used for monitoring the liquid level in the tank. However, this only provides one calibration point. Other calibration points can only be obtained by putting more liquid into the tank or by emptying some liquid from the

tank. Such variation of the liquid level may or may not be convenient. However, even if it can be done without too much disturbance to the normal use of the tank, the reading from the dipstick is of very limited accuracy because of the ambiguity in determining the exact point of contact between the dipstick and the meniscus of the liquid.

If the dipstick method is not accurate enough or is otherwise unsuitable, the alternative method of calibrating level is to use a calibration tank that has vertical sides and a flat bottom of known cross-sectional area. Tanks with circular bottoms and rectangular bottoms are both commonly used. With the level sensor in situ, measured quantities of liquid are emptied into the tank. This increases the level of liquid in the tank in steps, and each step creates a separate calibration point. The quantity of liquid added at each stage of the calibration process can be measured either in terms of its volume or in terms of its mass. If the volume of each quantity of liquid added is measured, knowledge of the cross-sectional area of the tank bottom allows the liquid level to be calculated directly. If the mass of each quantity of liquid added is measured, the specific gravity of the liquid has to be known in order to calculate its volume and hence the liquid level. In this case, use of water as the calibration liquid is beneficial since its specific gravity is unity and therefore the calculation of level is simplified.

To measure added water in terms of its volume, calibrated volumetric measures are used. If a 1 liter measure is used, this has a typical inaccuracy of $\pm 0.1\%$. Unfortunately, the errors in the measurement of each quantity of water added are cumulative, and therefore the possible error after 10 quantities of water have been added increases ten-fold to $\pm 1.0\%$. If 20 quantities are added to create 20 calibration points, the possible error is $\pm 2.0\%$ and so on.

Better accuracy can be obtained in the calibration process if the added water is measured in terms of its mass. This can be done conveniently by mounting the calibration tank on an electronic load cell. The typical inaccuracy of such a load cell is $\pm 0.05\%$ of its full-scale reading. This means that the inaccuracy of the level measurement when the tank is full is $\pm 0.05\%$ if the load cell is chosen such that it is giving its maximum output mass reading when the tank is full. Since the total mass of water in the tank is measured at each point in the calibration process, the measurement errors are not cumulative. However, the errors do increase for smaller volumes of water in the tank because the measurement uncertainty is expressed as a percentage of the full-scale reading of the load cell. Therefore, when the tank is only 10% full, the possible measurement error is $\pm 0.5\%$. This means that calibration inaccuracy increases for smaller quantities of water in the tank but the measurement uncertainty is always less than the case where measured volumes of water are added to the tank even for low levels.

Wherever possible, the liquid used in the calibration tank is water, since this avoids the cost involved in using any other liquid and it also makes the calculation of level simpler

when the quantities of water added to the tank are measured in terms of their volume. Unfortunately, the liquid used in the tank often has to be the same as that which the sensor being calibrated normally measures. For example, the specific gravity of the measured liquid is crucial to the operation of both hydrostatic systems and capacitive level sensors. Another example is level measurement using a radiation source, since the passage of radiation through the liquid between the source and detector is affected by the nature of the liquid.

17.13 Summary

We have seen that level sensors can be used to measure the position of the surface within some form of container of both solid materials in the form of powders and of liquids. We have looked at various types of level sensor, following which we considered how the various forms of level sensor available could be calibrated.

One very important observation that we made at the start of our discussion was that the accuracy requirements during level measurement vary widely, and this has an important effect on the type of sensor that is used in any given situation and the corresponding calibration requirements. For example, if the surface level of a liquid within a tank that is used for cooling purposes in an industrial process is being monitored, only a very approximate measurement of level is needed to allow a prediction about how long it will be before the tank needs refilling. However, if the level of liquid of a consumer product within a container is being monitored during the filling process, high accuracy is required in the measurement process.

Where only approximate measurements of liquid level are needed, we saw that dipsticks provide a suitable, low-cost method of measurement, although these require a human operator and so cannot be used as part of an automatic level control system. Float systems are also relatively low-cost instruments and have an electrical form of output that can be used as part of an automatic level control system, although the accuracy is little better than that of dipsticks.

Our discussion then moved on to sensors that provide greater measurement accuracy. First among these were hydrostatic systems. These are widely used in many industries for measuring liquid level, particularly in harsh chemical environments. Measurement uncertainty is usually about $\pm 0.5\%$ of full-scale reading, although this can be reduced to $\pm 0.1\%$ in the best hydrostatic systems. Since accurate knowledge of the liquid density is important in the operation of hydrostatic systems, serious measurement errors can occur if these systems are used to measure the level of mixtures of liquids since the density of such mixtures is rarely known to a sufficient degree of accuracy.

Moving on to look at capacitive level sensors, we observed that these were widely used for measuring the level of both liquids and solids in powdered or granular form, with a typical measurement uncertainty of 1%–2%. They are particularly useful for measuring the level of difficult materials such as liquid metals (high temperatures), liquid gases (low temperatures), and corrosive liquids (acids, etc.). However, they become inaccurate if the measured substance is prone to contamination by agents that change the dielectric constant.

Next on the list of devices studied was the ultrasonic level sensor. We noted that this is one of a number of noncontact techniques available. It is primarily used to measure the level of materials that are either in a highly viscous liquid form or in solid (powder or granular) form. We also observed that it is particularly useful for measuring the position of the interface between two immiscible liquids contained in the same vessel, and also for measuring the sludge or precipitate level at the bottom of a liquid-filled tank. The lowest measurement uncertainty achievable is $\pm 1\%$, but errors increase if the system is not properly calibrated, particularly with respect to the ambient temperature because of the changes in ultrasound speed that occur when the temperature changes.

The discussion then moved on to radar sensors, another noncontact measurement technique. We saw that this, albeit very expensive technique, provided a method for measuring level in conditions that are too difficult for most other forms of level sensor. Such conditions include measurement in closed tanks, measurement where the liquid is turbulent, and measurement in the presence of obstructions and steam condensate. Like ultrasonic sensors, they can also measure the level of solids in powder or granular form.

We then looked at nucleonic sensors. These provide yet another means of noncontact level measurement that finds niche applications in measuring the level of hot, molten metals and also in measuring the level of powdered or granular solids. However, apart from the high cost of nucleonic sensors, it is necessary to adhere to very strict safety regulations when using such sensors.

Having then briefly looked at two other less common level sensors, namely the vibrating level sensor and laser-based sensors, we went on to make brief comments about intelligent level sensors. We noted that most of the types of level sensor discussed are now available in an intelligent form that have quoted measurement uncertainty values down to $\pm 0.05\%$.

The final subject covered in this chapter was that of level sensor calibration. We noted that devices like a simple dipstick could be used to calibrate sensors that were only required to provide approximate measurements of level. However, for more accurate calibration, we observed that it was usual to use a calibrated tank in which quantities of liquid were added, measured either by weight or by volume, to create a series of calibration points. We concluded that greater accuracy could be achieved in the calibration points if each

quantity of liquid was weighed rather than measured with volumetric measures. We also noted that water was the cheapest liquid to use in the calibration tank but observed that it was necessary to use the same liquid as normally measured for certain sensors.

17.14 Problems

- 17.1 How do dipsticks and float systems work and what are their advantages and disadvantages in liquid level measurement?
- 17.2 Sketch three different kinds of hydrostatic level measurement systems. Briefly discuss the mode of operation and applications of each.
- 17.3 Discuss the mode of operation of the following, using a sketch to aid your discussion as appropriate: capacitive level sensor, ultrasonic level sensor.
- 17.4 What are the merits of microwave and radiometric level sensors? Discuss how each of these devices works.
- 17.5 What are the main things to consider when choosing a liquid level sensor for a particular application? What type of devices could you use for an application that required (a) low measurement accuracy, (b) high measurement accuracy where contact between the sensor and measured liquid is acceptable, (c) high measurement accuracy where there must not be any contact between the sensor and measured liquid?
- 17.6 Discuss the range of devices that are able to measure the level of the surface of solid material in powdered form contained within a hopper.
- 17.7 What procedures could you use to calibrate a sensor that is only required to provide approximate measurements of liquid level?
- 17.8 What is the best calibration procedure to use for sensors that are required to give high accuracy in level measurement?
- 17.9 Carry out an internet search into the types of intelligent (smart) level sensors that are now available.

Mass, force, and torque measurement

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18.1 Introduction

Mass, force, and torque are covered together within this chapter because they are closely related quantities. Mass describes the quantity of matter that a body contains. Force is the product of mass times acceleration, according to Newton's second law of motion:

$$Force = Mass \times acceleration$$

Forces can be applied in either a horizontal or vertical direction. A force applied in a downward, vertical direction gives rise to the term **weight**, which is defined as the downward force exerted by a mass that is subject to a gravitational force:

$$\text{Weight} = \text{Mass} \times \text{acceleration due to gravity.}$$

The final quantity covered in this chapter, torque, can be regarded as a rotational force. When applied to a body, torque causes the body to rotate about its axis of rotation. This is analogous to the horizontal motion of a body when a horizontal force is applied to it.

$$\text{Torque} = \text{Force applied}$$

$$\times \text{radial distance between application point of force and axis of rotation.}$$

18.2 Mass (weight) measurement

The **mass** of a body is always quantified in terms of a measurement of the **weight** of the body, this being the downward force exerted by the body when it is subject to gravity. Three methods are used to measure this force.

The first method of measuring the downward force exerted by a mass subject to gravity involves the use of a **load cell**. The load cell measures the downward force F , and then the mass M is calculated from the equation:

$$M = F/g,$$

where g is the acceleration due to gravity.

Since the values of g vary by small amounts at different points around the earth's surface, the value of M can only be calculated exactly if the value of g is known exactly. Nevertheless, load cells are in fact the most common instrument used to measure mass, especially in industrial applications. Several different forms of load cell are available. Most load cells are now electronic, although pneumatic and hydraulic types also exist. These types vary in features and accuracy, but all are easy to use since they are deflection-type instruments that give an output reading without operator intervention.

The second method of measuring mass is to use a spring balance. This also measures the downward force when the measured mass is subject to gravity. Hence, as in the case of load cells, the mass value can only be calculated exactly if the value of g is known exactly. Like a load cell, the spring balance is also a deflection-type instrument and therefore easy to use.

The final method of measuring mass is to use some form of mass-balance instrument. These provide an absolute measurement, since they compare the gravitational force on the

mass being measured with the gravitational force on a standard mass. Since the same gravitational force is applied to both masses, the exact value of g is immaterial. However, being a null-type instrument, any form of balance is tedious to use.

The following paragraphs consider these various forms of mass-measuring instrument in more detail.

18.2.1 Electronic load cell (Electronic balance)

The electronic load cell is now the preferred type of load cell in most applications. Within an electronic load cell, the gravitational force on the body being measured is applied to an elastic element. This deflects according to the magnitude of the body mass. Mass measurement is thereby translated into a displacement measurement task.

The elastic elements used are specially shaped and designed, some examples of which are shown in Fig. 18.1. The design aims are to obtain a linear output relationship between the

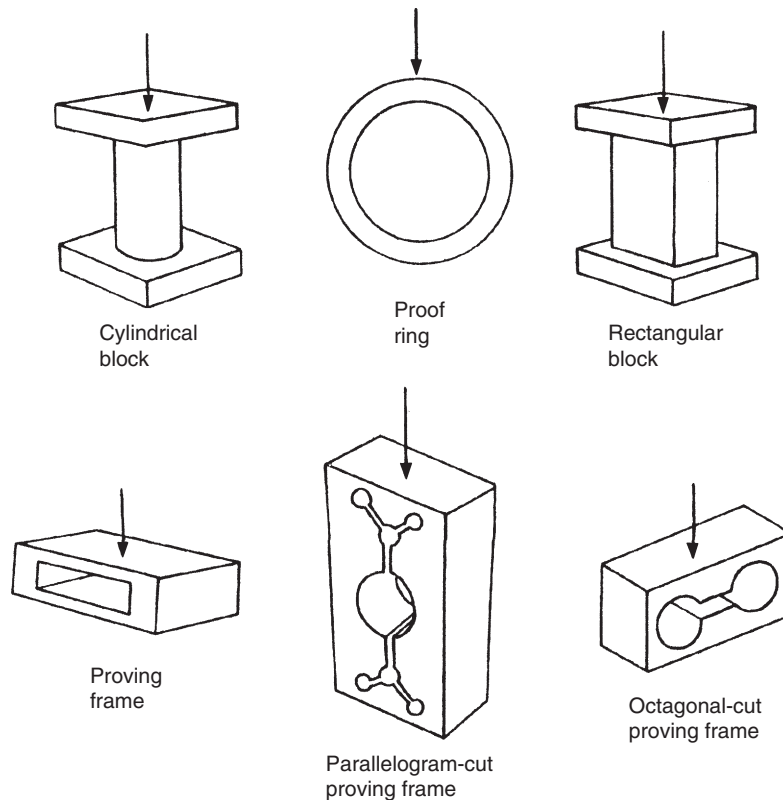


Figure 18.1
Elastic elements used in load cells.

applied force and the measured deflection and to make the instrument insensitive to forces that are not applied directly along the sensing axis. Load cells exist in both compression and tension forms. In the compression type, the measured mass is placed on top of a platform resting on the load cell, which therefore compresses the cell. In the alternative tension type, the mass is hung from the load cell, thereby putting the cell into tension.

Various types of displacement transducer are used to measure the deflection of the elastic elements. Of these, the strain gauge is used most commonly, since this gives the best measurement accuracy, with an inaccuracy figure less than $\pm 0.05\%$ of full-scale reading being obtainable. Load cells that include strain gauges are used to measure masses over a very wide range between 0 and 3000 tons. The measurement capability of an individual instrument designed to measure masses at the bottom end of this range would typically be 0.1–5 Kg, whereas instruments designed for the top of the range would have a typical measurement span of 10–3000 tons.

Elastic force transducers based on differential transformers (LVDTs) to measure deflections are used to measure masses up to 25 tons. Apart from having a lower maximum measuring capability, they are also inferior to strain gauge-based instruments in terms of their $\pm 0.2\%$ inaccuracy value. Their major advantage is their longevity and almost total lack of maintenance requirements.

The final type of displacement transducer used in this class of instrument is the piezoelectric device. Such instruments are used to measure masses in the range of 0–1000 tons. Piezoelectric crystals replace the specially designed elastic member normally used in this class of instrument, allowing the device to be physically small. As discussed previously, such devices can only measure dynamically changing forces because the output reading results from an induced electrical charge whose magnitude leaks away with time. The fact that the elastic element consists of the piezoelectric crystal means that it is very difficult to design such instruments to be insensitive to forces applied at an angle to the sensing axis. Therefore, special precautions have to be taken in applying these devices. Although such instruments are relatively cheap, their lowest inaccuracy is $\pm 1\%$ of full-scale reading, and they also have a high temperature coefficient.

Electronic load cells have significant advantages over most other forms of mass-measuring instruments in terms of their relatively low cost, wide measurement range, tolerance of dusty and corrosive environments, remote measurement capability, tolerance of shock loading, and ease of installation. However, one particular problem that can affect their performance is the phenomenon of creep. Creep describes the permanent deformation that an elastic element undergoes after it has been under load for a period of time. This can lead to significant measurement errors in the form of a bias on all readings if the instrument is not recalibrated from time to time. However, careful design and choice of materials can largely eliminate the problem.

Several compression-type load cells are often used together in a form of instrument known as the *electronic balance*. This is shown schematically in Fig. 18.2. Commonly, either three or four load cells are used in the balance, with the output mass measurement being formed from the sum of the outputs of each cell. Where appropriate, the upper platform can be replaced by a tank for weighing liquids, powders, etc.

18.2.2 Pneumatic and Hydraulic load cells

Pneumatic and hydraulic load cells translate mass measurement into a pressure measurement task, though they are now less common than the electronic load cell. A pneumatic load cell is shown schematically in Fig. 18.3. Application of a mass to the cell causes deflection of a diaphragm acting as a variable restriction in a nozzle-flapper mechanism. The output pressure measured in the cell is approximately proportional to the magnitude of the gravitational force on the applied mass. The instrument requires a flow of air at its input of around $0.25\text{m}^3/\text{h}$ at a pressure of 4 bar. Standard cells are available to measure a wide range of masses. For measuring small masses, instruments are available with a full-scale reading of 25 kg, while at the top of the range, instruments with a full-scale reading of 25 tons are obtainable. Inaccuracy is typically $\pm 0.5\%$ of full scale in pneumatic load cells.

The alternative, hydraulic load cell, is shown in Fig. 18.4. In this, the gravitational force due to the unknown mass is applied, via a diaphragm, to oil contained within an enclosed chamber. The corresponding increase in oil pressure is measured by a suitable pressure transducer. These instruments are designed for measuring much larger masses than pneumatic cells, with a load capacity of 500 tons being common. Special units can be obtained to measure masses as large as 50,000 tons. Besides their much greater measuring

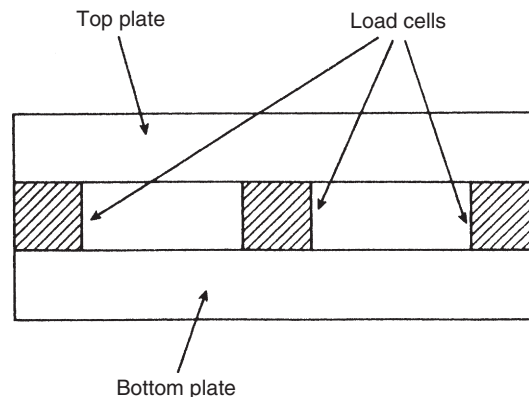


Figure 18.2
Load-cell-based electronic balance.

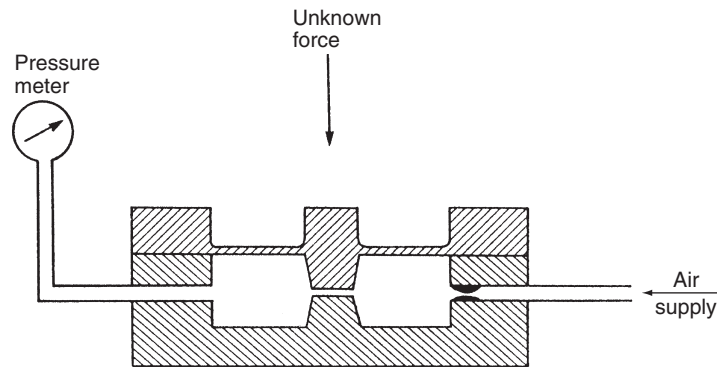


Figure 18.3
Pneumatic load cell.

range, hydraulic load cells are much more accurate than pneumatic cells, with an inaccuracy figure of $\pm 0.05\%$ of full scale being typical. However, in order to obtain such a level of accuracy, correction for the local value of g (acceleration due to gravity) is necessary. A measurement resolution of 0.02% is attainable.

18.2.3 Intelligent load cells

Intelligent load cells are formed by adding a microprocessor to a standard cell. This brings no improvement in accuracy because the load cell is already a very accurate device. What it does produce is an intelligent weighing system that can compute total cost from the measured weight, using stored cost per unit weight information, and provide an output in the form of a digital display. Cost per weight values can be prestored for a large number of substances, making such instruments very flexible in their operation.

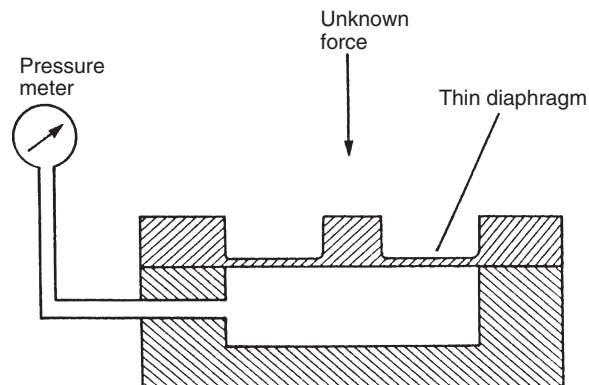


Figure 18.4
Hydraulic load cell.

In applications where the mass of an object is measured by several load cells used together (for example, load cells located at the corners of a platform in an electronic balance), the total mass can be computed more readily if the individual cells have a microprocessor providing digital output. In addition, it is also possible to use significant differences in the relative readings between different load cells as a fault detection mechanism in the system.

18.2.4 Mass balance (Weighing) instruments

Mass balance instruments are based on comparing the gravitational force on the measured mass with the gravitational force on another body of known mass. This principle of mass measurement is commonly known as *weighing*, and is used in instruments like the beam balance, weigh beam, and electromagnetic balance. Various forms of mass balance instruments are available, as discussed below.

Beam balance (Equal-arm balance): In the beam balance, shown in Fig. 18.5, standard masses are added to a pan on one side of a pivoted beam until the magnitude of the gravity force on them balances the magnitude of the gravitational force on the unknown mass acting at the other end of the beam. This equilibrium position is indicated by a pointer that moves against a calibrated scale.

Instruments of this type are capable of measuring a wide span of masses. Those at the top end of the range can typically measure masses up to 1000 g, whereas those at the bottom end of the range can measure masses of less than 0.01 g. Measurement resolution can be as good as 1 part in 10^7 of the full-scale reading if the instrument is designed and manufactured very carefully. The lowest measurement inaccuracy value attainable is $\pm 0.002\%$.

One serious disadvantage of this type of instrument is its lack of ruggedness. Continuous use and the inevitable shock loading that will occur from time to time both cause damage to the knife edges, leading to a deterioration in the measurement accuracy and

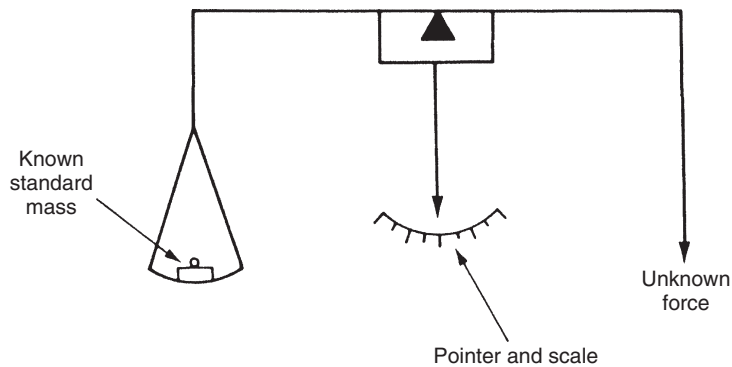


Figure 18.5
Beam balance (Equal arm balance).

measurement resolution. A further problem affecting their use in industrial applications is that it takes a relatively long time to make each measurement. For these reasons, the beam balance is normally reserved as a calibration standard and is not used in day-to-day production environments.

Weigh beam: The weigh beam, sketched in two alternative forms in Fig. 18.6, operates on similar principles to the beam balance but is much more rugged. In the first form, standard masses are added to balance the unknown mass and fine adjustment is provided by a known mass that is moved along a notched, graduated bar until the pointer is brought to the null, balance point. The alternative form has two or more graduated bars (3 bars shown in Fig. 18.6). Each bar carries a different standard mass and these are moved to appropriate positions on the notched bar to balance the unknown mass. Versions of these instruments are used to measure masses up to 50 tons.

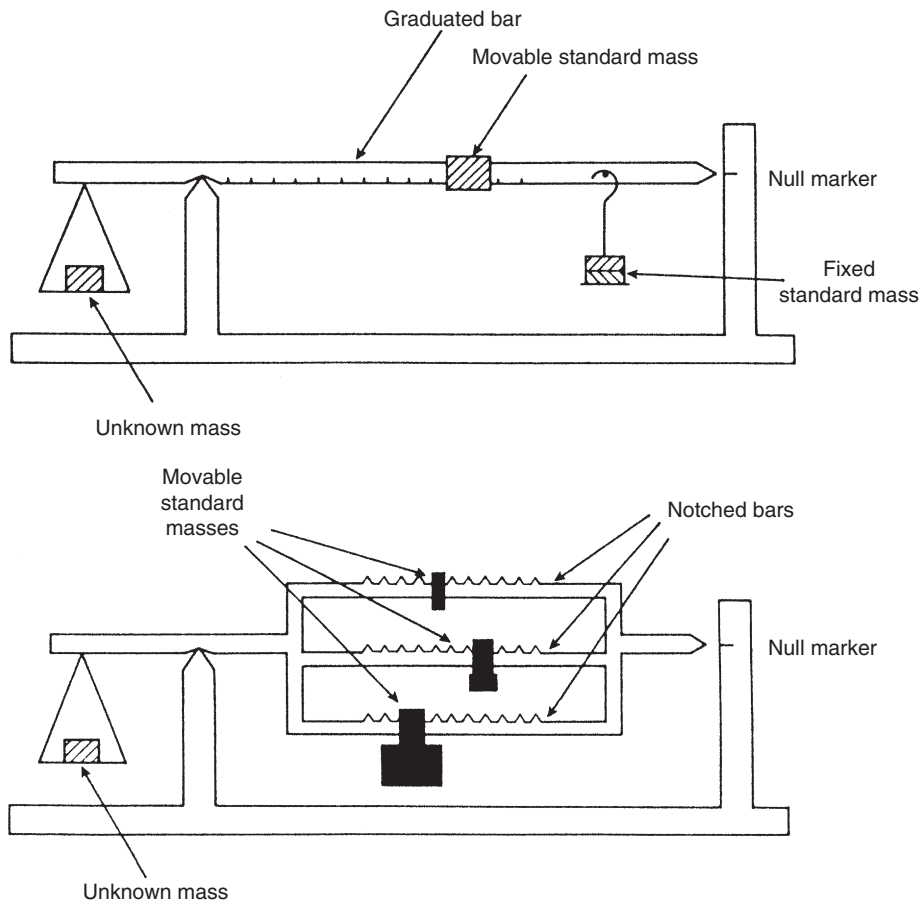


Figure 18.6
Two alternative forms of weigh beam.

Electromagnetic balance: The electromagnetic balance uses the torque developed by a current-carrying coil suspended in a permanent magnetic field to balance the unknown mass against the known gravitational force produced on a standard mass, as shown in Fig. 18.7. A light source and detector system is used to determine the null balance point. The voltage output from the light detector is amplified and applied to the coil, thus creating a servosystem where the deflection of the coil in equilibrium is proportional to the applied force. Its advantages over beam balances and weigh beams include its smaller size, its insensitivity to environmental changes (modifying inputs), and its electrical form of output. Despite these apparent advantages, it is no longer in common use because of the development of other instruments, particularly electronic balances.

18.2.5 Spring balance

Spring balances provide a method of mass measurement that is both simple and cheap. The mass is hung on the end of a spring and the deflection of the spring due to the downwards gravitational force on the mass is measured against a scale. Because the characteristics of the spring are very susceptible to environmental changes, measurement accuracy is usually relatively poor. However, if compensation is made for the changes in spring characteristics, then a measurement inaccuracy less than $\pm 0.2\%$ is achievable. According to the design of the instrument, masses between 0.5 Kg and 10 tons can be measured.

18.3 Force measurement

This section is concerned with the measurement of horizontal forces that either stretch or compress the body that they are applied to according to the direction of the force with

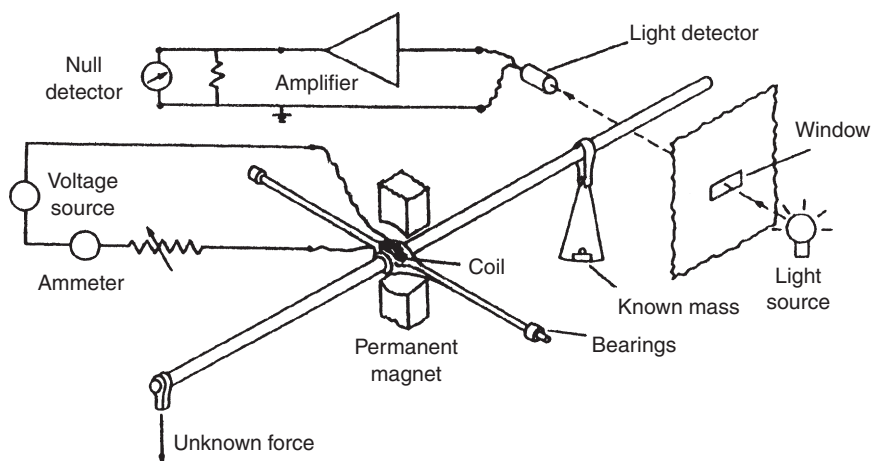


Figure 18.7
Electromagnetic balance.

respect to the body. If a force of magnitude, F , is applied to a body of mass, M , the body will accelerate at a rate, A , according to the equation:

$$F = M A$$

The standard unit of force is the *Newton*, this being the force that will produce an acceleration of 1 meter per second squared in the direction of the force when it is applied to a mass of 1 kilogram. One way of measuring an unknown force is therefore to measure the acceleration when it is applied to a body of known mass. An alternative technique is to measure the variation in the resonant frequency of a vibrating wire as it is tensioned by an applied force. Finally, forms of load cell that deform in the horizontal direction when horizontal forces are applied can also be used as force sensors. These techniques are discussed below.

18.3.1 Use of accelerometers

The technique of applying a force to a known mass and measuring the acceleration produced can be carried out using any type of accelerometer. Unfortunately, the method is of very limited practical value because, in most cases, forces are not free entities but are part of a system (from which they cannot be decoupled) in which they are acting on some body that is not free to accelerate. However, the technique can be of use in measuring some transient forces, and also for calibrating the forces produced by thrust motors in space vehicles.

18.3.2 Vibrating wire sensor

This instrument, illustrated in [Fig. 18.8](#), consists of a wire that is kept vibrating at its resonant frequency by a variable-frequency oscillator. The resonant frequency of a wire under tension is given by:

$$f = \frac{0.5}{L} \sqrt{\left(\frac{M}{T}\right)}$$

where M is the mass per unit length of the wire, L is the length of the wire, and T is the tension due to the applied force, F . Thus, measurement of the output frequency of the oscillator allows the force applied to the wire to be calculated.

18.3.3 Use of load cells

Special forms of electronic load cell that are designed to deflect in the horizontal direction are used to measure horizontal forces that are applied to them.

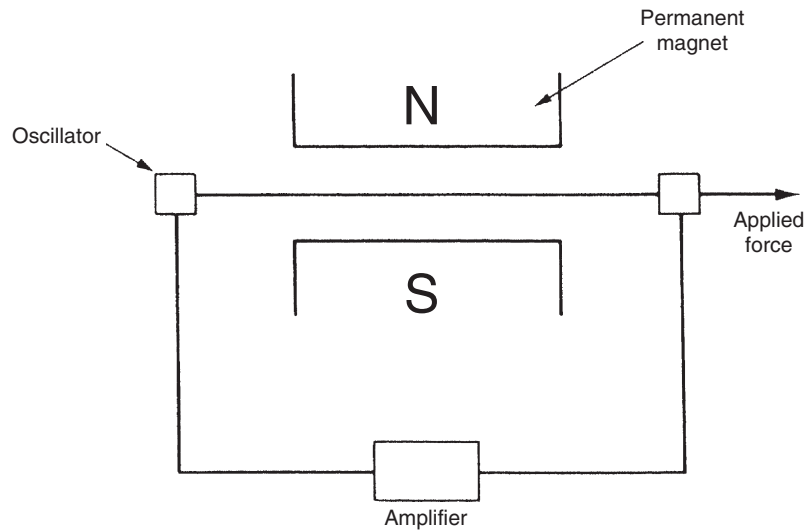


Figure 18.8
Vibrating-wire sensor.

18.4 Torque measurement

Torque can be thought of as a rotational force. If a circumferential force F is applied to a shaft of radius r , the torque produced in the shaft is given by: $T = F \times r$.

The effect of the torque applied to a shaft is to produce a shear stress in the shaft, this stress being maximum at the surface of the shaft. The maximum shear stress in a rotating shaft is given by $\tau_{\max} = \frac{T \times r}{J}$, where J is the moment of inertia of the shaft.

It is obviously important that the design of any rotating shaft, or any other form of rotating element, is such that it can withstand the maximum shear stress applied to it. If this condition is not met, the shaft will fail during service. Torque measurement is therefore of fundamental importance in all rotating bodies and applies to the rotation of shafts in many things like pumps, rotational cutting equipment, gearbox shafts, vehicle axles, and electric motors. Torque measurement is also a necessary part of measuring the power transmitted by rotating shafts.

The three methods of measuring torque now commonly used consist of either (1) measuring the strain produced in a rotating body due to an applied torque, (2) measuring torque by an optical method, and (3) measuring torque using surface acoustic wave devices. The latter are MEMS-scale devices. Historically, two other methods were also used that were mentioned in the first edition of this book. These were (1) measuring the reaction force in cradled shaft bearings and (2) using equipment known as the “Prony brake.” These have been omitted from this edition since they are rarely used nowadays.

18.4.1 Measurement of induced strain

Measuring the strain induced in a shaft due to an applied torque has been the most common method used for torque measurement in recent years. The method involves bonding four strain gauges onto the shaft as shown in Fig. 18.9, where the strain gauges are arranged in a DC bridge circuit. The output from the bridge circuit is a function of the strain in the shaft and hence of the torque applied. It is very important that the positioning of the strain gauges on the shaft is precise, and the difficulty in achieving this makes the instrument relatively expensive.

The technique is ideal for measuring the stalled torque in a shaft before rotation commences. However, a problem is encountered in the case of rotating shafts because a suitable method then has to be found for making the electrical connections to the strain gauges. One solution to this problem found in many commercial instruments is to use a system of slip rings and brushes for this, although this increases the cost of the instrument still further. An alternative solution recently developed is to use wireless telemetry to transmit the signals from the strain gauges.

18.4.2 Optical torque measurement

Optical techniques for torque measurement have been enabled by the development of laser diodes and fiber-optic light transmission systems. One such system is shown in Fig. 18.10. Two black-and-white striped wheels are mounted at either end of the rotating shaft and are in alignment when no torque is applied to the shaft. Light from a laser diode light source is directed by a pair of fiber-optic cables onto the wheels. The rotation of the wheels causes pulses of reflected light and these are transmitted back to a receiver by a second pair of fiber-optic cables. Under zero torque conditions, the two pulse trains of reflected

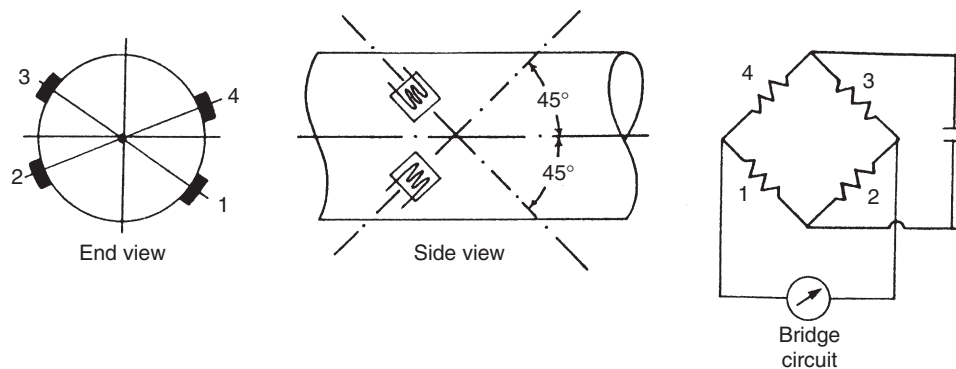


Figure 18.9
Position of torque-measuring strain gauges on shaft.

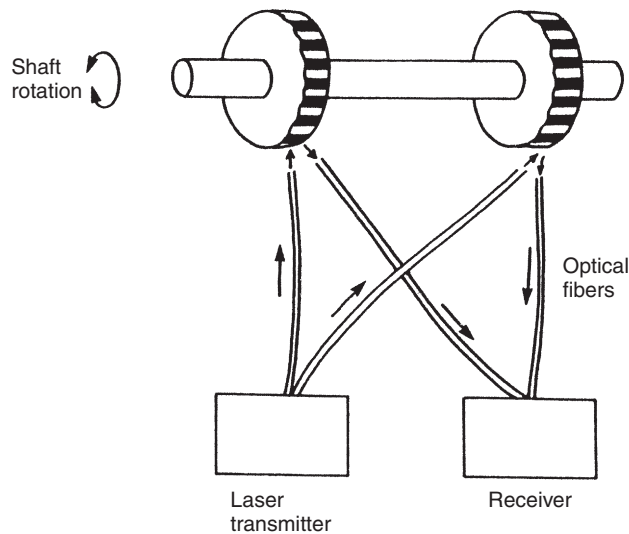


Figure 18.10
Optical torque measurement.

light are in phase with each other. If torque is now applied to the shaft, the reflected light is modulated. Measurement by the receiver of the phase difference between the reflected pulse trains therefore allows the magnitude of torque in the shaft to be calculated. The cost of such instruments is relatively low, and an additional advantage in many applications is their small physical size.

18.4.3 Torque measurement using surface acoustic wave MEMS devices

This is a relatively new technique based on surface acoustic wave (SAW) technology in microelectrical mechanical systems (MEMS) devices that is patented. Devices based on the technology are only available from one company at the present time ([TorqSense, 2013](#)). The technique involves measuring the resonant frequency change between two SAW devices when torque is applied to the shaft that the SAW devices are attached to. The SAW device consists of an array of thin metal electrodes deposited on a quartz piezoelectric crystal substrate. The electrodes are arranged such that their polarities are alternate, and this sets up a surface wave on the crystal when an RF signal of an appropriate frequency is applied to them. This is facilitated by the spacing of the electrodes, which is arranged to be either one-half or one-quarter of the frequency of the applied RF signal. When a torque is applied to the shaft, this causes strain within it and a corresponding change in the electrode spacing, and this in turn causes a change in the resonant frequency of the SAW devices. The two resonant frequencies are transmitted to a pickup outside the shaft via wireless transmission. The shaft torque is then computed by appropriate electronic signal processing and analysis.

SAW devices are part of the class of MEMS devices, and therefore have obvious size advantages. They have a high immunity to magnetic fields, and are thus applicable in applications where electronic interference is a serious problem, such as close to electric motors. They also have a fast electrical and mechanical response time. The fact that the measurement technique is a noncontact one is also advantageous in many applications.

18.5 Calibration of mass, force and torque measuring sensors

One particular difficulty that arises in the calibration of mass, force, and torque measuring instruments is the variability in the value of g (the acceleration due to gravity). Apart from instruments like the beam balance that directly compares two masses, all other instruments have an output reading that depends on the value of g .

The value of g is given by Helmert's formula:

$$g = 980.6 - 2.6 \cos \varphi - 0.000309h$$

where φ is the latitude and h is the altitude in meters.

It can be seen from this formula that g varies with both latitude and altitude. At the equator ($\cos \phi = 0^\circ$) $g = 978.0$, whereas at the poles ($\cos \phi = 0^\circ$) $g = 983.2$. In Britain, a working value of 980.7 is normally used for g , and very little error can normally be expected when using this value. Where necessary, the exact value of g can be established by measuring the period and length of a pendulum.

Another difficulty that arises in calibrating mass, force, and torque sensors is the presence of an upwards force generated by the air medium in which the instruments are tested and used. According to Archimedes' principle, when a body is immersed in a fluid (air in this case), there is an upwards force proportional to the volume of fluid displaced. Even in pure mass-balance instruments, an error is introduced because of this unless both the body of unknown mass and the standard mass have the same density. This error can be quantified as:

$$Error = \frac{SG_a}{SG_u} - \frac{SG_a}{SG_m}$$

where SG_a is the specific gravity of air, SG_u is the specific gravity of the substance being measured, and SG_m is the specific gravity of the standard mass.

Fortunately, the maximum error due to this upwards force (which has the largest magnitude when weighing low-density liquids such as gasoline) will not exceed 0.2%. Therefore, in most circumstances, the error due to air buoyancy can be neglected. However, for calibrations at the top of the calibration tree, where the highest levels of accuracy are demanded, either correction must be made for this factor or it must be avoided by carrying out the calibration in vacuum conditions.

18.5.1 Mass calibration

The primary requirement in mass calibration is the maintenance of a set of standard masses that are applied to the mass sensor being calibrated. Provided that this set of standard masses is protected from damage, there is little reason for the value of the masses to change. Despite this, the values of the masses must be checked at prescribed intervals, typically annually, in order to maintain the traceability of the calibration to reference standards. The instrument used to provide this calibration check on the standard masses is either a beam balance, a weigh beam, an electromagnetic balance, or a proof ring-based load cell.

Beam balance

The beam balance is used for calibrating masses in the range between 10 mg and 1 Kg. The measurement resolution and accuracy achieved depends on the quality and sharpness of the knife edge that the pivot is formed from. For high measurement resolution, friction at the pivot must be as close to zero as possible, and hence a very sharp and clean knife-edge pivot is demanded. The two halves of the beam on either side of the pivot are normally of equal length and are measured from the knife edge. Any bluntness, dirt, or corrosion in the pivot can cause these two lengths to become unequal, causing consequent measurement errors. Similar comments apply about the knife edges on the beam that the two pans are hung from. It is also important that all knife edges are parallel, otherwise displacement of the point of application of the force over the line of the knife edge can cause further measurement errors. This last form of error also occurs if the mass is not placed centrally on the pan.

Great care is therefore required in the use of such an instrument, and, provided that it is kept in good condition, particularly with regard to keeping the knife edges sharp and clean, high measurement accuracy is achievable. Such good condition can be confirmed by applying calibrated masses to each side of the balance. If the instrument is then exactly balanced, all is well.

Weigh beam

In order to use it as a calibration standard, a weigh beam has to be manufactured and maintained to a high standard. Provided that these conditions are met, it can be used as a standard for calibrating masses up to 50 tons.

Electromagnetic balance

Various forms of electromagnetic balance exist as alternatives to the three instruments just described for calibration duties. A particular advantage of the electromagnetic balance is its use of an optical system to magnify motion around the null point, leading to higher

measurement accuracy. Consequently, this type of instrument is often preferred for calibration duties, particularly for higher measurement ranges. The actual degree of accuracy achievable depends on the magnitude of the mass being measured. In the range between 100g and 10 kg, an inaccuracy of $\pm 0.0001\%$ is achievable. Above and below this range, the inaccuracy is worse, increasing to $\pm 0.002\%$ measuring 5 tons and $\pm 0.03\%$ measuring 10 mg.

Proof-ring-based load cell

The proof-ring-based load cell is used for calibration in the range between 150 kg and 2000 tons. When used for calibration, the displacement of the proof ring in the instrument is measured by either an LVDT or a micrometer. As the relationship between the applied mass/force and the displacement is not a straight-line one, a force/deflection graph has to be used to interpret the output. The lowest measurement inaccuracy achievable is $\pm 0.1\%$.

18.5.2 Force sensor calibration

Force sensors are calibrated by using special machines that apply a set of known force values to the sensor. The machines involved are very large and expensive. For this reason, force sensor calibration is normally devolved to either specialist calibration companies or manufacturers of the measurement devices being calibrated, both of which will give advice about the frequency of calibration necessary to maintain traceability of measurements to national reference standards.

18.5.3 Calibration of torque-measuring systems

As for the case of force sensor calibration, special machines are required for torque measurement system calibration that can apply accurately known torque values to the system being calibrated. Such machines are very expensive. It is therefore normal to use the services of specialist calibration companies or to use similar services provided by the manufacturer of the torque measurement system. Again, the company to which the calibration task is assigned will give advice on the required frequency of calibration.

18.6 Summary

We have covered the measurement of all three quantities—mass, force, and torque—in this chapter since the three quantities are closely related. We also learned that weight is another related quantity since this describes the force exerted on a mass that is subject to gravity.

Mass is measured in one of three distinct ways, using a load cell, using a spring balance, and using one of several instruments working on the mass-balance principle. Of these,

load cells and spring balances are deflection-type instruments, whereas the mass balance is a null-type instrument. This means that a balance is somewhat tedious to use compared with other forms of mass-measuring instruments.

With respect to load cells, we looked first at the electronic load cell since this is now the type of load cell that is preferred in most applications where masses between 0.1 kg and 3000 tons in magnitude are measured. We learned that pneumatic and hydraulic load cells represent somewhat older technology that is used much less frequently nowadays. However, special types of hydraulic load cells still find a significant number of applications in measuring large masses, where the maximum capability is 50,000 tons. We noted that variations in the local value of g (the acceleration due to gravity) has some effect on the accuracy of load cells but observed that the magnitude of this error was usually small. Before leaving the subject of load cells, we also made some mention of intelligent load cells.

Looking next at mass balance instruments, we saw that a particular advantage that they had was their immunity to variations in the value of g . We studied the various types of balances available in the form of the beam balance, weigh beam, and electromagnetic balance.

We then ended the review of mass-measuring instruments by looking at the spring balance. Our conclusion about this was that, while simple and cheap, its measurement accuracy is usually relatively poor.

Moving on to force measurement, we noted that transient forces could be measured by an accelerometer. However, static forces were measured either by a vibrating wire sensor or by a special form of load cell.

Finally, looking at torque measurement, we saw that the main methods currently available for measuring torque were to measure the induced strain in a rotating shaft, to measure the torque optically, and to measure torque using surface acoustic wave technology. It was noted that two older techniques that were in the first edition of this text (measuring the reaction forces in the bearings supporting a rotating shaft, and using a Prony brake) are now largely obsolete and have not been covered in this revised edition. We then concluded the chapter by examining the techniques used for calibrating the measuring devices covered in the chapter. We noted that calibration of mass-measuring sensors involved the use of a set of standard masses. Regarding the calibration of force and torque sensors, we saw that both of these required the use of special machines that generate a set of known force or torque values. Because such machines are very expensive, we noted that it was normal to use the services of either specialist calibration companies or the manufacturers of the measurement devices being calibrated.

18.7 Problems

- 18.1 What is the difference between mass and weight? Discuss briefly the three main methods of measuring the mass of a body.
- 18.2 Explain, using a sketch as appropriate, how each of the following forms of load cell work: (a) electronic, (b) pneumatic, (c) hydraulic, (d) intelligent.
- 18.3 Discuss the main characteristics of the four kinds of load cell mentioned in question 18.2. Which form is most common, and why?
- 18.4 Briefly discuss the working characteristics of the following: (a) beam balance, (b) weigh beam.
- 18.5 How does a spring balance work? What are its advantages and disadvantages compared with other forms of mass-measuring instruments?
- 18.6 Carry out an internet search into the development of MEMS sensors in mass measurement.
- 18.7 What are the available techniques for measuring a force acting in a horizontal direction?
- 18.8 Discuss briefly the three main methods used to measure torque.
- 18.9 Carry out an internet search into the development of MEMS sensors in torque measurement.
- 18.10 Discuss the general principles of calibrating mass-measuring instruments.
- 18.11 Which instruments are used as a reference standard in mass calibration? What special precautions have to be taken in manufacturing and using such reference instruments?

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Translational motion, vibration, and shock measurement

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Movement is an integral part of many systems, and therefore sensors to measure motion are an important tool for engineers. Motion occurs in many forms. Simple movement causes a *displacement* in the body affected by it, although this can take two alternative forms according to whether it is motion in a straight line (*translational displacement*) or angular motion about an axis (*rotational displacement*). Displacement only describes the fact that a body has moved but does not define the speed at which the motion occurs. Speed is defined by the term *velocity*. As for displacement, velocity occurs in two forms. *Translational velocity* describes the speed at which a body changes position when moving in a straight line, and *rotational velocity* (sometimes called *angular velocity*) describes the speed at which a body turns about the axis of rotation. Finally, it is clear that changes in velocity occur during the motion of a body. To start, the body is at rest and the velocity is zero. At the start of motion, there is a change in velocity from zero to some nonzero value. The term *acceleration* is used to describe the rate at which the velocity changes. As for displacement and velocity, acceleration also comes in two forms. *Translational acceleration* describes the rate of change of translational velocity, and *rotational acceleration* (sometimes called *angular acceleration*) describes the rate of change of rotational velocity.

With motion occurring in so many different forms, a review of the various sensors used to measure these different forms of motion would not conveniently fit within a single chapter. Therefore, this chapter reviews only sensors used to measure translational motion; those used to measure rotational motion are deferred to the next chapter. The following sections therefore look in turn at the measurement of translational displacement, velocity, and acceleration.

The subjects of vibration and shock are included in final sections of this chapter. Both of these are related to translational acceleration and therefore properly belong within this chapter on translational motion. Vibrations consist of linear harmonic motion, and measurement of the accelerations involved in this motion is important in many industrial and other environments. Shock is also related to acceleration and characterizes the motion involved when a moving body is suddenly brought to rest, often when a falling body hits the floor. This normally involves large magnitude deceleration (negative acceleration).

19.2 Displacement

Translational displacement transducers are instruments that measure the motion of a body in a straight line between two points. Apart from their use as a primary transducer measuring the motion of a body, translational displacement transducers are also widely used as a secondary component in measurement systems, in which some other physical quantity such as pressure, force, acceleration, or temperature is translated into a translational motion by the primary measurement transducer. Many different types of translational displacement transducer exist and these, along with their relative merits and characteristics, are discussed in the following sections of this chapter. The factors governing the choice of a suitable type of instrument in any particular measurement situation are considered in the final section at the end of the chapter.

19.2.1 Resistive potentiometer

The resistive potentiometer is perhaps the best-known displacement-measuring device. It consists of a resistance element with a movable contact, as shown in Fig. 19.1. A voltage V_s is applied across the two ends, A and B , of the resistance element and an output voltage V_o is measured between the point of contact C of the sliding element and the end of the resistance element A . A linear relationship exists between the output voltage V_o and the distance AC , which can be expressed by:

$$\frac{V_o}{V_s} = \frac{AC}{AB} \quad (19.1)$$

The body whose motion is being measured is connected to the sliding element of the potentiometer, so that translational motion of the body causes a motion of equal magnitude of the slider along the resistance element and a corresponding change in the output voltage V_o .

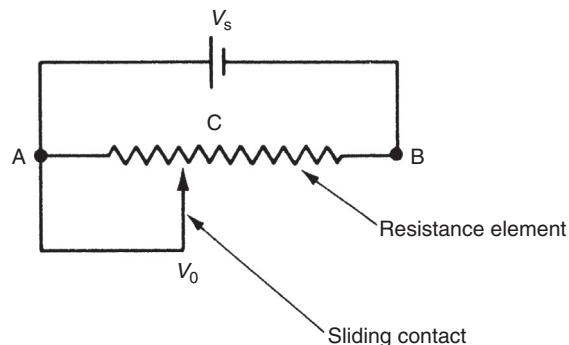


Figure 19.1
The resistive potentiometer.

Three different types of potentiometer exist, wire-wound, carbon-film and plastic-film, so named according to the material used to construct the resistance element. Wire-wound potentiometers consist of a coil of resistance wire wound on a nonconducting former. As the slider moves along the potentiometer track, it makes contact with successive turns of the wire coil. This limits the resolution of the instrument to the distance from one coil turn to the next. Much better measurement resolution is obtained from potentiometers using either a carbon film or a conducting plastic film for the resistance element. Theoretically, the resolution of these is limited only by the grain size of the particles in the film, suggesting that measurement resolutions up to 10^{-4} mm ought to be attainable. In practice, the resolution is limited by mechanical difficulties in constructing the spring system that maintains the slider in contact with the resistance track, although these types are still considerably better than wire-wound types.

Operational problems of potentiometers all occur at the point of contact between the sliding element and the resistance track. The most common problem is dirt under the slider, which increases the resistance and thereby gives a false output voltage reading, or in the worst case, causes a total loss of output. High-speed motion of the slider can also cause the contact to bounce, giving an intermittent output. Friction between the slider and the track can also be a problem in some measurement systems in which the body whose motion is being measured is moved by only a small force of a similar magnitude to these friction forces.

The life expectancy of potentiometers is normally quoted as a number of reversals (i.e., as the number of times the slider can be moved backward and forward along the track). The values quoted for wire-wound, carbon-film, and plastic-film types are typically 1 million, 5 million, and 30 million, respectively. In terms of both life expectancy and measurement resolution, therefore, the carbon and plastic film types are clearly superior, although wire-wound types do have one advantage with respect to their lower temperature coefficient. This means that wire-wound types exhibit much less variation in their characteristics in the presence of varying ambient temperature conditions.

A typical inaccuracy value that is quoted for translational motion resistive potentiometers is $\pm 1\%$ of full-scale reading. Manufacturers produce potentiometers to cover a large span of measurement ranges. At the bottom end of this span, instruments with a range of ± 2 mm are available whereas at the top end, instruments with a range of ± 1 m are produced.

The resistance of the instrument measuring the output voltage at the potentiometer slider can affect the value of the output reading, as discussed in Chapter 3. As the slider moves along the potentiometer track, the ratio of the measured resistance to that of the measuring instrument varies, and thus the linear relationship between the measured displacement and the voltage output is distorted as well. This effect is minimized when the potentiometer

resistance is small relative to that of the measuring instrument. This is achieved first by using a high-impedance measuring instrument and second, by keeping the potentiometer resistance as small as possible. Unfortunately, the latter is incompatible with achieving high measurement sensitivity because this requires a high potentiometer resistance. A compromise between these two factors is therefore necessary. The alternative strategy of obtaining high measurement sensitivity by keeping the potentiometer resistance low and increasing the excitation voltage is impossible in practice because of the power rating limitation. This restricts the allowable power loss in the potentiometer to its heat dissipation capacity.

The process of choosing the best potentiometer from a range of instruments that are available, considering power rating and measurement linearity considerations, is illustrated in the below example.

■ Example 19.1

The output voltage from a translational motion potentiometer of stroke length 0.1 m is to be measured by an instrument whose resistance is 10 k Ω . The maximum measurement error, which occurs when the slider is positioned two-thirds of the way along the element (i.e., when $AC = 2AB/3$ in Fig. 19.1), must not exceed 1% of the full-scale reading. The highest possible measurement sensitivity is also required. A family of potentiometers with a power rating of 1 W per 0.01 m and resistances ranging from 100 Ω to 10 k Ω in 100- Ω steps are available. Choose the most suitable potentiometer from this range and calculate the sensitivity of measurement that it gives.

■ Solution

Referring to the labeling used in Fig. 19.1, let the resistance of portion AC of the resistance element be R_i and that of the whole length AB of the element be R_t . Also, let the resistance of the measuring instrument be R_m and the output voltage measured by it be V_m . When the voltage-measuring instrument is connected to the potentiometer, the net resistance across AC is the sum of two resistances in parallel (R_i and R_m) given by:

$$R_{AC} = \frac{R_i R_m}{R_i + R_m}.$$

Let the excitation voltage applied across the ends AB of the potentiometer be V and the resultant current flowing between A and B be I . Then, I and V are related by:

$$I = \frac{V}{R_{AC} + R_{CB}} = \frac{V}{[R_i R_m / R_i + R_m] + R_t - R_i}$$

V_m can now be calculated as:

$$V_m = IR_{AC} = \frac{VR_iR_m}{\{[R_iR_m/(R_i + R_m)] + R_t - R_i\}\{R_i + R_m\}}$$

If we express the voltage that exists across AC in the absence of the measuring instrument as V_0 , we can express the error due to the loading effect of the measuring instrument as: $\text{Error} = V_0 - V_m$.

From Eq. (19.1), $V_0 = (R_iV)/R_t$. Thus,

$$\begin{aligned} \text{Error} = V_0 - V_m &= V \left(\frac{R_i}{R_t} \right) \left(\frac{R_iR_m}{\{[R_iR_m/(R_i + R_m)] + R_t - R_i\}\{R_i + R_m\}} \right) \\ &\times \left(\frac{R_i^2(R_i - R_t)}{R_t[R_iR_t + R_mR_t - R_i^2]} \right) \end{aligned} \quad (19.2)$$

Substituting $R_i = 2R_t/3$ into Eq. (19.2) to find the maximum error:

$$\text{Maximum error} = \frac{2R_t}{2R_t + 9R_m}$$

For a maximum error of 1%:

$$\frac{2R_t}{2R_t + 9R_m} = 0.01 \quad (19.3)$$

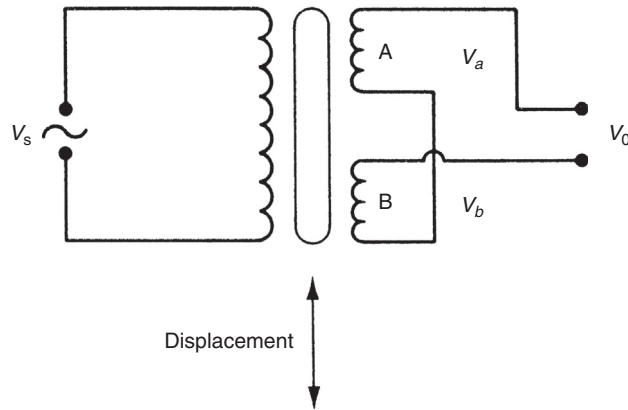
Substituting $R_m = 10,000 \Omega$ into Eq. (19.3) gives $R_t = 454 \Omega$. The nearest resistance values in the range of potentiometers available are 400 and 500 Ω . The value of 400 Ω has to be selected, because this is the only one that gives a maximum measurement error of less than 1%.

The thermal rating of the potentiometers is quoted as 1 W/0.01 m (i.e., 10 W for the total length of 0.1 m). By Ohm's law, maximum supply voltage = $\sqrt{\text{power} \times \text{resistance}} = \sqrt{10 \times 400} = 63.25$ Volts

Thus, the measurement sensitivity = $63.25/0.1$ V/m = 632.5 V/m.

19.2.2 Linear variable differential transformer

The linear variable differential transformer, which is commonly known by the abbreviation LVDT, consists of a transformer with a single primary winding and two secondary windings connected in the series opposing manner shown in Fig. 19.2. The object whose translational displacement is to be measured is physically attached to the central iron core of the transformer, so that all motions of the body are transferred to the core.


Figure 19.2

The linear variable differential transformer (LVDT).

For an excitation voltage V_s given by $V_s = V_p \sin(\omega t)$, the electromotive forces (emfs) induced in the secondary windings V_a and V_b are given by:

$$V_a = K_a \sin(\omega t - \phi) \quad ; \quad V_b = K_b \sin(\omega t - \phi)$$

Parameters K_a and K_b depend on the amount of coupling between the respective secondary and primary windings and hence on the position of the iron core. With the core in the central position, $K_a = K_b$, and we have: $V_a = V_b = K \sin(\omega t - \phi)$.

Because of the series opposition mode of connection of the secondary windings, $V_o = V_a - V_b$, and hence with the core in the central position, $V_o = 0$. Suppose the core is displaced upward (i.e., toward winding A) by a distance x . If $K_a = K_1$ and $K_b = K_2$, we have:

$$V_o = (K_1 - K_2) \sin(\omega t - \phi).$$

If, alternatively, the core were displaced downward from the null position (i.e., toward winding B) by a distance x , the values of K_a and K_b would then be $K_a = K_2$ and $K_b = K_1$, and we would have:

$$V_o = (K_2 - K_1) \sin(\omega t - \phi) = (K_1 - K_2) \sin(\omega t + [\pi - \phi]).$$

Thus, for equal magnitude displacements $+x$ and $-x$ of the core away from the central (null) position, the magnitude of the output voltage V_o is the same in both cases. The only information about the direction of movement of the core is contained in the phase of the output voltage, which differs between the two cases by 180 degrees. If, therefore, measurements of core position on both sides of the null position are required, it is necessary to measure the phase as well as the magnitude of the output voltage. The relationship between the magnitude of the output voltage and the core position is

approximately linear over a reasonable range of movement of the core on either side of the null position and is expressed using a constant of proportionality C as: $V_o = Cx$.

The only moving part in an LVDT is the central iron core. Because the core is only moving in the air gap between the windings, there is no friction or wear during operation. For this reason, the instrument is a very popular one for measuring linear displacements and has a quoted life expectancy of 200 years. The typical inaccuracy is $\pm 0.5\%$ of full-scale reading and measurement resolution is almost infinite. Instruments are available to measure a wide span of measurements from $\pm 100 \mu\text{m}$ to $\pm 100 \text{mm}$. The instrument can be made suitable for operation in corrosive environments by enclosing the windings within a nonmetallic barrier, which leaves the magnetic flux paths between the core and windings undisturbed. An epoxy resin is commonly used to encapsulate the coils for this purpose. One further operational advantage of the instrument is its insensitivity to mechanical shock and vibration.

Some problems that affect the accuracy of the LVDT are the presence of harmonics in the excitation voltage and stray capacitances, both of which cause a nonzero output of low magnitude when the core is in the null position. It is also impossible in practice to produce two identical secondary windings, and the small asymmetry that invariably exists between the secondary windings adds to this nonzero null output. The magnitude of this is always less than 1% of the full-scale output and in many measurement situations is of little consequence. When necessary, the magnitude of these effects can be measured by applying known displacements to the instrument. After this, appropriate compensation can be applied to subsequent measurements.

19.2.3 Variable capacitance transducers

Like variable inductance, the principle of variable capacitance is used in displacement measuring transducers in various ways. The three most common forms of variable capacitance transducer are shown in Fig. 19.3. In Fig. 19.3a, the capacitor plates are formed by two concentric, hollow metal cylinders. The displacement to be measured is applied to the inner cylinder, which alters the capacitance. The second form, Fig. 19.3b, consists of two flat, parallel, metal plates, one of which is fixed and one of which is movable. Displacements to be measured are applied to the movable plate, and the capacitance changes as this moves. Both of these first two forms use air as the dielectric medium between the plates. The final form, Fig. 19.3c, has two flat, parallel, metal plates with a sheet of solid dielectric material between them. The displacement to be measured causes a capacitance change by moving the dielectric sheet.

Inaccuracies as low as $\pm 0.01\%$ are possible with these instruments, with measurement resolutions of $1 \mu\text{m}$. Individual devices can be selected from manufacturers' ranges that measure displacements as small as 10^{-11}m or as large as 1m . That such instruments consist only of two simple conducting plates means that it is possible to fabricate devices

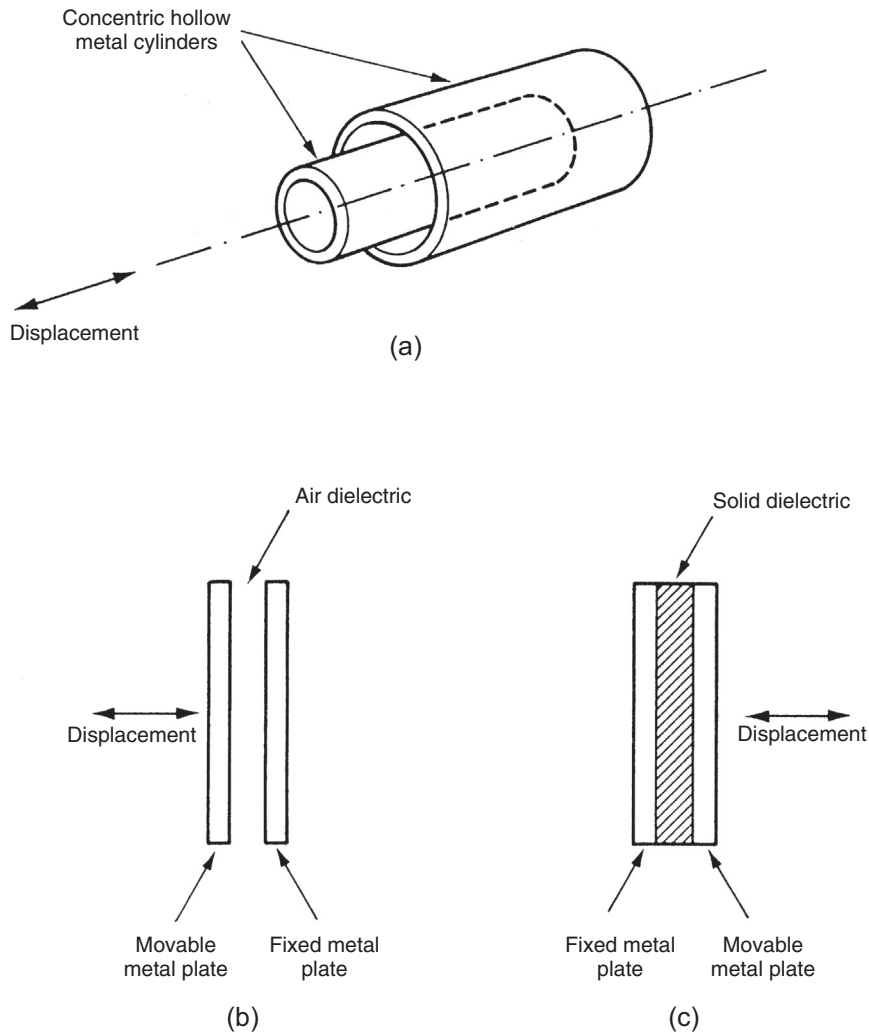


Figure 19.3
Variable-capacitance transducer.

tolerant to a wide range of environmental hazards such as extreme temperatures, radiation, and corrosive atmospheres. Because there are no contacting moving parts, there is no friction or wear in operation and the life expectancy quoted is 200 years. The major problem with variable capacitance transducers is their high impedance. This makes them very susceptible to noise and means that the length and position of connecting cables need to be chosen carefully. In addition, high-impedance instruments need to be used to measure the value of the capacitance. Because of these difficulties, use of these devices tends to be limited to the few applications for which the high accuracy and measurement resolution of the instrument are required.

19.2.4 Variable inductance transducers

One simple type of variable inductance transducer was shown earlier in Fig. 13.4. This has a typical measurement range of 0–10 mm. An alternative form of variable inductance transducer shown in Fig. 19.4a has a similar size and physical appearance to the LVDT, but has a center-tapped single winding. The two halves of the winding are connected, as shown in Fig. 19.4b, to form two arms of a bridge circuit that is excited with an alternating voltage. With the core in the central position, the output from the bridge is zero. Displacements of the core either side of the null position cause a net output voltage that is approximately proportional to the displacement for small movements of the core. Instruments in this second form are available to cover a wide span of displacement measurements. At the lower end of this span, instruments with a range of 0–2 mm are available, whereas at the top end, instruments with a range of 0–5 m can be obtained.

19.2.5 Strain gauges and piezoresistive sensors

The principles of strain gauges and piezoresistive sensors were covered earlier in Chapter 13. Because of their very small range of measurement (typically 0–50 μm), these devices are normally used only to measure displacements within devices such as diaphragm-based pressure sensors rather than as a primary sensor in their own right for direct displacement measurement. However, they can be used to measure larger displacements if the range of displacement measurement is extended by the scheme illustrated in Fig. 19.5. In this, the

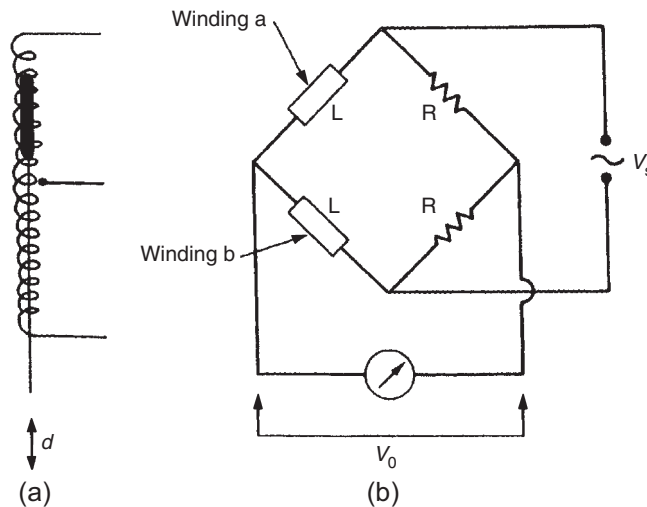


Figure 19.4

(a) Variable inductance transducers. (b) Connection in bridge circuit.

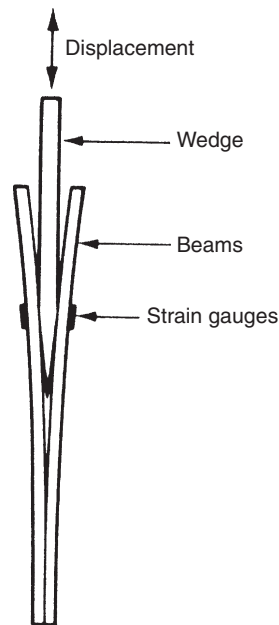


Figure 19.5

Strain gauges measuring large displacements.

displacement to be measured is applied to a wedge fixed between two beams carrying strain gauges. As the wedge is displaced downward, the beams are forced apart and strained, causing an output reading on the strain gauges. Using this method, displacements up to about 50 mm can be measured.

19.2.6 Piezoelectric transducers

The piezoelectric transducer is effectively a force-measuring device used within many instruments that are designed to measure either force itself or the force-related quantities of pressure and acceleration. It is included within this discussion of linear displacement transducers because its mode of operation is to generate an emf that is proportional to the distance by which it is compressed. The device is manufactured from a crystal, which can be either a natural material such as quartz or a synthetic material such as lithium sulfate. The crystal is mechanically stiff (i.e., a large force is required to compress it), and consequently piezoelectric transducers can only be used to measure the displacement of mechanical systems that are stiff enough themselves to be unaffected by the stiffness of the crystal. When the crystal is compressed, a charge is generated on the surface that is measured as the output voltage. Unfortunately, as is normal with any induced charge, the charge leaks away over a period of time. Consequently, the output voltage—time characteristic is as shown in [Fig. 19.6](#).

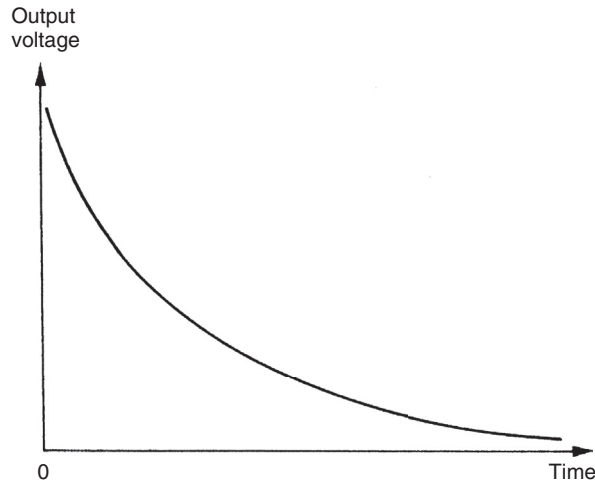


Figure 19.6

Voltage—time characteristic of piezoelectric transducer after step displacement.

Because of this characteristic, piezoelectric transducers are not suitable for measuring static or slowly varying displacements, even though the time constant of the charge-decay process can be lengthened by adding a shunt capacitor across the device.

As a displacement-measuring device, the piezoelectric transducer has a high sensitivity, about 1000 times better than the strain gauge. Its typical inaccuracy is $\pm 1\%$ of full-scale reading and its life expectancy is three million reversals.

19.2.7 Nozzle flapper

The nozzle flapper is a displacement transducer that translates displacements into a pressure change. A secondary pressure-measuring device is therefore required within the instrument. The general form of a nozzle flapper is shown schematically in Fig. 19.7. Fluid at a known supply pressure, P_s , flows through a fixed restriction and then through a variable restriction formed by the gap, x , between the end of the main vessel and the flapper plate. The body whose displacement is being measured is connected physically to the flapper plate. The output measurement of the instrument is the pressure P_o in the chamber shown in Fig. 19.7, and this is almost proportional to x over a limited range of movement of the flapper plate. The instrument typically has a first-order response characteristic. Air is commonly used as the working fluid and this gives the instrument a time constant of about 0.1 seconds. The instrument has extremely high sensitivity, but its range of measurement is small. A typical measurement range is ± 0.05 mm with a measurement resolution of ± 0.01 μm . One common application of nozzle flappers is measuring displacements within a load cell, which are typically small.

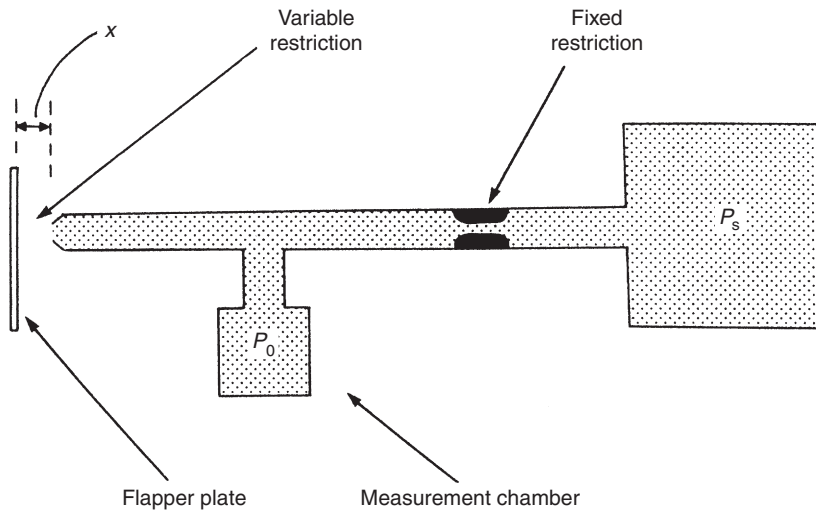


Figure 19.7
Nozzle flapper.

19.2.8 Other methods of measuring small- to medium-sized displacements

Apart from the methods outlined above, several other techniques for measuring small translational displacements exist, as discussed next. Some of these involve special instruments that have a limited sphere of application, such as for measuring machine tool displacements.

Linear inductosyn

The linear Inductosyn is an extremely accurate instrument that is widely used for axis measurement and control within machine tools. Typical measurement resolution is $2.5 \mu\text{m}$. The instrument consists of two magnetically coupled parts that are separated by an air gap, typically 0.125 mm wide, as shown in Fig. 19.8. One part, the track, is attached to the axis along which displacements are to be measured. This would generally be the bed of a machine tool. The other part, the slider, is attached to the body that is to be measured or positioned. This would usually be a cutting tool.

The track, which may be several meters long, consists of a fine metal wire formed into the pattern of a continuous rectangular waveform and deposited onto a glass base. The typical pitch (cycle length), s , of the pattern is 2 mm , and this extends over the full length of the track. The slider is usually about 50 mm wide and carries two separate wires formed into continuous rectangular waveforms that are displaced with respect to each other by one-quarter of the cycle pitch (i.e., by 90 electrical degrees). The wire waveform on the track is excited by an applied voltage given by: $V_s = V \sin(\omega t)$

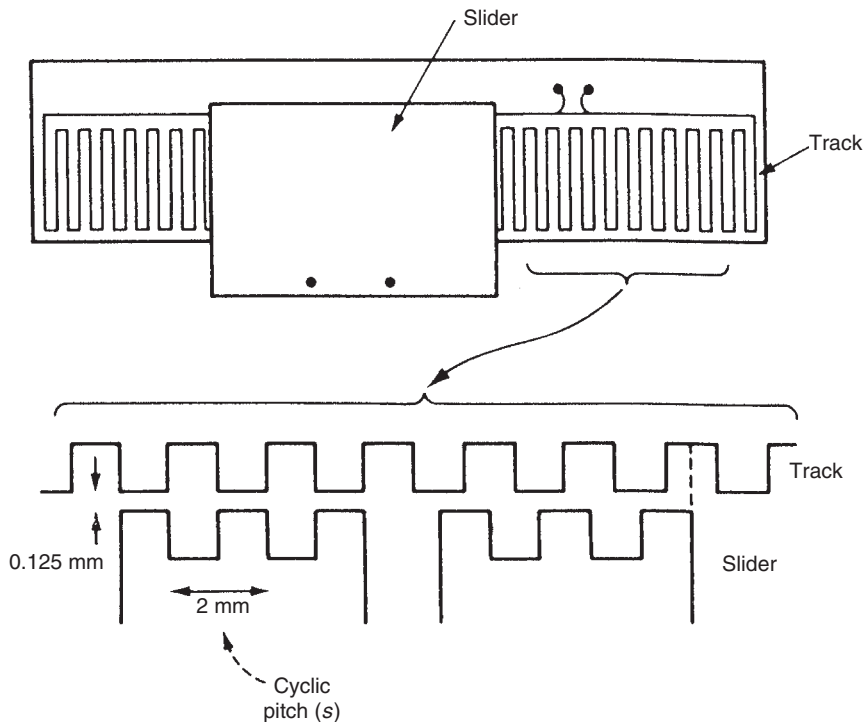


Figure 19.8
Linear inductosyn

This excitation causes induced voltages in the slider windings. When the slider is positioned in the null position such that its first winding is aligned with the winding on the track, the output voltages on the two slider windings are given by: $V_1 = 0$; $V_2 = V \sin(\omega t)$

For any other position, the slider winding voltages are given by:

$$V_1 = V \sin(\omega t) \sin(2\pi x / s) \quad ; \quad V_2 = V \sin(\omega t) \cos(2\pi x / s)$$

where x is the displacement of the slider away from the null position.

Consideration of these equations for the slider-winding outputs shows that the pattern of output voltages repeats every cycle pitch. Therefore, the instrument can discriminate only displacements of the slider within one cycle pitch of the windings. This means that the typical measurement range of an Inductosyn is only 2 mm. This is of no use in normal applications, and therefore an additional displacement transducer with coarser resolution but larger measurement range has to be used as well. This coarser measurement is commonly made by translating the linear displacements by suitable gearing into rotary motion, which is then measured by a rotational displacement transducer.

One slight problem with the Inductosyn is the relatively low level of electromagnetic coupling between the track and slider windings. Compensation for this is made by using a high-frequency excitation voltage (5–10 kHz is common).

Translation of linear displacements into rotary motion

In some applications, it is inconvenient to measure linear displacements directly, either because there is insufficient space to mount a suitable transducer or because it is inconvenient for other reasons. A suitable solution in such cases is to translate the translational motion into rotational motion by suitable gearing. Any of the rotational displacement transducers discussed in the next chapter can then be applied.

Integration of output from velocity transducers and accelerometers

If velocity transducers or accelerometers already exist in a system, displacement measurements can be obtained by integrating the output from these instruments. However, this gives information only about the relative position with respect to some arbitrary starting point. It does not yield a measurement of the absolute position of a body in space unless all motions away from a fixed starting point are recorded.

Laser interferometer

The standard interferometer has been used for over 100 years for accurate measurement of displacements. The laser interferometer is a relatively recent development that uses a laser light source instead of the conventional light source used in a standard interferometer. The laser sources extends the measurement range of the instrument by a significant amount while maintaining the same measurement resolution found in a standard interferometer. In the particular form of laser interferometer shown in Fig. 19.9, a dual-frequency

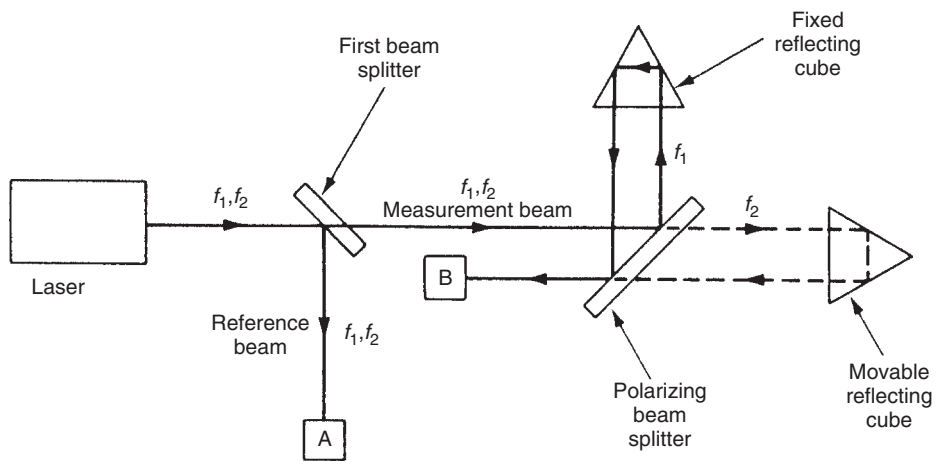


Figure 19.9
Laser interferometer.

helium–neon (He–Ne) laser is used that gives an output pair of light waves at a nominal frequency of 5×10^{14} Hz. The two waves differ in frequency by 2×10^6 Hz and have opposite polarization. This dual-frequency output waveform is split into a measurement beam and a reference beam by the first beam splitter.

The reference beam is sensed by the polarizer and photodetector, *A*, which converts both waves in the light to the same polarization. The two waves interfere constructively and destructively alternately, producing light–dark flicker at a frequency of 2×10^6 Hz. This excites a 2-MHz electrical signal in the photodetector.

The measurement beam is separated into the two component frequencies by a polarizing beam splitter. Light of the first frequency, f_1 , is reflected by a fixed reflecting cube into a photodetector and polarizer, *B*. Light of the second frequency, f_2 , is reflected by a movable reflecting cube and also enters *B*. The displacement to be measured is applied to the movable cube. With the movable cube in the null position, the light waves entering *B* produce an electrical signal output at a frequency of 2 MHz, which is the same frequency as the reference signal output from *A*. Any displacement of the movable cube causes a Doppler shift in the frequency f_2 and changes the output from *B*. The frequency of the output signal from *B* varies between 0.5 and 3.5 MHz according to the speed and direction of movement of the movable cube. The outputs from *A* and *B* are amplified and subtracted. The resultant signal is fed to a counter whose output indicates the magnitude of the displacement in the movable cube and whose rate of change indicates the velocity of motion.

This technique is used in applications requiring high-accuracy measurement, such as machine tool control. Such systems can measure displacements over ranges of up to 2 m with an inaccuracy of only a few parts per million. They are therefore an attractive alternative to the Inductosyn, in having both high measurement resolution and a large measurement range within one instrument.

Fotonic sensor

The Fotonic sensor is one of many recently developed instruments that make use of fiber-optic techniques. It consists of a light source, a light detector, a fiber-optic light transmission system, and a plate that moves with the body whose displacement is being measured, as shown on [Fig. 19.10](#). Light from the outward fiber-optic cable travels across the air gap to the plate and some of it is reflected back into the return fiber-optic cable. The amount of light reflected back from the plate is a function of the air gap length, x , and hence of the plate displacement. Measurement of the intensity of the light carried back along the return cable to the light detector allows the displacement of the plate to be calculated. Common applications of Fotonic sensors are measuring diaphragm displacements in pressure sensors and measuring the movement of bimetallic temperature sensors.

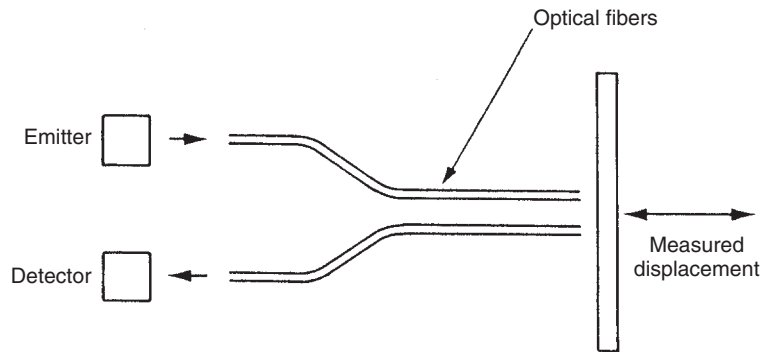


Figure 19.10
Fotonic sensor.

Noncontacting optical sensor

Fig. 19.11 shows an optical technique that is used to measure small displacements. The motion to be measured is applied to a vane whose displacement progressively shades one of a pair of monolithic photodiodes that are exposed to infrared radiation. A displacement measurement is obtained by comparing the output of the reference (unshaded) photodiode with that of the shaded one. The typical range of measurement is ± 0.5 mm with an inaccuracy of $\pm 0.1\%$ of full scale. Such sensors are used in some intelligent pressure-measuring instruments based on Bourdon tubes or diaphragms, as described in Chapter 15.

19.2.9 Measurement of large displacements (range sensors)

One final class of instruments that has not yet been mentioned consists of those designed to measure relatively large translational displacements. These are usually known as range sensors and measure the motion of a body with respect to some fixed datum point. Most range sensors use an energy source and energy detector, but measurement using a rotary potentiometer and a spring-loaded drum provides an alternative method.

Energy source/detector-based range sensors

The fundamental components in energy source/detector-based range sensors are an energy source, an energy detector, and an electronic means of timing the time of flight of the energy between the source and detector. The form of energy used is either ultrasonic or light. In some systems, both energy source and detector are fixed on the moving body, and operation depends on the energy being reflected back from the fixed boundary, as in Fig. 19.12a. In other systems, the energy source is attached to the moving body and the energy detector is located within the fixed boundary, as shown in Fig. 19.12b.

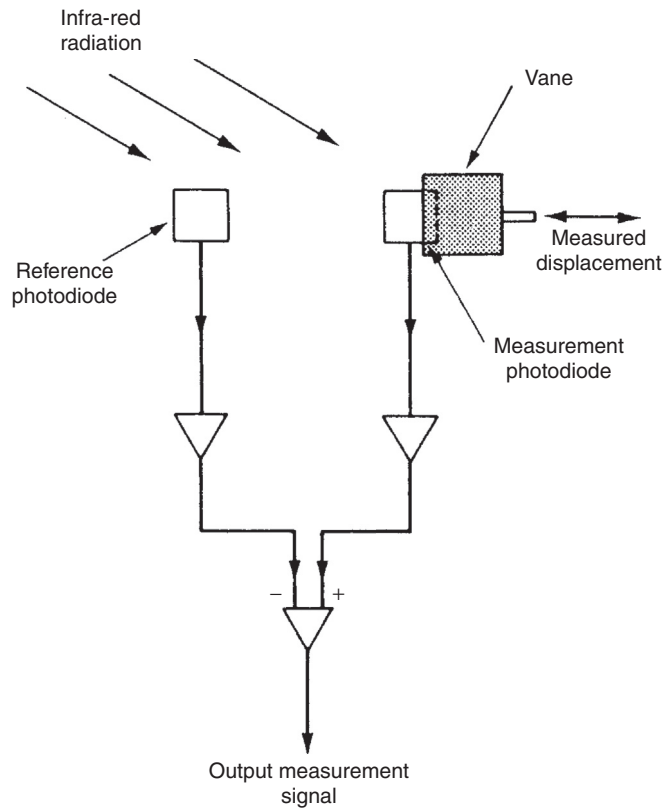


Figure 19.11
Noncontacting optical sensor.

In ultrasonic systems, the energy is transmitted from the source in high-frequency bursts. A frequency of at least 20 kHz is usual, and 40 kHz is common for measuring distances up to 5 m. By measuring the time of flight of the energy, the distance of the body from the fixed boundary can be calculated, using the fact that the speed of sound in air is 340 m/s. Because of difficulties in measuring the time of flight with sufficient accuracy, ultrasonic systems are not suitable for measuring distances of less than about 300 mm. Measurement resolution is limited by the wavelength of the ultrasonic energy and can be improved by operating at higher frequencies. At higher frequencies, however, attenuation of the magnitude of the ultrasonic wave as it passes through air becomes significant. Therefore, only low frequencies are suitable if large distances are to be measured. The typical inaccuracy of ultrasonic range-finding systems is $\pm 0.5\%$ of full scale. Fuller information about ultrasonic range measurement was given earlier in Section 13.10.5.

Optical range-finding systems generally use a laser light source. The speed of light in air is about 3×10^8 m/s, so that light takes only a few nanoseconds to travel a meter. In

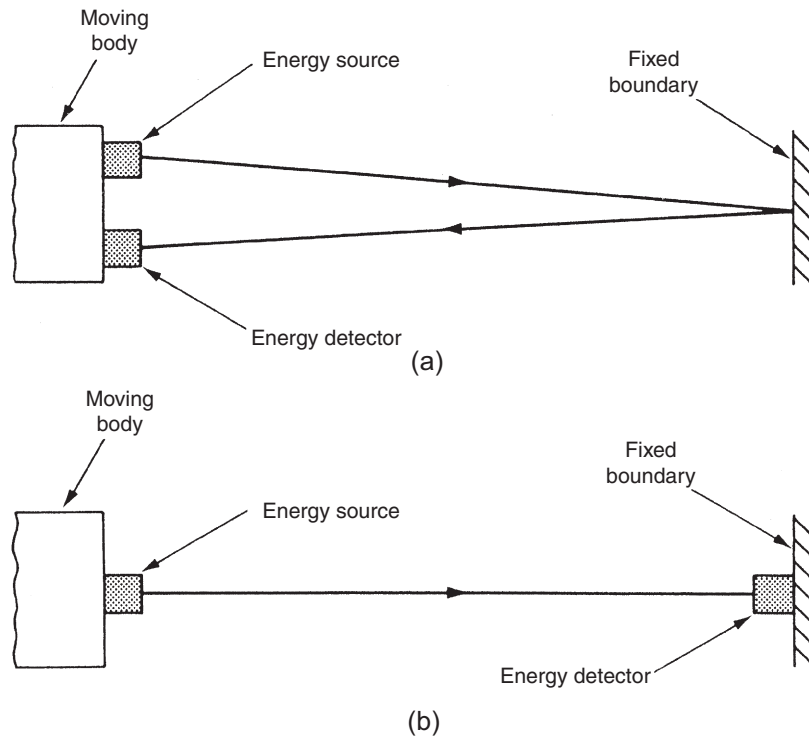


Figure 19.12
Range sensors.

consequence, such systems are suitable for measuring only large displacements for which the time of flight is long enough to be measured with reasonable accuracy.

Rotary potentiometer and spring-loaded drum

Another method for measuring large displacements that are beyond the measurement range of common displacement transducers is shown in Fig. 19.13. This consists of a steel wire attached to the body whose displacement is being measured: the wire passes round a pulley and onto a spring-loaded drum whose rotation is measured by a rotary potentiometer. A multiturn potentiometer is usually required for this to give an adequate measurement resolution. With this measurement system, it is possible to reduce measurement uncertainty to as little as $\pm 0.01\%$ of full-scale reading.

19.2.10 Proximity sensors

For the sake of completeness, it is proper to conclude this chapter on translational displacement transducers with a consideration of proximity sensors. Proximity detectors provide information on the displacement of a body with respect to some boundary, but

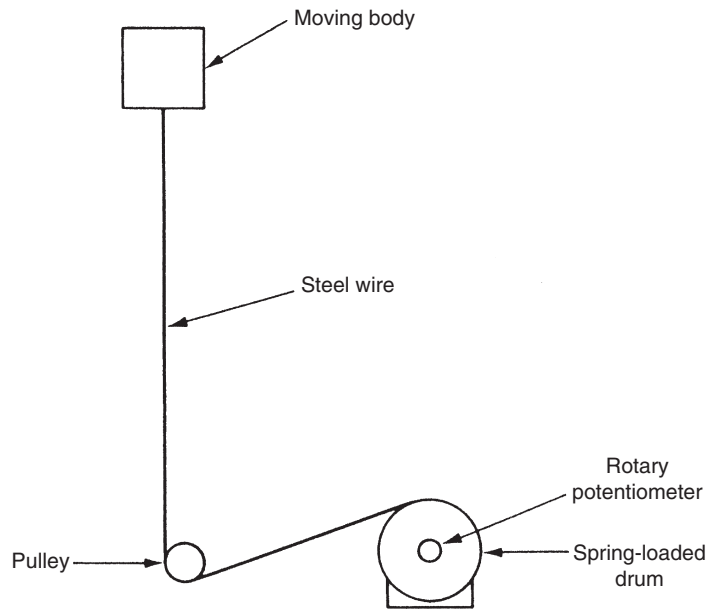


Figure 19.13

System for measuring large displacements.

only insofar as to say whether the body is less than or greater than a certain distance away from the boundary. The output of a proximity sensor is thus binary in nature: the body is or is not close to the boundary.

Like range sensors, proximity detectors use an *energy source and detector*. The detector is a device whose output changes between two states when the magnitude of the incident reflected energy exceeds a certain threshold level. A common form of proximity sensor uses an infrared light-emitting diode (LED) source and a phototransistor. Light triggers the transistor into a conducting state when the LED is within a certain distance from a reflective boundary and the reflected light exceeds a threshold level. This system is physically small, occupying a volume of only a few cubic centimeters. If even this small volume is obtrusive, fiber-optic cables can be used to transmit light from a remotely mounted LED and phototransistor. The threshold displacement detected by optical proximity sensors can be varied between 0 and 2 m.

Another form of proximity sensor uses the principle of *varying inductance*. Such devices are particularly suitable for operation in aggressive environmental conditions and can be made vibration and shock resistant by vacuum encapsulation techniques. The sensor contains a high-frequency oscillator whose output is demodulated and fed via a trigger circuit to an amplifier output stage. The oscillator output radiates through the surface of the sensor, and when the sensor surface becomes close to an electrically or magnetically conductive boundary, the output voltage is reduced because of the interference with the flux paths. At a certain point, the output voltage is reduced sufficiently for the trigger

circuit to change state and reduce the amplifier output to zero. Inductive sensors can be adjusted to change state at displacements in the range of 1–20 mm. A *Hall-effect sensor* is another form of inductive proximity sensor.

A third form of proximity sensor uses the *capacitive principle*. These can operate under conditions similar to those of inductive types. The threshold level of displacement detected can be varied between 5 and 40 mm.

Fiber-optic proximity sensors also exist in which the amount of reflected light varies with the proximity of the fiber ends to a boundary, as shown earlier in Fig. 13.8C. This is particularly useful for proximity detection in confined spaces.

19.2.11 Choosing translational measurement transducers

The choice between the various translational motion transducers available for any particular application depends mainly on the magnitude of the displacement to be measured, although the operating environment is also relevant.

The requirement to measure displacements of less than 2 mm usually occurs as part of an instrument that is measuring some other physical quantity such as pressure, and several types of device have evolved to fulfill this task. The LVDT, strain gauges, the Fotonic sensor, variable-capacitance transducers, and the noncontacting optical transducer all find application in measuring diaphragm or Bourdon-tube displacements within pressure transducers. Load cell displacements are also small and are commonly measured by nozzle flapper devices.

For measurements within the range of 2 mm to 2 m, the number of suitable instruments grows. Both the relatively cheap potentiometer and the LVDT, which is more expensive, are commonly used for such measurements. Variable-inductance and variable-capacitance transducers are also used in some applications. In addition, strain gauges measuring the strain in two beams forced apart by a wedge (see [Section 19.2.5](#)) can measure displacements up to 50 mm. If high measurement resolution is required, either the linear Inductosyn or the laser interferometer is used.

Finally, range sensors are normally used if the displacement to be measured exceeds 2 m.

As well as choosing sensors according to the magnitude of displacement to be measured, sometimes the measurement environment is also relevant. If the environmental operating conditions are severe (for example, hot, radioactive, or corrosive atmospheres), devices that can be easily protected from these conditions must be chosen, such as the LVDT, variable inductance, and variable capacitance instruments.

19.2.12 Calibration of translational displacement measurement transducers

Most translational displacement transducers measuring displacements up to 50 mm can be calibrated at the workplace level by using standard micrometers to measure a set of displacements and compare the reading from the displacement transducer being calibrated when it is reading the same set of displacements. Such micrometers can provide a reference standard with an inaccuracy of $\pm 0.003\%$ of full-scale reading. If better accuracy is required, micrometer-based calibrators are available from several manufacturers that reduce the measurement inaccuracy down to $\pm 0.001\%$ of full-scale reading.

For sensors that measure displacements exceeding 50 mm (including those classified as range sensors), the usual calibration tool is a laser interferometer. This can provide measurement uncertainty down to $\pm 0.0002\%$ of full-scale reading. According to which laser interferometer model is chosen, a measurement range up to 50 m is possible. Obviously, laser interferometers are expensive devices that are also physically large for a model measuring up to 50 m, and therefore calibration services using these are usually given to specialist calibration companies or instrument manufacturers.

19.3 Velocity

Translational velocity cannot be measured directly and therefore must be calculated indirectly by other means, as set out below.

19.3.1 Differentiation of displacement measurements

Differentiation of position measurements obtained from any of the translational displacement transducers described in [Section 19.2](#) can be used to produce a translational velocity signal. Unfortunately, the process of differentiation always amplifies noise in a measurement system. Therefore, if this method has to be used, a low-noise instrument such as a DC excited carbon film potentiometer or laser interferometer should be chosen. In the case of potentiometers, AC excitation must be avoided because of the problem that harmonics in the power supply would cause.

19.3.2 Integration of the output of an accelerometer

Where an accelerometer is included within a system, integration of its output can be performed to yield a velocity signal. The process of integration attenuates rather than amplifies measurement noise, and this is therefore an acceptable technique in terms of measurement accuracy.

19.3.3 Conversion to rotational velocity

Conversion from translational to rotational velocity is the final measurement technique open to the system designer and it is the one that is most commonly used. This conversion enables any of the rotational velocity measuring instruments described in Chapter 20 to be applied.

19.3.4 Calibration of velocity measurement systems

Because translational velocity is never measured directly, the calibration procedure used depends on the system used for velocity measurement. If a velocity measurement is being calculated from a displacement or acceleration measurement, the traceability of system calibration requires the associated displacement or acceleration transducer used to be correctly calibrated. The only other measurement technique is conversion of the translational velocity into rotational velocity, in which case the system calibration depends on the calibration of the rotational velocity transducer used.

19.4 Acceleration

The only class of device available for measuring acceleration is the accelerometer. These are available in a wide variety of types and ranges designed to meet particular measurement requirements. They have a frequency response between zero and a high value and have a form of output that can be readily integrated to give displacement and velocity measurements. The frequency response of accelerometers can be improved by altering the level of damping in the instrument. Such adjustment must be done carefully, however, because frequency response improvements are achieved only at the expense of degrading the measurement sensitivity. Besides their use for general-purpose motion measurement, accelerometers are widely used to measure mechanical shocks and vibrations.

Most forms of accelerometer consist of a mass suspended by a spring and damper inside a housing, as shown in Fig. 19.14. The accelerometer is rigidly fastened to the body undergoing acceleration. Any acceleration of the body causes a force, F_a , on the mass, M , given by: $F_a = M\ddot{x}$.

This force is opposed by the restraining effect, F_s , of a spring with spring constant K , and the net result is that the mass is displaced by a distance x from its starting position such that: $F_s = Kx$.

In steady-state, when the mass inside is accelerating at the same rate as the case of the accelerometer, $F_a = F_s$ and so:

$$Kx = M\ddot{x} \quad \text{or} \quad \ddot{x} = (Kx)/M \quad (19.4)$$

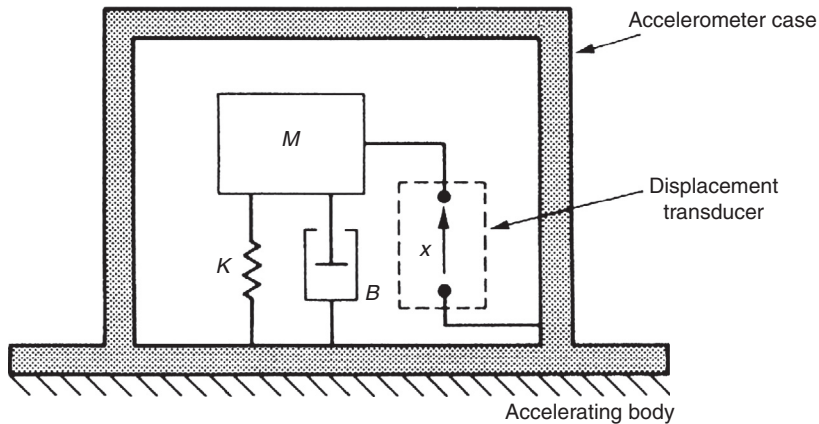


Figure 19.14
Structure of an accelerometer.

This is the equation of motion of a second order system, and, in the absence of damping, the output of the accelerometer would consist of nondecaying oscillations. A damper is therefore included within the instrument, which produces a damping force, F_d , proportional to the velocity of the mass M given by: $F_d = B\dot{x}$

This modifies the previous equation of motion (Eq. 19.4) to the following:

$$Kx + B\dot{x} = M\ddot{x} \quad (19.5)$$

One important characteristic of accelerometers is their sensitivity to accelerations at right angles to the sensing axis (the direction along which the instrument is designed to measure acceleration). This is defined as the **cross-sensitivity** and is specified in terms of the output, expressed as a percentage of the full-scale output, when an acceleration of some specified magnitude (e.g., 30g) is applied at 90 degrees to the sensing axis.

The acceleration reading is obtained from the instrument by measurement of the displacement of the mass within the accelerometer. Many different displacement-measuring techniques are used in the various types of accelerometer that are commercially available. Different types of accelerometer also vary in terms of the type of spring element and form of damping used.

Resistive potentiometers are one such displacement-measuring instrument used in accelerometers. These are used mainly to measure slowly varying accelerations and low-frequency vibrations in the range of 0–50g. The measurement resolution obtainable is about 1 in 400 and typical values of cross-sensitivity are $\pm 1\%$. Inaccuracy is about $\pm 1\%$ and life expectancy is quoted at two million reversals. A typical size and weight are 125 cm³ and 500 g.

Strain gauges and piezoresistive sensors are also used in accelerometers to measure accelerations up to 200g. These serve as the spring element and also measure the mass displacement, thus simplifying the instrument's construction. Their typical characteristics are a resolution of 1 in 1000, inaccuracy of $\pm 1\%$, and cross-sensitivity of 2%. They have a major advantage over potentiometer-based accelerometers in terms of their much smaller size and weight (3 cm³ and 25 g).

Another displacement transducer found in accelerometers is the LVDT. This device can measure accelerations up to 700g with a typical inaccuracy of $\pm 1\%$ of full scale. They are of a similar physical size to potentiometer-based instruments but are lighter-weight (100 g).

The other common displacement transducer used in accelerometers is the piezoelectric type. The major advantage of using piezoelectric crystals is that they also act as the spring and damper within the instrument. In consequence, the device is small (15 cm³) and has a low mass (50 g), but because of the nature of piezoelectric crystal operation, such instruments are not suitable for measuring constant or slowly time-varying accelerations. Because the electrical impedance of a piezoelectric crystal is itself high, the output voltage must be measured with a high-impedance instrument to avoid loading effects. Many piezoelectric crystal-based accelerometers incorporate a high-impedance charge amplifier within the body of the instrument. This simplifies the signal conditioning requirements external to the accelerometer but can lead to problems in certain operational environments because these internal electronics are exposed to the same environmental hazards as the rest of the accelerometer. Typical measurement resolution of this class of accelerometer is 0.1% of full scale with an inaccuracy of $\pm 1\%$. Individual instruments are available to cover a wide range of measurements from 0.03g full-scale up to 1000g full scale.

Intelligent accelerometers are also available that give even better performance through inclusion of processing power to compensate for environmentally induced errors.

Recently, Microelectromechanical system (MEMS) sensors are available for measuring acceleration. These find application in things such as crash sensing in vehicle airbags. They are also used in computer game controllers and cell phones, and for shake detection in digital cameras. Currently, two forms of MEMS accelerometer exist. The **cantilever beam accelerometer** (Fig. 19.15) consists of a small mass subject to the measured acceleration that is mounted on a thin silicon cantilever-shaped membrane. In the example shown in Fig. 19.15, displacement of the membrane owing to the acceleration force is measured by piezoresistors deposited on the membrane. However, a variation on this type of accelerometer uses varying capacitance to measure the displacement by etching a variable capacitor plate into the membrane. The **balanced force accelerometer** (Fig. 19.16) is the alternative form of MEMS device available for measuring acceleration. This consists of a thin silicon structure attached to springs at both ends. The silicon structure acts as the accelerometer mass. The device has three electrodes: one movable

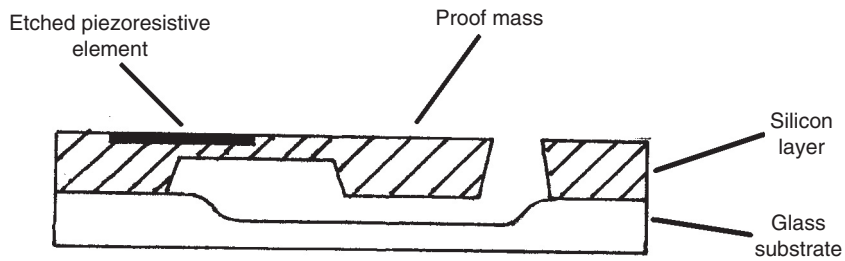


Figure 19.15
Cantilever beam accelerometer.

central electrode attached to one side of the silicon structure and two fixed electrodes attached to the casing of the device and located on either side of the central electrode. When the device becomes subject to an acceleration force, the mass moves to one side until the opposing spring forces balance the acceleration force. As the mass moves, the central electrode attached to it moves closer to one of the fixed electrodes and farther away from the other. The amount of movement of the mass, and hence the acceleration measured, is then calculated from the change in capacitance between the electrodes.

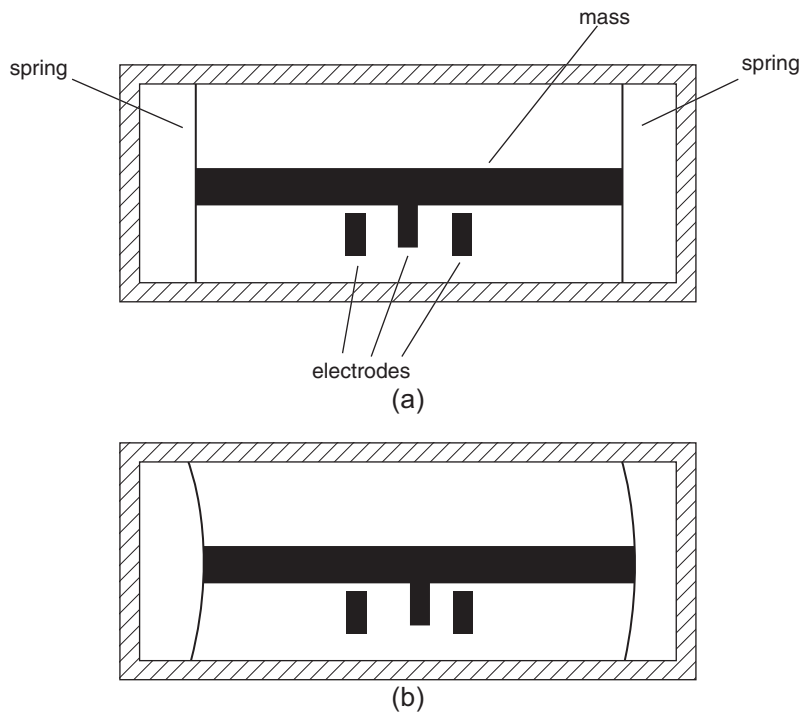


Figure 19.16
Balanced force accelerometer. (a) Sensor with zero acceleration applied. (b) Sensor with deflection of mass owing to applied acceleration.

Two forms of *fiber optic–based accelerometer* also exist. One form measures the effect on light transmission intensity caused by a mass subject to acceleration resting on a multimode fiber. The other form measures the change in phase of light transmitted through a monomode fiber that has a mass subject to acceleration resting on it.

19.4.1 Selection of accelerometers

In choosing between the different types of accelerometer for a particular application, the mass of the instrument is particularly important. This should be much less than that of the body whose motion is being measured, to avoid loading effects that affect the accuracy of the readings obtained. In this respect, instruments based on strain gauges are best.

19.4.2 Calibration of accelerometers

The primary method of calibrating accelerometers is to mount them on a table rotating about a vertical axis such that the sensing axis of the accelerometer is pointing toward the axis of rotation of the table. The acceleration, a , is then given by:

$$a = r(2\pi v)^2$$

where r is the radius of rotation measured from the center of the rotating table to the center of the accelerometer mass, and v is the velocity of rotation of the table (in revolutions per second).

This obviously requires the rotational speed of the table to be measured accurately by a calibrated sensor. Provided this condition is met, various reference acceleration values can be generated by changing the rotational speed of the table.

19.5 Vibration

19.5.1 Nature of vibration

Vibrations are commonly encountered in the operation of machinery and industrial plants; therefore, measurement of the accelerations associated with such vibrations is extremely important in industrial environments. The peak accelerations involved in such vibrations can be of 100g or greater in magnitude, although both the frequency of oscillation and the magnitude of displacements from the equilibrium position in vibrations tend to vary randomly. Vibrations normally consist of linear harmonic motion that can be expressed mathematically as:

$$X = X_o \sin(\omega t) \tag{19.6}$$

where X is the displacement from the equilibrium position at any general point in time, X_o is the peak displacement from the equilibrium position, and ω is the angular frequency of

the oscillations. By differentiating Eq. (19.6) with respect to time, an expression for the velocity v of the vibrating body at any general point in time is obtained as:

$$v = -\omega X_o \cos(\omega t) \quad (19.7)$$

Differentiating Eq. (19.7) again with respect to time, we obtain an expression for the acceleration, α , of the body at any general point in time as:

$$\alpha = -\omega^2 X_o \sin(\omega t) \quad (19.8)$$

Inspection of Eq. (19.8) shows that the peak acceleration is given by:

$$\alpha_{peak} = \omega^2 X_o \quad (19.9)$$

This square law relationship between peak acceleration and oscillation frequency is the reason why high values of acceleration occur during relatively low-frequency oscillations. For example, an oscillation at 10 Hz produces peak accelerations of $2g$.

■ Example 19.2

A pipe carrying a fluid vibrates at a frequency of 60 Hz with displacements of 8 mm from the equilibrium position. Calculate the peak acceleration. ■

■ Solution

From Eq. (19.9), $\alpha_{peak} = \omega^2 X_o = (2\pi 60)^2 \times (0.008) = 1137.0 \text{ m/s}^2$

Using the fact that the standard acceleration due to gravity, g , is 9.81 m/s^2 , this answer can be expressed alternatively as: $\alpha_{peak} = 1137.0/9.81 = 115.9g$. ■

19.5.2 Vibration measurement

It is apparent that the intensity of vibration can be measured in terms of displacement, velocity, or acceleration. Acceleration is clearly the best parameter to measure at high frequencies. However, because displacements are large at low frequencies according to Eq. (19.9), it would seem that measuring either displacement or velocity would be best at low frequencies. The amplitude of vibrations can be measured by various forms of displacement transducer. Fiber optic-based devices are particularly attractive and can give measurement resolution as good as $1 \mu\text{m}$. Unfortunately, there are considerable practical difficulties in mounting and calibrating displacement and velocity transducers; therefore, they are rarely used. Because of this, vibration is usually measured by accelerometers at all frequencies. The most common type of

transducer used is the piezoaccelerometer, which has typical inaccuracy levels of $\pm 2\%$. Capacitive MEMS accelerometers are also available and are becoming popular.

The frequency response of accelerometers is particularly important in vibration measurement in view of the inherently high-frequency characteristics of the measurement situation. The bandwidth of potentiometer-based accelerometers goes up to only 25 Hz. Accelerometers that include strain gauges can measure frequencies up to 150 Hz and the latest instruments using piezoresistive strain gauges have bandwidths up to 2 kHz. Finally, inclusion of piezoelectric crystal displacement transducers yields an instrument with a bandwidth that can be as high as 7 kHz.

When measuring vibration, consideration must be given to the fact that attaching an accelerometer to the vibrating body will significantly affect the vibration characteristics if the body has a small mass. The effect of such loading of the measured system can be quantified by following the equation:

$$a_1 = a_b \left(\frac{m_b}{m_b + m_a} \right).$$

where a_1 is the acceleration of the body with accelerometer attached, a_b is the acceleration of the body without the accelerometer, m_a is the mass of the accelerometer, and m_b is the mass of the body. Such considerations emphasize the advantage of piezoaccelerometers and capacitive MEMS accelerometers for measuring vibration, because these have a lower mass than other forms of accelerometer and so contribute least to this system-loading effect.

As well as an accelerometer, a vibration measurement system requires other elements to translate the accelerometer output into a recorded signal, as shown in [Fig. 19.17](#). The three other necessary elements are a signal conditioning element, a signal analyzer, and a signal recorder. The signal-conditioning element amplifies the relatively weak output signal from the accelerometer and transforms the high output impedance of the accelerometer to a lower impedance value. The signal analyzer then converts the signal into the form required for output. The output parameter may be displacement, velocity, or acceleration, and this may be expressed as the peak value, root-mean-square value, or average absolute value. The final element of the measurement system is the signal recorder. All elements of the measurement system, and especially the signal recorder, must be chosen carefully to avoid distortion of the vibration waveform. The bandwidth should be such that it is at least a factor of 10 better than the bandwidth of the vibration frequency components at both ends. Thus, its lowest frequency limit should be less than or equal to 0.1 times the fundamental frequency of vibration and its upper frequency limit should be greater than or equal to 10 times the highest significant vibration frequency component.

If the frequency of vibration has to be known, the stroboscope is a suitable instrument to measure this. If the stroboscope is made to direct light pulses at the body at the same frequency as the vibration, the body will apparently stop vibrating.

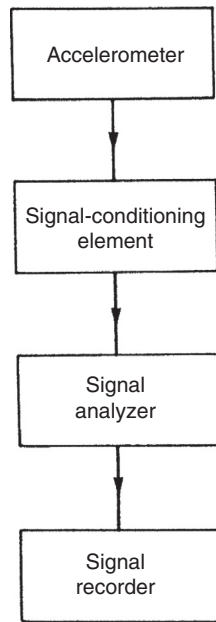


Figure 19.17
Vibration measurement system.

19.5.3 Calibration of vibration sensors

Calibration of the accelerometer used within a vibration measurement system is normally carried out by mounting the accelerometer in a back-to-back configuration with a reference calibrated accelerometer on an electromechanically excited vibrating table.

19.6 Shock

Shock describes a type of motion in which a moving body is brought suddenly to rest, often because of a collision. This is very common in industrial situations and usually involves a body being dropped and hitting the floor. Shocks characteristically involve large-magnitude deceleration (e.g., 500g) that lasts for a very short time (e.g., 5 ms). An instrument with a high-frequency response is required for shock measurement, and for this reason, piezoelectric crystal-based accelerometers are commonly used. Other elements for analyzing and recording the signal are required, as shown in Fig. 19.18 and described in the last section. A storage oscilloscope is a suitable instrument for recording the output signal, because this allows the time duration as well as the acceleration levels in the shock to be measured. Alternatively, if a permanent record is required, the screen of a standard oscilloscope can be photographed.

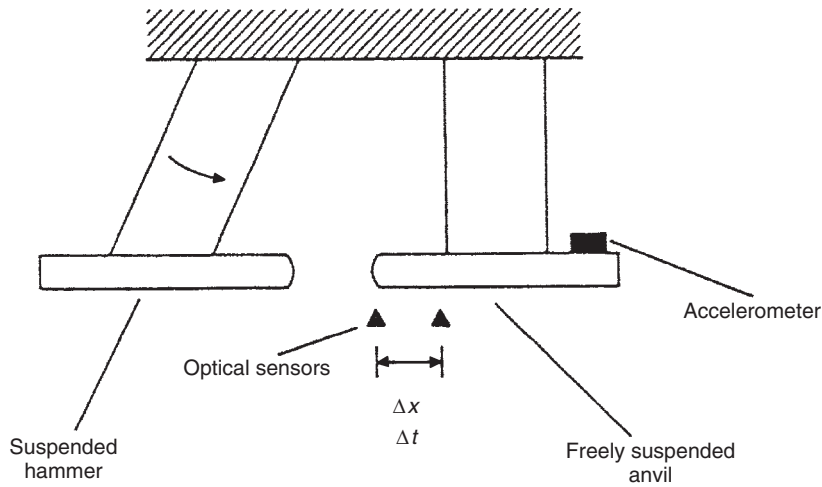


Figure 19.18
Shock measurement.

■ Example 19.3

A body is dropped from a height of 10 m and experiences a shock when it hits the ground. If the duration of the shock is 5 ms, calculate the magnitude of the shock in terms of g .

■ Solution

The equation of motion for a body falling under gravity gives the following expression for the terminal velocity: v :

$$v = \sqrt{2gx}$$

where x is the height through which the body falls. Having calculated v , the average deceleration during the collision can be calculated as:

$$\alpha = v/t$$

where t is the time duration of the shock. Substituting the appropriate numerical values into these expressions:

$$v = \sqrt{(2 \times 9.81 \times 10)} = 14.0 \text{ m/s};$$

$$\alpha = 14.0/0.005 = 2801 \text{ m/s} = 286 \text{ g}.$$

19.6.1 Calibration of shock sensors

Calibration of the accelerometer used within a shock sensor is frequently carried out using a pneumatic shock exciter. This device consists of a piston within a circular tube. High-pressure air is applied to one face of the piston, but it does not move initially because it is held at the end of the tube by a mechanical latching mechanism. When the latch is released, the piston accelerates at a high rate until it is brought to rest by a padded anvil at the other end of the tube. The accelerometer being calibrated and a calibrated reference accelerometer are both mounted on the anvil. By varying the characteristics of the padding, the deceleration level and hence magnitude of the shock produced on the anvil can be varied.

19.7 Summary

This chapter has been concerned with the measurement of translational (straight-line) motion. This can take the form of displacement, velocity (rate of change of displacement), or acceleration (rate of change of velocity). We have looked at sensors for measuring each of these, and in the case of acceleration, we have also studied vibration and shock measurement, because both involve acceleration measurement.

Our study of displacement sensors started with the resistive potentiometer, where we learned that potentiometers came in three different forms: wire-wound, carbon-film, and plastic-film. We then look at the LVDT, variable capacitance, and variable inductance sensors. We noted that strain gauges were used to measure small displacements (typically up to 50 μm in magnitude). We also noted that force-measuring piezoelectric sensors could be regarded as displacement sensors because their mode of operation is to generate an emf that is proportional to the distance by which they are compressed by the applied force. We also discussed the nozzle flapper, which measures displacements by converting them into a pressure change. We then summarized some other techniques for measuring small and medium-sized displacements, including translating linear motion into rotational motion, integrating the output from velocity and acceleration sensors, and using specialist devices such as the linear Inductosyn, laser interferometer, Fotonic sensor, and noncontacting optical sensor. In a discussion on the measurement of relatively large displacements, we showed that this could be achieved by several devices commonly called range sensors. We included some mention of proximity sensors, because these properly belong within the classification of displacement sensors, although they are a special case in that their binary form of output merely indicates whether the sensor is or is not within some threshold distance of a boundary. Finally, before leaving the subject of displacement measurement, we looked at the techniques used to calibrate them.

Our discussion of translational velocity measurement introduced us to the fact that this cannot be measured directly. We then examined the only three ways to measure it: differentiation of position measurements, integration of the output of an accelerometer,

and conversion from translational to rotational velocity. Finally, we considered how measurements obtained via each of the techniques could be calibrated.

In the case of acceleration measurement, we observed that this could be measured only by some form of accelerometer. We noted that attributes such as frequency response and cross-sensitivity were important as well as measurement accuracy in accelerometers. We discovered that almost all accelerometers work on the principle that a mass inside them displaces when subject to acceleration. Accelerometers differ mainly in the technique used to measure this mass displacement, and we looked in turn at devices that use the resistive potentiometer, strain gauge, piezoresistive sensor, LVDT, and variable capacitance sensor, respectively. We then discussed the one exception to the rule that accelerometers contain a moving mass: the piezoelectric accelerometer. Finally, we looked at the primary method of calibrating accelerometers using a rotating table.

We then concluded the chapter by looking at vibration and shock measurement. Both of these involve accelerations, and therefore both need an accelerometer to quantify their magnitude. Starting with vibration, we noted that this was a common phenomenon, especially in industrial situations. We learned that vibration consists of a linear harmonic motion in which the peak acceleration can exceed 100g and in which the oscillation frequency and peak amplitude can vary randomly. We noted that the amplitude of vibration could be calculated from a measurement of the peak acceleration, and we went on to look at the suitability of various forms of accelerometer for such a measurement.

Finally, we considered shock measurement. This revealed that large-magnitude decelerations are involved in the phenomenon of shock, which typically occurs when a falling body hits the floor or a collision occurs between two solid objects. High-frequency response is particularly important in shock measurement, and the most suitable device to measure this is a piezoelectric crystal-based accelerometer.

19.8 Problems

- 19.1 Discuss the mode of operation and characteristics of a linear motion resistive potentiometer.
- 19.2 What is an LVDT? How does it work?
- 19.3 Explain how the following two instruments work and discuss their main operating characteristics and uses: (a) variable capacitance transducer, (b) variable inductance transducer.
- 19.4 Discuss the mode of operation and characteristics of the following: (a) piezoelectric displacement transducer, (b) nozzle flapper (in the case of the nozzle flapper, provide a schematic sketch of the device to aid your description of its operation).
- 19.5 Sketch a linear Inductosyn. How does it work? What are its main characteristics?
- 19.6 What is a laser interferometer and what are its principle characteristics? With the aid of a sketch, explain how it works.

- 19.7 What are range sensors? Describe two main types of range sensor.
- 19.8 Discuss the main types of proximity sensor available, mentioning particularly their suitability for operation in harsh environments.
- 19.9 What are the main considerations in choosing a translational motion transducer for a particular application? Give examples of some types of translational motion transducer and the applications for which they are suitable.
- 19.10 Discuss the usual calibration procedures for translational motion transducers.
- 19.11 What are the main ways of measuring translational velocities? How are such measurements calibrated?
- 19.12 What are the principles of operation of a linear motion accelerometer? What features would you expect to see in a high-quality accelerometer?
- 19.13 What types of displacement sensor are used within accelerometers? What are the relative merits of these alternative displacement sensors?
- 19.14 Describe the different forms of MEMS sensor that are used in accelerometers and sketch their structure.
- 19.15 Write a mathematical equation that describes the phenomenon of vibration.
- 19.16 Explain briefly the three main ways of measuring vibration.
- 19.17 When an accelerometer is attached to a vibrating body, it has a loading effect that alters the characteristics of the vibration. Write a mathematical equation that describes this loading effect. How can this loading effect be minimized?
- 19.18 A translational motion potentiometer of stroke length 200 mm is to be used in a particular measurement situation. The output voltage from the potentiometer is to be measured by an instrument whose resistance is 15 k Ω . The maximum measurement error, which occurs when the slider is positioned two-thirds of the way along the element (i.e., when $AC = 2AB/3$ in Fig. 19.1), must not exceed 0.5% of the full-scale reading. The highest possible measurement sensitivity is also required. A family of potentiometers with a power rating of 0.1 W/mm and resistances ranging from 100 Ω to 10 k Ω in 100- Ω steps are available. Choose the most suitable potentiometer from this range and calculate the measurement sensitivity that it provides.

- 19.19 (a) Show that the expression for peak acceleration α in a vibrating body is given by:

$$\alpha = -\omega^2 X_o \sin(\omega t)$$

where X_o is the peak displacement from the equilibrium position, and ω is the angular frequency of the oscillations.

- (b) A pipe carrying a fluid vibrates at a frequency of 50 Hz with displacements of 7 mm from the equilibrium position. Calculate the peak acceleration.
- 19.20 A body is dropped from a height of 15 m and experiences a shock when it hits the ground. If the duration of the shock is 3 ms, calculate the magnitude of the shock in terms of g . (Assume $g = 9.81 \text{ m/s}^2$).

Rotational motion transducers

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20.1 Introduction

The different forms of motion have already been explained in the introduction to the last chapter. In that introduction, it was explained that motion occurred in two forms. These are translational motion, which describes the movement of a body along a single axis, and rotational motion, which describes the motion of a body about a single axis. Because the number of sensors involved in motion measurement is quite large, the review of them is divided into two chapters. The last chapter described translational motion sensors and now this chapter describes rotational motion sensors. Again, as for translational motion, rotational motion can occur in the form of either displacement, velocity, or acceleration, and these are considered separately in the following sections.

20.2 Rotational displacement

Rotational displacement transducers measure the angular motion of a body about some rotation axis. They are important not only for measuring the rotation of bodies such as shafts, but also as part of systems that measure translational displacement by converting the translational motion to a rotary form. The various devices available for measuring rotational displacements are presented below, and the arguments for choosing a particular form in any given measurement situation are considered later in section 20.2.9.

20.2.1 Circular and helical potentiometers

The *circular potentiometer* is the cheapest device available for measuring rotational displacements. It works on almost exactly the same principles as the translational motion potentiometer, except that the track is bent round into a circular shape. The measurement range of individual devices varies from $0\text{--}10^\circ$ to $0\text{--}360^\circ$ depending on whether the track forms a full circle or only part of a circle. Where a greater measurement range than $0\text{--}360^\circ$ is required, a *helical potentiometer* is used. The helical potentiometer accommodates multiple turns of the track by forming the track into a helix shape, and some devices are able to measure up to 60 full revolutions. Unfortunately, the greater mechanical complexity of a helical potentiometer makes the device more expensive than a circular potentiometer. The two forms of device are shown in [Fig. 20.1](#).

Both kinds of device give a linear relationship between the measured quantity and the output reading because the output voltage measured at the sliding contact is proportional to the angular displacement of the slider from its starting position. However, as with linear track potentiometers, all rotational potentiometers can give performance problems if dirt on the track causes loss of contact. They also have a limited life because of wear between

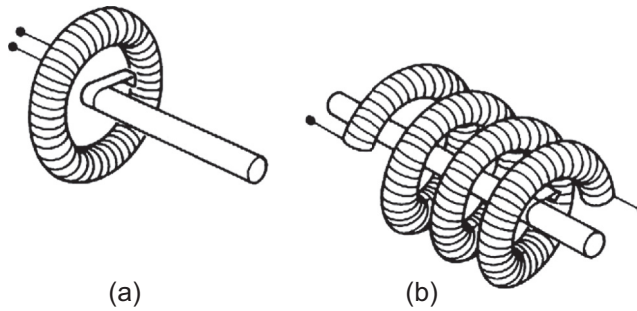


Figure 20.1

Rotary motion potentiometers: (A) circular; (B) helical.

the sliding surfaces. The typical inaccuracy of this class of devices varies from $\pm 1\%$ of full scale for circular potentiometers down to $\pm 0.002\%$ of full scale for the best helical potentiometers.

20.2.2 Rotational variable differential transformer

The rotational variable differential transformer (RVDT) is a special form of differential transformer that measures rotational rather than translational motion. The method of construction and connection of the windings is exactly the same as for the linear variable differential transformer (LVDT), except that a specially shaped core is used that varies the mutual inductance between the windings as it rotates, as shown in Fig. 20.2. Like its linear equivalent, the instrument suffers no wear in operation and therefore has a very long life with almost no maintenance requirements. It can also be modified for operation in harsh

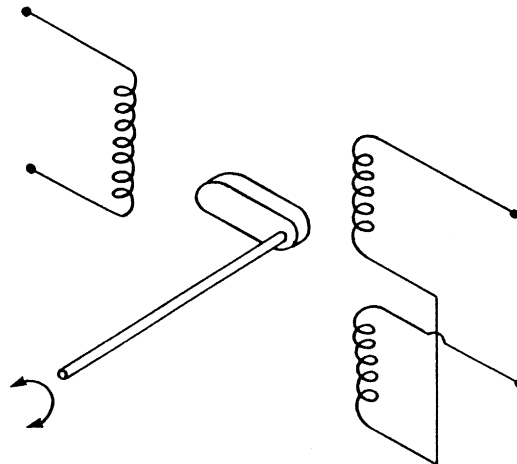


Figure 20.2

Rotary differential transformer.

environments by enclosing the windings inside a protective enclosure. However, apart from the difficulty of avoiding some asymmetry between the secondary windings, great care has to be taken in these instruments to machine the core to exactly the right shape. In consequence, the inaccuracy cannot be reduced below $\pm 1\%$, and even this level of accuracy is only obtained for limited excursions of the core of ± 40 degrees away from the null position. For angular displacements of ± 60 degrees, the typical inaccuracy rises to $\pm 3\%$, and the instrument is unsuitable for measuring displacements greater than this.

20.2.3 Incremental shaft encoders

Incremental shaft encoders are one of a class of encoder devices that give an output in digital form. They measure the instantaneous angular position of a shaft relative to some arbitrary datum point, but are unable to give any indication about the absolute position of a shaft. The principle of operation is to generate pulses as the shaft whose displacement is being measured rotates. These pulses are counted and the total angular rotation inferred from the pulse count. The pulses are generated either by optical or by magnetic means and are detected by suitable sensors. Of the two, the optical system is considerably cheaper and therefore much more common. Such instruments are very convenient for computer control applications, as the measurement is already in the required digital form and therefore the analog to digital signal conversion process required for an analog sensor is avoided.

An example of an optical incremental shaft encoder is shown in [Fig. 20.3](#). It can be seen that the instrument consists of a pair of disks, one of which is fixed and one of which rotates with the body whose angular displacement is being measured. Each disk is basically opaque but has a pattern of windows cut into it. The fixed disk has only one window and the light source is aligned with this so that the light shines through all the time. The second disk has two tracks of windows cut into it that are spaced equidistantly around the disk, as shown in [Fig. 20.4](#). Two light detectors are positioned beyond the second disk so that one is aligned with each track of windows. As the second disk rotates, light alternately enters and does not enter the detectors, as windows and then opaque regions of the disk pass in front of them. These pulses of light are fed to a counter, with the final count after motion has ceased corresponding to the angular position of the moving body relative to the starting position. The primary information about the magnitude of rotation is obtained by the detector aligned with the outer track of windows. However, the pulse count obtained from this gives no information about the direction of rotation. The necessary direction information is provided by the second, inner track of windows, which have an angular displacement with respect to the outer set of windows of half a window width. The pulses from the detector aligned with the inner track of windows therefore lag or lead the primary set of pulses according to the direction of rotation.

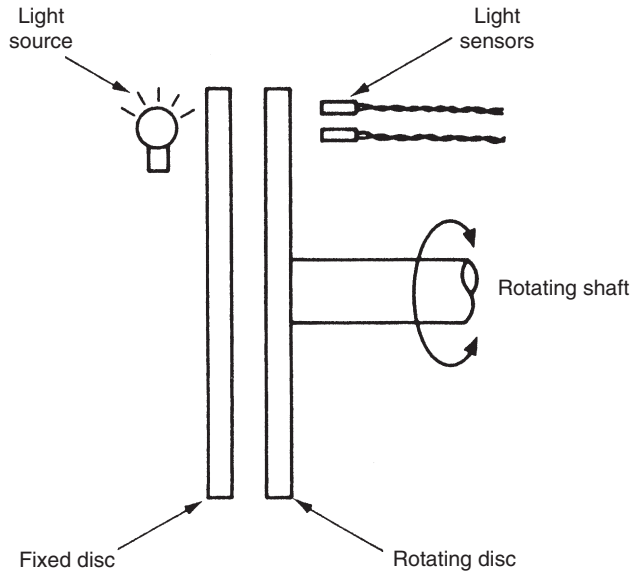


Figure 20.3
Optical incremental shaft encoder.

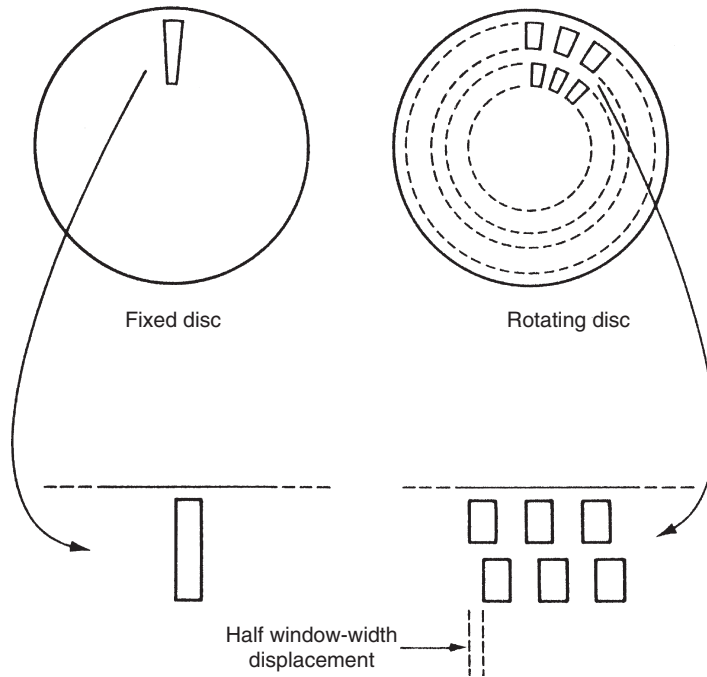


Figure 20.4
Window arrangement in incremental shaft encoder.

The maximum measurement resolution obtainable is limited by the number of windows that can be machined onto a disk. The maximum number of windows per track for a 150 mm-diameter disk is 5000, which gives a basic angular measurement resolution of 1 in 5000. By using more sophisticated circuits that increment the count on both the rising and falling edges of the pulses through the outer track of windows, it is possible to double the resolution to a maximum of 1 in 10,000. At the expense of even greater complexity in the counting circuit, it is possible also to include the pulses from the inner track of windows in the count, so giving a maximum measurement resolution of 1 in 20,000.

Optical incremental shaft encoders are a popular instrument for measuring relative angular displacements and are very reliable. Problems of noise in the system giving false counts can sometimes cause difficulties, although this can usually be eliminated by squaring the output from the light detectors. Such instruments are found in many applications where rotational motion has to be measured. Incremental shaft encoders are also commonly used in circumstances where a translational displacement has been transformed to a rotational one by suitable gearing. One example of this practice is in measuring the translational motions in numerically controlled drilling machines. Typical gearing used for this would give one revolution per mm of translational displacement. By using an incremental shaft encoder with 1000 windows per track in such an arrangement, a measurement resolution of 1 micron is obtained.

20.2.4 Coded-disk shaft encoders

Unlike the incremental shaft encoder that gives a digital output in the form of pulses that have to be counted, the digital shaft encoder has an output in the form of a binary number of several digits that provides an absolute measurement of shaft position. Digital encoders provide high accuracy and reliability. They are particularly useful for computer control applications, but they have a significantly higher cost than incremental encoders. Three different forms exist, using optical, electrical, and magnetic energy systems, respectively.

Optical digital shaft encoder

The optical digital shaft encoder is the cheapest form of encoder available and is the one used most commonly. It is found in a variety of applications, and one where it is particularly popular is in measuring the position of rotational joints in robot manipulators. The instrument is similar in physical appearance to the incremental shaft encoder. It has a pair of disks (one movable and one fixed) with a light source on one side and light detectors on the other side, as shown in Fig. 20.5. The fixed disk has a single window, and the principal way in which the device differs from the incremental shaft encoder is in the design of the windows on the movable disk, as shown in Fig. 20.6. These are cut in four or more tracks instead of two and are arranged in sectors as well as tracks. An energy

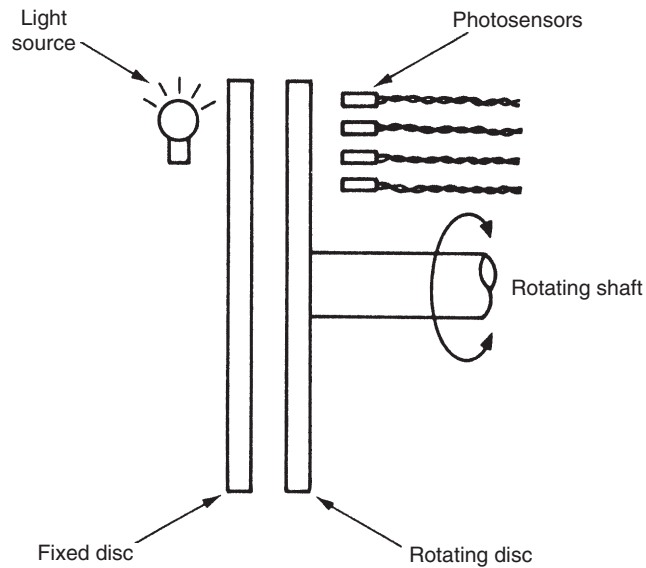


Figure 20.5
Coded-disk shaft encoder.

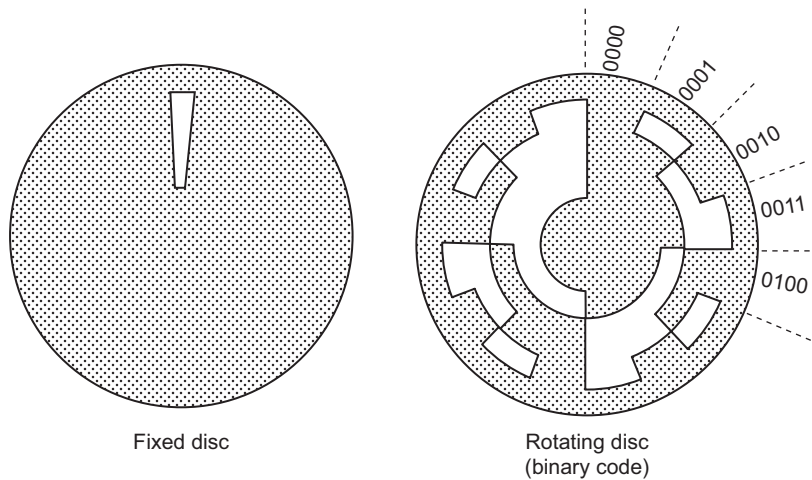


Figure 20.6
Window arrangement for coded-disk shaft encoder.

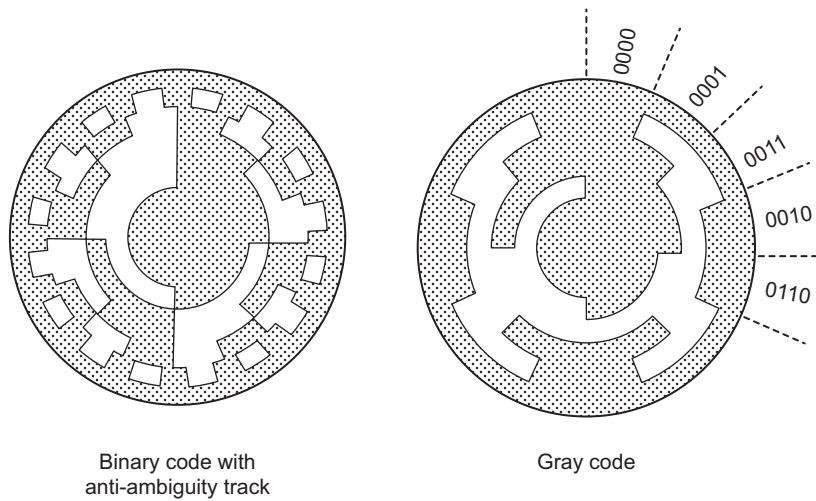
detector is aligned with each track, and these give an output of “1” when energy is detected and an output of “0” otherwise. The measurement resolution obtainable depends on the number of tracks used. For a four-track version, the resolution is 1 in 16, with progressively higher measurement resolution being attained as the number of tracks is increased. These binary outputs from the detectors are combined together to give a binary

number of several digits. The number of digits corresponds to the number of tracks on the disk, which in the example shown in Fig. 20.6 is four. The pattern of windows in each sector is cut such that, as that particular sector passes across the window in the fixed disk, the four energy detector outputs combine to give a unique binary number. In the binary-coded example shown in Fig. 20.6, the binary number output increments by one as each sector in the rotating disk passes in turn across the window in the fixed disk. Thus the output from sector 1 is 0001, from sector 2 is 0010, from sector 3 is 0011, etc.

While this arrangement is perfectly adequate in theory, serious problems can arise in practice due to the manufacturing difficulty involved in machining the windows of the movable disk such that the edges of the windows in each track are exactly aligned with each other. Any misalignment means that, as the disk turns across the boundary between one sector and the next, the outputs from each track will switch at slightly different instants of time, and therefore the binary number output will be incorrect over small angular ranges corresponding to the sector boundaries. The worst error can occur at the boundary between sectors 7 and 8, where the output is switching from 0111 to 1000. If the energy sensor corresponding to the first digit switches before the others, then the output will be 1111 for a very small angular range of movement, indicating that sector 15 is aligned with the fixed disk rather than sector 7 or 8. This represents an error of 100% in the indicated angular position.

In practice, there are two ways that are used to overcome this difficulty. Both of these solutions involve an alteration to the manner in which windows are machined on the movable disk, as shown in Fig. 20.7. The first method adds an extra outer track on the disk, known as an *anti-ambiguity track*, which consists of small windows that span a small angular range on either side of each sector boundary of the main track system. When energy sensors associated with this extra track sense energy, this is used to signify that the disk is aligned on a sector boundary and the output is unreliable. The second method is somewhat simpler and cheaper, because it avoids the expense of machining the extra anti-ambiguity track. It does this by using a special code, known as the *Gray code*, to cut the tracks in each sector on the moveable disk. The Gray code is a special binary representation where only one binary digit changes in moving from one decimal number representation to the next, i.e., from one sector to the next in the digital shaft encoder. The code is illustrated in Table 20.1.

It is possible to manufacture optical digital shaft encoders with up to 21 tracks, which gives a measurement resolution of 1 part in 10^6 (about 1 second of arc). Unfortunately, there is a high cost involved in the special photolithography techniques used to cut the windows in order to achieve such a measurement resolution, and very high-quality mounts and bearings are needed. Hence, such 21 track devices are very expensive.


Figure 20.7

Modified window arrangements for the rotating disk.

Table 20.1: The gray code.

Decimal number	Binary code	Gray code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

Contacting (electrical) digital shaft encoder

The contacting digital shaft encoder consists of only one disk, which rotates with the body whose displacement is being measured. The disk has conducting and nonconducting segments rather than the transparent and opaque areas found on the movable disk of the optical form of instrument, but these are arranged in an identical pattern of sectors and

tracks. The disk is charged to a low potential by an electrical brush in contact with one side of the disk, and a set of brushes on the other side of the disk measures the potential in each track. The output of each detector brush is interpreted as a binary value of “1” or “0” according to whether the track in that particular segment is conducting or not and hence whether a voltage is sensed or not. As for the case of the optical form of instrument, these outputs are combined together to give a multibit binary number. Contacting digital shaft encoders have a similar cost to the equivalent optical instruments and have operational advantages in severe environmental conditions of high temperature or mechanical shock. They suffer from the usual problem of output ambiguity at the sector boundaries, but this problem is overcome by the same methods as used in optical instruments.

A serious problem in the application of contacting digital shaft encoders arises from their use of brushes. These introduce friction into the measurement system, and the combination of dirt and brush wear causes contact problems. Consequently, problems of intermittent output can occur, and such instruments generally have limited reliability and a high maintenance cost. Measurement resolution is also limited because of the lower limit on the minimum physical size of the contact brushes. The maximum number of tracks possible is 10, which limits the resolution to 1 part in 1000. Thus, contacting digital shaft encoders are normally only used where the environmental conditions are too severe for optical instruments.

Magnetic digital shaft encoder

Magnetic digital shaft encoders consist of a single rotatable disk, as in the contacting form of encoder discussed in the previous section. The pattern of sectors and tracks consists of magnetically conducting and nonconducting segments, and the sensors aligned with each track consist of small toroidal magnets. Each of these sensors has a coil wound on it that has a high or low voltage induced in it according to the magnetic field close to it. This field is dependent on the magnetic conductivity of that segment of the disk that is closest to the toroid.

These instruments have no moving parts in contact and therefore have a similar reliability to optical devices. Their major advantage over optical equivalents is an ability to operate in very harsh environmental conditions. Unfortunately, the process of manufacturing and accurately aligning the toroidal magnet sensors required makes such instruments very expensive. Their use is therefore limited to a few applications where both high measurement resolution and also operation in harsh environments are required.

20.2.5 The resolver

The resolver is an electromechanical device that gives an analog output by transformer action. It is more rugged than encoders in adverse operating environments. Resolver-to-

digital converters are available and provide a simple way of interfacing resolvers to digital systems. Physically, resolvers resemble a small AC motor and have a diameter ranging from 10 to 100 mm. They are frictionless and reliable in operation because they have no contacting moving surfaces, and consequently they have a long life. The best devices give measurement resolutions of 0.1%.

Resolvers have two stator windings, which are mounted at right angles to one another, and a rotor, which can have either one or two windings. As the angular position of the rotor changes, the output voltage changes. The simpler configuration of a resolver with only one winding on the rotor is illustrated in Fig. 20.8. This exists in two separate forms that are distinguished according to whether the output voltage changes in amplitude or changes in phase as the rotor rotates relative to the stator winding.

Varying amplitude output resolver

The stator of this type of resolver is excited with a single-phase sinusoidal voltage of frequency ω , where the amplitudes in the two windings are given by:

$$V_1 = V \sin(\beta); \quad V_2 = V \cos(\beta)$$

where $V = V_s \sin(\omega t)$

The effect of this is to give a field at an angle of $(\beta + \pi/2)$ relative to stator winding 1.

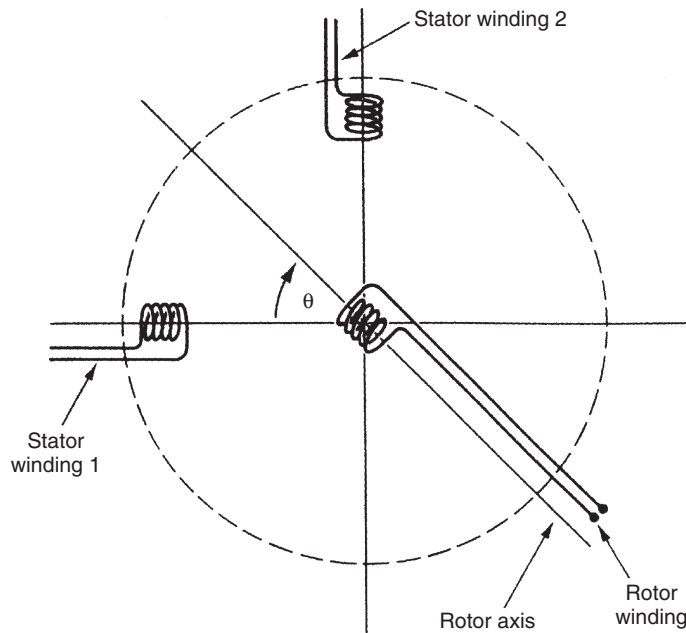


Figure 20.8

Schematic representation of resolver windings.

Suppose that the angle of the rotor winding relative to that of the stator winding is given by θ . Then the magnetic coupling between the windings is a maximum for $\theta = (\beta + \pi/2)$ and a minimum for $(\theta = \beta)$. The rotor output voltage is of fixed frequency and varying amplitude given by:

$$V_o = KV_s \sin(\beta - \theta)\sin(\omega t)$$

This relationship between shaft angle position and output voltage is nonlinear, but approximate linearity is obtained for small angular motions where $|\beta - \theta| < 15^\circ$.

Intelligent versions of this type of resolver are available that use a microprocessor to process the sine and cosine outputs. This can improve the measurement resolution to 2 minutes of arc.

Varying phase output resolver

This is a less common form of resolver that is only used in a few applications. The stator windings are excited with a two-phase sinusoidal voltage of frequency ω , and the instantaneous voltage amplitudes in the two windings are given by:

$$V_1 = V_s \sin(\omega t) \quad ; \quad V_2 = V_s \sin(\omega t + \pi/2) = V_s \cos(\omega t)$$

The net output voltage in the rotor winding is the sum of the voltages induced due to each stator winding. This is given by:

$$\begin{aligned} V_o &= KV_s \sin(\omega t)\cos(\theta) + KV_s \cos(\omega t)\cos(\pi/2 - \theta) \\ &= KV_s[\sin(\omega t)\cos(\theta) + \cos(\omega t)\sin(\theta)] \\ &= KV_s \sin(\omega t + \theta) \end{aligned}$$

This represents a linear relationship between shaft angle and the phase shift of the rotor output relative to the stator excitation voltage. The accuracy of shaft rotation measurement depends on the accuracy with which the phase shift can be measured. This can be improved by increasing the excitation frequency, ω , and it is possible to reduce inaccuracy down to $\pm 0.1\%$. However, increasing the excitation frequency also increases magnetizing losses. Consequently, a compromise excitation frequency of about 400Hz is used.

20.2.6 The synchro

Like the resolver, the synchro is a motor-like, electromechanical device with an analog output. It is also known as a **Selsyn** and by other brand names. Apart from having three stator windings instead of two, the instrument is similar in appearance and operation to the resolver and has the same range of physical dimensions. The rotor usually has a dumbbell shape and, like the resolver, can have either one or two windings.

Synchros have been in use for many years for the measurement of angular positions, especially in military applications, and achieve similar levels of accuracy and measurement resolution to digital encoders. One common application is axis measurement in machine tools, where the translational motion of the tool is translated into a rotational displacement by suitable gearing. Synchros are tolerant to high temperatures, high humidity, shock, and vibration, and are therefore suitable for operation in such harsh environmental conditions. Some maintenance problems are associated with the slip ring and brush system used to supply power to the rotor. However, the only major source of error in the instrument is asymmetry in the windings, and reduction of measurement inaccuracy down to $\pm 0.5\%$ is easily achievable.

Fig. 20.9 shows the simpler form of synchro with a single-rotor winding. If an AC excitation voltage is applied to the rotor via slip rings and brushes, this sets up a certain pattern of fluxes and induced voltages in the stator windings by transformer action. For a rotor excitation voltage, V_r , given by:

$$V_r = V \sin(\omega t)$$

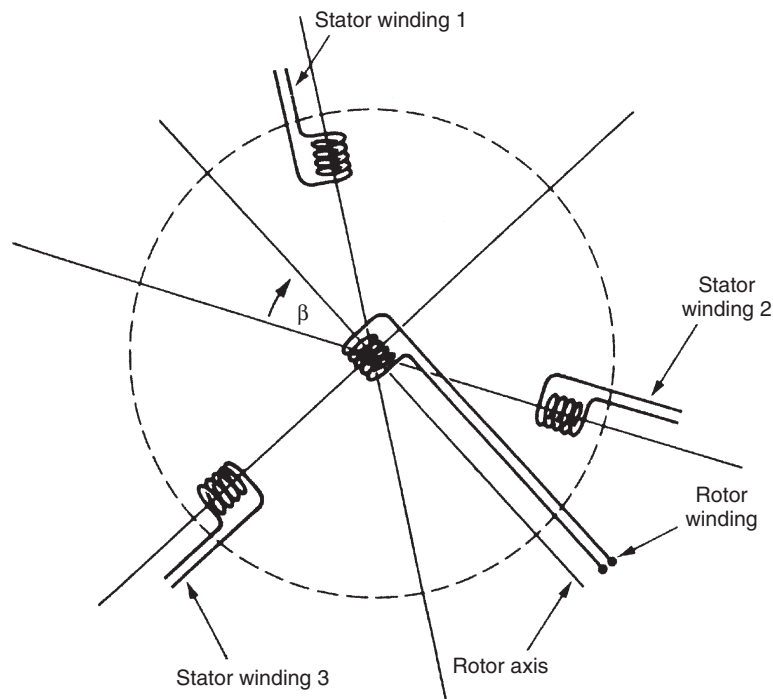


Figure 20.9
Schematic representation of synchro windings.

the voltages induced in the three stator windings are:

$$V_1 = V \sin(\omega t) \sin(\beta); \quad V_2 = V \sin(\omega t) \sin(\beta + 2\pi/3); \quad V_3 = V \sin(\omega t) \sin(\beta - 2\pi/3)$$

where β is the angle between the rotor and stator windings.

If the rotor is turned at constant velocity through one full revolution, the voltage waveform induced in each stator winding is as shown in Fig. 20.10. This has the form of a carrier-modulated waveform, in which the carrier frequency corresponds to the excitation frequency, ω . It follows that if the rotor is stopped at any particular angle, β' , the peak-to-peak amplitude of the stator voltage is a function of β' . If therefore the stator winding voltage is measured, generally as its root-mean-squared value, this indicates the magnitude of the rotor rotation away from the null position. The direction of rotation is determined by the phase difference between the stator voltages, which is indicated by their relative instantaneous magnitudes.

Although a single synchro is able to measure an angular displacement by itself, it is much more common to find a pair of them used for this purpose. When used in pairs, one member of the pair is known as the synchro transmitter and the other as the synchro transformer, and the two sets of stator windings are connected together, as shown in Fig. 20.11. Each synchro is of the form shown in Fig. 20.9, but the rotor of the transformer is fixed for displacement-measuring applications. A sinusoidal excitation voltage is applied to the rotor of the transmitter, setting up a pattern of fluxes and induced voltages in the transmitter stator windings. These voltages are transmitted to the transformer stator windings, where a similar flux pattern is established. This in turn causes a sinusoidal voltage to be induced in the fixed transformer rotor winding. For an excitation voltage, $V \sin(\omega t)$, applied to the transmitter rotor, the voltage measured in the transformer rotor is given by:

$$V_o = V \sin(\omega t) \sin(\theta)$$

where θ is the relative angle between the two rotor windings.

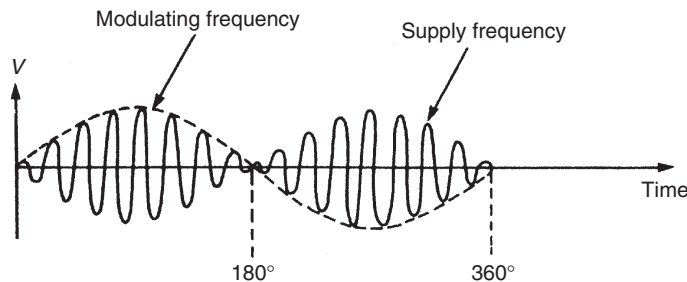


Figure 20.10
Synchro stator voltage waveform.

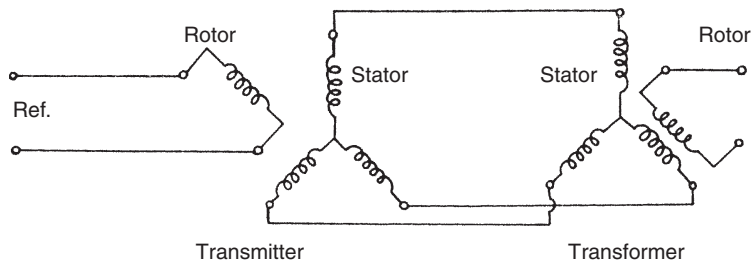


Figure 20.11

Synchro transmitter-transformer pair.

Apart from their use as a displacement transducer, such synchro pairs are commonly used to transmit angular displacement information over some distance, for instance to transmit gyro compass measurements in an aircraft to remote meters. They are also used for load positioning, allowing a load connected to the transformer rotor shaft to be controlled remotely by turning the transmitter rotor. For these applications, the transformer rotor is free to rotate, and it is also damped to prevent oscillatory motions. In the simplest arrangement, a common sinusoidal excitation voltage is applied to both rotors. If the transmitter rotor is turned, this causes an imbalance in the magnetic flux patterns and results in a torque on the transformer rotor that tends to bring it into line with the transmitter rotor. This torque is typically small for small displacements, and so this technique is only useful if the load torque on the transformer shaft is very small. In other circumstances, it is necessary to incorporate the synchro pair within a servomechanism, where the output voltage induced in the transformer rotor winding is amplified and applied to a servomotor that drives the transformer rotor shaft until it is aligned with the transmitter shaft.

20.2.7 The rotary inductosyn

This instrument is similar in operation to the linear inductosyn, except that it measures rotary displacements and has tracks that are arranged radially on two circular disks, as shown in Fig. 20.12. Typical diameters of the instrument vary between 75 and 300 mm. The larger versions give a measurement resolution of up to 0.05 seconds of arc. However, like its linear equivalent, the rotary inductosyn has a very small measurement range. Therefore, a lower-resolution, rotary displacement transducer with a larger measurement range must be used in conjunction with it.

20.2.8 Gyroscopes

Gyroscopes measure both absolute angular displacement and absolute angular velocity. Until recently, the mechanical, spinning-wheel gyroscope had a dominant position in the

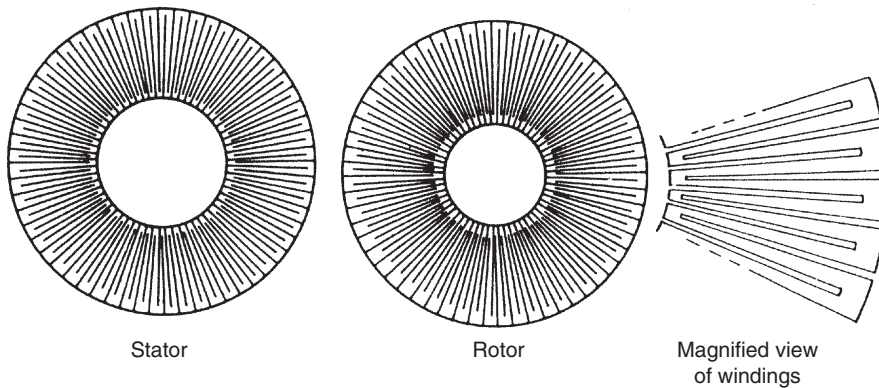


Figure 20.12
Rotary inductosyn.

marketplace. However, this position is now being challenged by optical gyroscopes and microelectromechanical system (MEMS) gyroscopes.

Mechanical gyroscopes

Mechanical gyroscopes consist essentially of a large, motor-driven wheel whose angular momentum is such that the axis of rotation tends to remain fixed in space, thus acting as a reference point. The gyro frame is attached to the body whose motion is to be measured. The output is measured in terms of the angle between the frame and the axis of the spinning wheel. Two different forms of mechanical gyroscope are used for measuring angular displacement, the free gyro and the rate-integrating gyro. A third type of mechanical gyroscope, the rate gyro, measures angular velocity and is described in [Section 20.3](#).

The *free gyroscope* is illustrated in [Fig. 20.13](#). This measures the absolute angular rotation about two perpendicular axes of the body to which its frame is attached. Two alternative methods of driving the wheel are used in different versions of the instrument. One of these is to enclose the wheel in stator-like coils that are excited with a sinusoidal voltage. A voltage is applied to the wheel via slip rings at both ends of the spindle carrying the wheel. The wheel behaves as a rotor, and motion is produced by motor action. The other, less common, method is to fix vanes on the wheel that is then driven by directing a jet of air onto the vanes.

The free gyroscope can measure angular displacements of up to 10 degrees with a high accuracy. For greater angular displacements, interaction between the measurements on the two perpendicular axes starts to cause a serious loss of accuracy. The physical size of the coils in the motor-action driven system also limits the measurement range to 10 degrees. For these reasons, this type of gyroscope is only suitable for measuring rotational

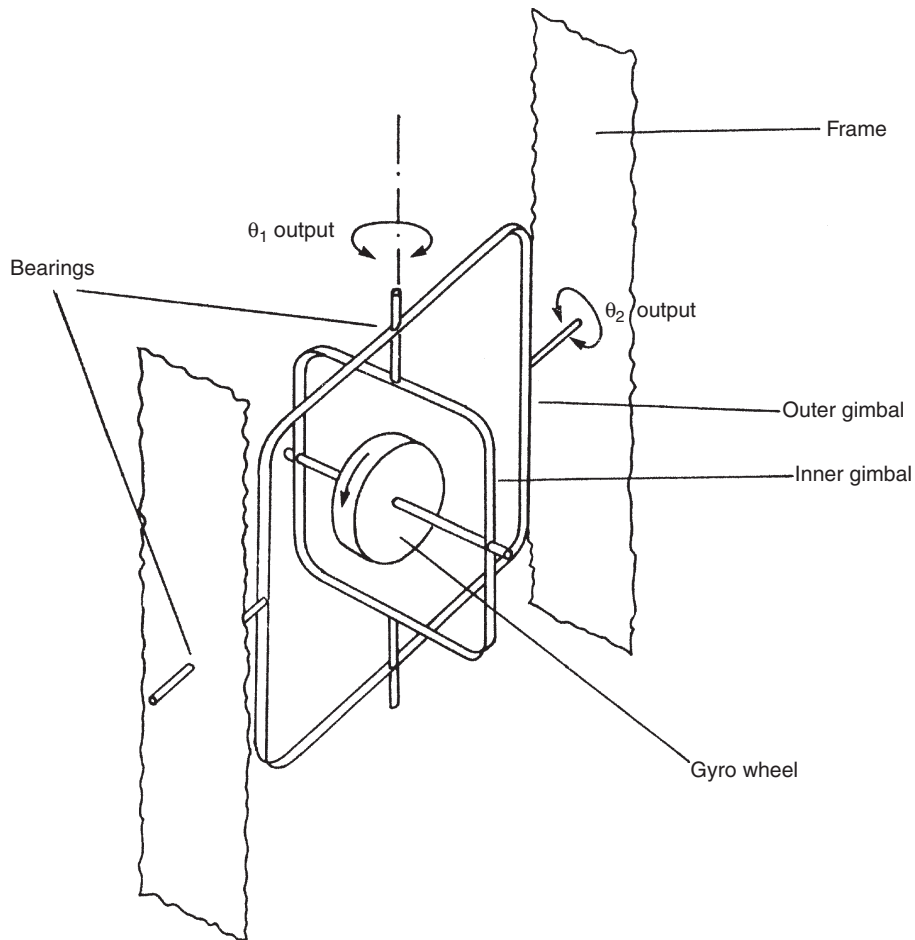


Figure 20.13
Free gyroscope.

displacements of up to 10 degrees. A further operational problem of free gyroscopes is the presence of angular drift (precession) due to bearing friction torque. This has a typical magnitude of 0.5 degrees per minute and means that the instrument can only be used over short time intervals of, say, 5 minutes. This time duration can be extended if the angular momentum of the spinning wheel is increased.

A major application of the free gyroscope is in inertial navigation systems. Only two free gyros mounted along orthogonal axes are needed to monitor motions in three dimensions, because each instrument measures displacement about two axes. The limited angular range of measurement is not usually a problem in such applications, as control action prevents the error in the direction of motion about any axis ever exceeding one or two degrees.

However, precession is a much greater problem, and, for this reason, the rate-integrating gyro is used much more commonly.

The *rate-integrating gyroscope*, or *integrating gyro* as it is commonly known, is illustrated in Fig. 20.14. It measures angular displacements about a single axis only, and therefore three instruments are required in a typical inertial navigation system. The major advantage of the instrument over the free gyro is the almost total absence of precession, with typical specifications quoting drifts of only 0.01 degrees/h. The instrument has a first-order type of response given by:

$$\frac{\theta_o}{\theta_i}(D) = \frac{K}{\tau D + 1} \quad (20.1)$$

where $K = H/\beta$, $\tau = M/\beta$, θ_i is the input angle, θ_o is the output angle, D is the D-operator, H is the angular momentum, M is the moment of inertia of the system about the measurement axis, and β is the damping coefficient.

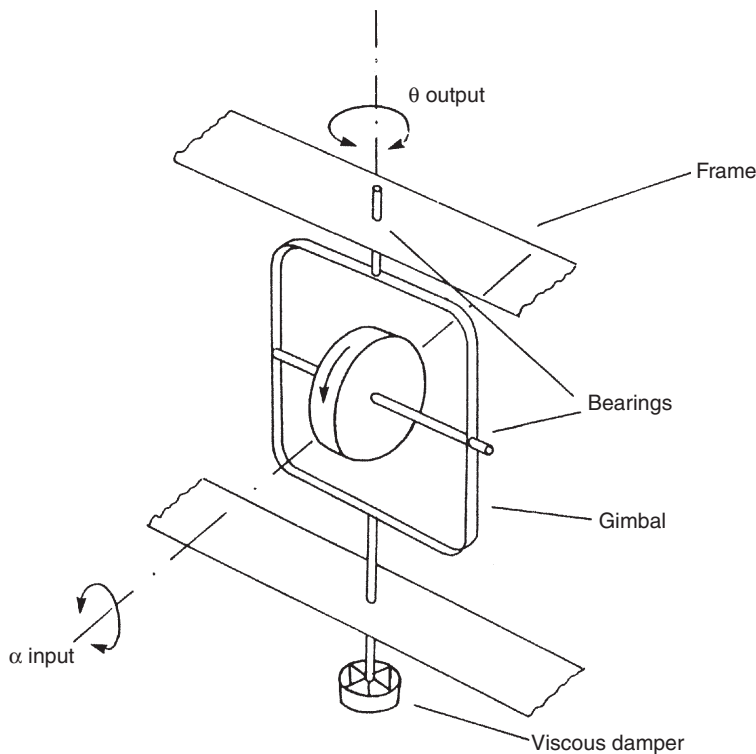


Figure 20.14
Rate-integrating gyroscope.

Inspection of Eq. (20.1) shows that to obtain a high value of measurement sensitivity, K , a high value of H and low value of β are required. A large H is normally obtained by driving the wheel with a hysteresis-type motor revolving at high speeds of up to 24,000 rpm. However, the damping coefficient β can only be reduced so far, because a small value of β results in a large value for the system time constant, τ , and an unacceptably low speed of system response. Therefore, the value of β has to be chosen as a compromise between these constraints.

Besides their use as a fixed reference in inertial guidance systems, integrating gyros are also commonly used within aircraft autopilot systems and in military applications such as stabilizing weapon systems in tanks.

Optical gyroscopes

Optical gyroscopes are a relatively recent development and come in two forms, the ring laser gyroscope and the fiber-optic gyroscope.

The *ring laser gyroscope* consists of a glass ceramic chamber containing a helium-neon gas mixture in which two laser beams are generated by a single anode/twin cathode system, as shown in Fig. 20.15. Three mirrors, supported by the ceramic block and mounted in a triangular arrangement, direct the pair of laser beams around the cavity in

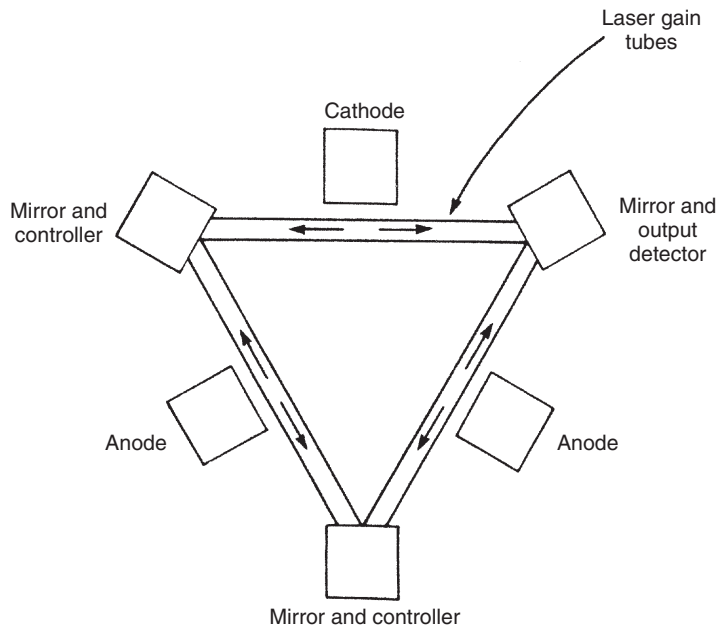


Figure 20.15
Ring laser gyroscope.

opposite directions. Any rotation of the ring affects the coherence of the two beams, raising the frequency of one and lowering the frequency of the other. The clockwise and anticlockwise beams are directed into a photodetector that measures the beat frequency according to the frequency difference, which is proportional to the angle of rotation. The advantages of the ring laser gyroscope over traditional, mechanical gyroscopes are considerable. The measurement accuracy obtained is substantially better than that afforded by mechanical gyros in a similar price range. The device is also considerably smaller physically, which is of considerable benefit in many applications.

The *fiber-optic gyroscope* is a rate gyro that measures angular velocity. It is described later in [Section 20.3.5](#).

20.2.9 Choice between rotational displacement transducers

Choice between the various rotational displacement transducers that might be used in any particular measurement situation depends first of all upon whether absolute measurement of angular position is required or whether the measurement of rotation relative to some arbitrary starting point is acceptable. Other factors affecting the choice between instruments are the required measurement range, the resolution of the transducer, and the measurement accuracy afforded.

Where only measurement of relative angular position is required, the incremental encoder is a very suitable instrument. The best commercial instruments of this type can measure rotations to a resolution of 1 part in 20,000 of a full revolution, and the measurement range is an infinite number of revolutions. Instruments with such a high measurement resolution are very expensive, but much cheaper versions are available according to what lower level of measurement resolution is acceptable.

All the other instruments presented in this chapter provide an absolute measurement of angular position. The required measurement range is a dominant factor in the choice between these. If this exceeds one full revolution, then the only instrument available is the helical potentiometer. Such devices can measure rotations of up to 60 full turns, but they are expensive because the procedure involved in manufacturing a helical resistance element to a reasonable standard of accuracy is difficult.

For measurements of less than one full revolution, the range of available instruments widens. The cheapest one available is the circular potentiometer, but much better measurement accuracy and resolution is obtained from coded-disk encoders. The cheapest of these is the optical form, but certain operating environments necessitate the use of the alternative contacting (electrical) and magnetic versions. All types of coded-disk encoder are very reliable and are particularly attractive in computer control schemes, as the output is in digital form. A varying phase output resolver is yet another instrument that can

measure angular displacements up to one full revolution in magnitude. Unfortunately, this instrument is expensive because of the complicated electronics incorporated to measure the phase variation and convert it to a varying-amplitude output signal, and hence it is no longer in common use.

An even greater range of instruments becomes available as the required measurement range is reduced further. These include the synchro (± 90 degrees), the varying amplitude output resolver (± 90 degrees), and the rotational variable differential transformer (± 40 degrees). All these instruments have a high reliability and a long service life.

Finally, two further instruments are available for satisfying special measurement requirements, the rotary inductosyn and the gyroscope. The rotary inductosyn is used in applications where very high measurement resolution is required, although the measurement range afforded is extremely small and a coarser-resolution instrument must be used in parallel with it to extend the measurement range. Gyroscopes, in both mechanical and optical forms, are used to measure small angular displacements up to ± 10 degrees in magnitude in inertial navigation systems and similar applications.

20.2.10 Calibration of rotational displacement transducers

The coded disk shaft encoder is normally used for the calibration of rotary potentiometers and rotational variable differential transformers. A typical model provides a reference standard with measurement uncertainty of $\pm 0.1\%$ of the full-scale reading. If greater accuracy is required, for example, in calibrating encoders of lesser accuracy, encoders with measurement uncertainty down to $\pm 0.0001\%$ of the full-scale reading can be obtained and used as a reference standard, although these have a very high associated cost.

20.3 Rotational velocity

The main application of rotational velocity transducers is in speed control systems. They also provide the usual means of measuring translational velocities, which are transformed into rotational motions for measurement purposes by suitable gearing. Many different instruments and techniques are available for measuring rotational velocity as presented below. MEMS gyroscopes have also been added to the list in this new edition. These measure transient rather than continuous velocities, but are now widely used in many applications.

20.3.1 Digital tachometers

Digital tachometers, or to give them their proper title, digital *tachometric generators*, are usually noncontact instruments that sense the passage of equally spaced marks on the

surface of a rotating disk or shaft. Measurement resolution is governed by the number of marks around the circumference. Various types of sensors are used, such as optical, inductive, and magnetic ones. As each mark is sensed, a pulse is generated and input to an electronic pulse counter. Usually, velocity is calculated in terms of the pulse count in unit time, which of course only yields information about the mean velocity. If the velocity is changing, instantaneous velocity can be calculated at each instant of time that an output pulse occurs, using the scheme shown in Fig. 20.16. In this circuit, the pulses from the transducer gate the train of pulses from a 1 MHz clock into a counter. Control logic resets the counter and updates the digital output value after receipt of each pulse from the transducer. The measurement resolution of this system is highest when the speed of rotation is low.

Optical sensing

Digital tachometers with optical sensors are often known as *optical tachometers*. Optical pulses can be generated by one of the two alternative photoelectric techniques illustrated in Fig. 20.17. In the scheme shown in Fig. 20.17A, the pulses are produced as the windows in a slotted disk pass in sequence between a light source and a detector. The alternative scheme, shown in Fig. 20.17B, has both light source and detector mounted on the same side of a reflective disk that has black sectors painted onto it at regular angular intervals. Light sources are normally either lasers or LEDs, with photodiodes and phototransistors being used as detectors. Optical tachometers yield better accuracy than other forms of digital tachometer. However, they are less reliable than other forms because dust and dirt can block light paths.

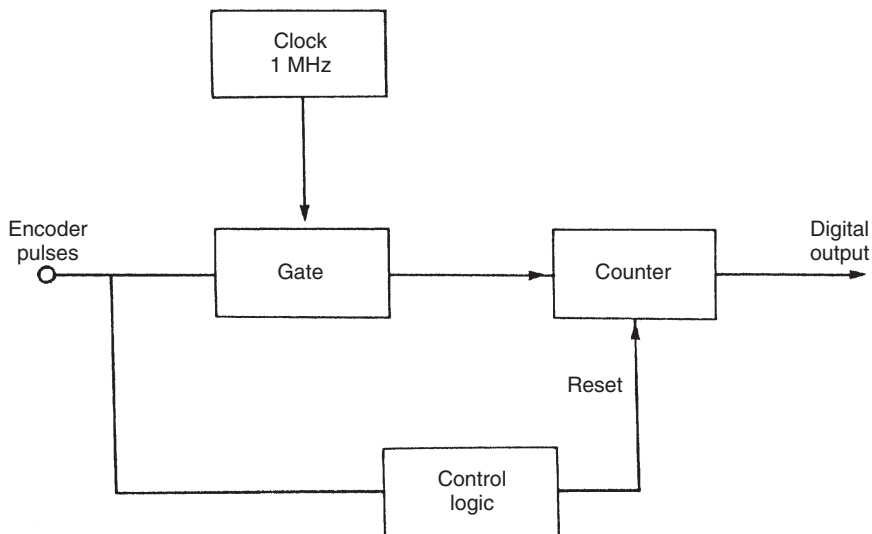


Figure 20.16

Scheme to measure instantaneous angular velocities.

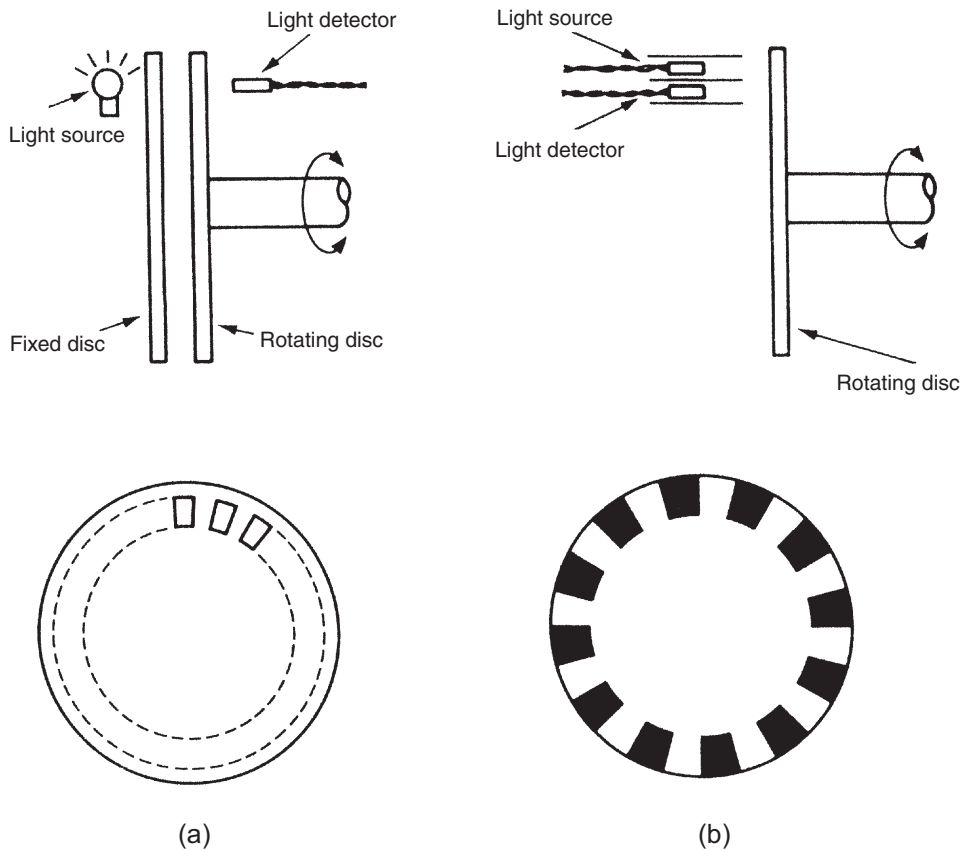


Figure 20.17
Photoelectric pulse generation techniques.

Inductive sensing

Variable reluctance velocity transducers, also known as **induction tachometers**, are a form of digital tachometer that use inductive sensing. They are widely used in the automotive industry within antiskid devices, antilock braking systems, and traction control. One relatively simple and cheap form of this type of device was described earlier in Section 13.4 (Fig. 13.2). A more sophisticated version, shown in Fig. 20.18, has a rotating disk that is constructed from a bonded-fiber material into which soft iron poles are inserted at regular intervals around its periphery. The sensor consists of a permanent magnet with a shaped pole piece, which carries a wound coil. The distance between the pickup and the outer perimeter of the disk is typically 0.5 mm. As the disk rotates, the soft iron inserts on the disk move in turn past the pickup unit. As each iron insert moves toward the pole piece, the reluctance of the magnetic circuit increases and hence the flux in the pole piece also increases. Similarly, the flux in the pole piece decreases as the each iron insert moves

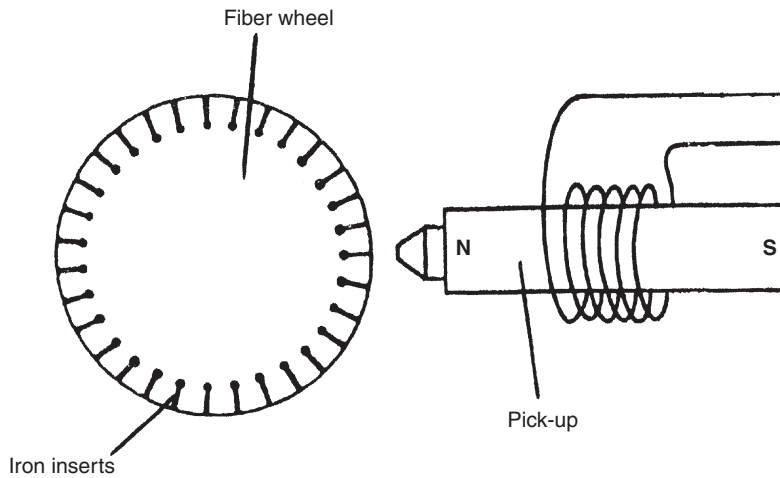


Figure 20.18
Variable reluctance transducer.

away from the sensor. The changing magnetic flux inside the pickup coil causes a voltage to be induced in the coil whose magnitude is proportional to the rate of change of flux. This voltage is positive while the flux is increasing and negative while it is decreasing. Thus, the output is a sequence of positive and negative pulses whose frequency is proportional to the rotational velocity of the disk. The maximum angular velocity that the instrument can measure is limited to about 10,000 r.p.m. because of the finite width of the induced pulses. As the velocity increases, the distance between the pulses is reduced, and at a certain velocity, the pulses start to overlap. At this point, the pulse counter ceases to be able to distinguish the separate pulses. The optical tachometer has significant advantages in this respect, since the pulse width is much narrower, allowing measurement of higher velocities.

A simpler and cheaper form of variable reluctance transducer also exists that uses a ferromagnetic gear wheel in place of a fiber disk. The motion of the tip of each gear tooth toward and away from the pickup unit causes a similar variation in the flux pattern to that produced by the iron inserts in the fiber disk. However, the pulses produced by these means are less sharp, and, consequently, the maximum angular velocity measurable is lower.

Magnetic (Hall-effect) sensing

The rotating element in *Hall-effect* or *magnetostrictive tachometers* has a very simple design in the form of a toothed metal gearwheel. The sensor is a solid-state, Hall-effect device that is placed between the gear wheel and a permanent magnet. When an inter-tooth gap on the gear wheel is adjacent to the sensor, the full magnetic field from the

magnet passes through it. Later, as a tooth approaches the sensor, the tooth diverts some of the magnetic field, and so the field through the sensor is reduced. This causes the sensor to produce an output voltage that is proportional to the rotational speed of the gear wheel.

20.3.2 Stroboscopic methods

The stroboscopic technique of rotational velocity measurement operates on a similar physical principle to digital tachometers except that the pulses involved consist of flashes of light generated electronically and whose frequency is adjustable so that it can be matched with the frequency of occurrence of some feature on the rotating body being measured. This feature can either be some naturally occurring one such as gear teeth or the spokes of a wheel, or it can be an artificially created pattern of black and white stripes. In either case, the rotating body appears stationary when the frequencies of the light pulses and body features are in synchronism. Flashing rates available in commercial stroboscopes vary from 110 up to 150,000 per minute according to the range of velocity measurement required, and typical measurement inaccuracy is $\pm 1\%$ of the reading. The instrument is usually in the form of a handheld device that is pointed toward the rotating body.

It must be noted that measurement of the flashing rate at which the rotating body appears stationary does not automatically indicate the rotational velocity, because synchronism also occurs when the flashing rate is some integral submultiple of the rotational speed. The practical procedure followed is therefore to adjust the flashing rate until synchronism is obtained at the largest flashing rate possible, R_1 . The flashing rate is then carefully decreased until synchronism is again achieved at the next lower flashing rate, R_2 . The rotational velocity is then given by:

$$V = \frac{R_1 R_2}{R_1 - R_2}$$

20.3.3 Analog tachometers

Analog tachometers are less accurate than digital tachometers but are nevertheless still used successfully in many applications. Various forms exist:

The *DC tachometer* has an output that is approximately proportional to its speed of rotation. Its basic structure is identical to that found in a standard DC generator used for producing power, and is shown in Fig. 20.19. Both permanent-magnet types and separately excited field types are used. However, certain aspects of the design are optimized to improve its accuracy as a speed-measuring instrument. One significant design modification is to reduce the weight of the rotor by constructing the windings on a hollow fiberglass

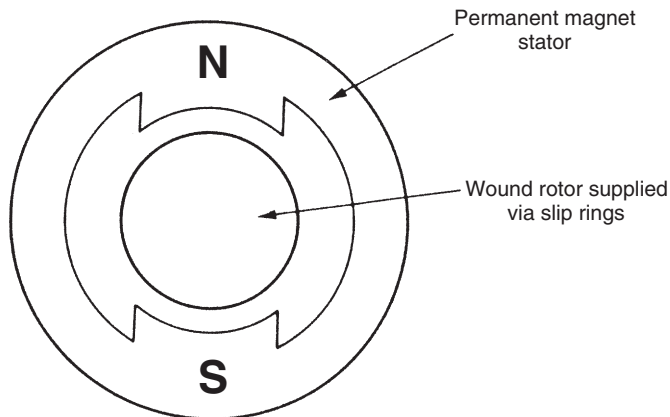


Figure 20.19
DC tachometer.

shell. The effect of this is to minimize any loading effect of the instrument on the system being measured. The DC output voltage from the instrument is of a relatively high magnitude, giving a high measurement sensitivity that is typically 5 V per 1000 r.p.m. The direction of rotation is determined by the polarity of the output voltage. A common range of measurement is 0–6000 r.p.m. Maximum nonlinearity is usually about $\pm 1\%$ of the full-scale reading. One problem with these devices that can cause difficulties under some circumstances is the presence of an AC ripple in the output signal. The magnitude of this can be up to 2% of the output DC level.

The *AC tachometer* has an output approximately proportional to rotational speed like the DC tachogenerator. Its mechanical structure takes the form of a two-phase induction motor, with two stator windings and (usually) a drag-cup rotor, as shown in Fig. 20.20.

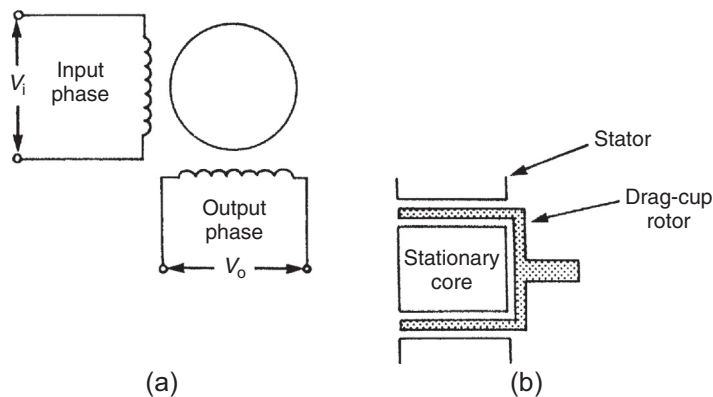


Figure 20.20

AC tachometer.: (a) rotor and stator winding arrangement, (b) detail of drag-cup rotor.

One of the stator windings is excited with an AC voltage and the measurement signal is taken from the output voltage induced in the second winding. The magnitude of this output voltage is zero when the rotor is stationary, and otherwise is proportional to the angular velocity of the rotor. The direction of rotation is determined by the phase of the output voltage, which switches by 180 degrees as the direction reverses. Therefore, both the phase and magnitude of the output voltage have to be measured. A typical range of measurement is 0–4000 r.p.m., with an inaccuracy of $\pm 0.05\%$ of full-scale reading. Cheaper versions with a squirrel-cage rotor also exist, but measurement inaccuracy in these is typically $\pm 0.25\%$.

The drag-cup tachometer, also known as an **eddy-current tachometer**, has a central spindle carrying a permanent magnet that rotates inside a nonmagnetic drag cup consisting of a cylindrical sleeve of electrically conductive material, as shown in Fig. 20.21. As the spindle and magnet rotate, a voltage is induced that causes circulating eddy currents in the cup. These currents interact with the magnetic field from the permanent magnet and produce a torque. In response, the drag cup turns until the induced torque is balanced by the torque due to the restraining springs connected to the cup. When equilibrium is reached, the angular displacement of the cup is proportional to the rotational velocity of

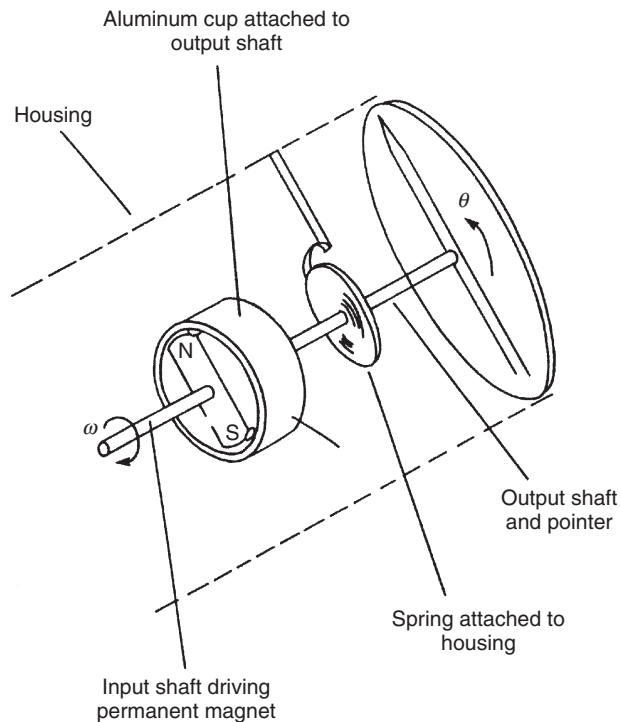


Figure 20.21
Drag-cup tachometer.

the central spindle. The instrument has a typical measurement inaccuracy of $\pm 0.5\%$ and is commonly used in the speedometers of motor vehicles and also as a speed indicator for aero-engines. It is capable of measuring velocities up to 15,000 r.p.m.

Analog-output forms of the *variable reluctance velocity transducer* (see Section 20.3.1) also exist in which the output voltage pulses are converted into an analog, varying-amplitude, DC voltage by means of a frequency-to-voltage converter circuit. However, the measurement accuracy is inferior to digital output forms.

20.3.4 The rate gyroscope

The rate gyro, illustrated in Fig. 20.22, has an almost identical construction to the rate-integrating gyro (Fig. 20.14), and differs only by including a spring system that acts as an additional restraint on the rotational motion of the frame. The instrument measures the absolute angular velocity of a body and is widely used for generating stabilizing signals within vehicle navigation systems. The typical measurement resolution given by the instrument is 0.01 degrees/s and rotation rates up to 50 degrees/s can be measured. The angular velocity, α , of the body is related to the angular deflection of the gyroscope, θ , by the equation:

$$\frac{\theta}{\alpha} (D) = \frac{H}{MD^2 + \beta D + K} \quad (20.2)$$

where H is the angular momentum of the spinning wheel, M is the moment of inertia of the system, β is the viscous damping coefficient, K is the spring constant, and D is the D -operator.

This relationship (20.2) is a second-order differential equation, and we must consequently expect the device to have a response typical of second-order instruments, as discussed in Chapter 2. Therefore, the instrument must be designed carefully so that the output response is neither oscillatory nor too slow in reaching a final reading. To assist in the design process, it is useful to reexpress Eq. (20.2) in the following form:

$$\frac{\theta}{\alpha} (D) = \frac{K'}{D^2/\omega^2 + 2\xi D/\omega + 1} \quad (20.3)$$

where $K' = H/K$, $\omega = \sqrt{K/M}$ and $\xi = \frac{\beta}{2\sqrt{KM}}$

The static sensitivity of the instrument, K' , is made as large as possible by using a high-speed motor to spin the wheel and so make H high. Reducing the spring constant K further improves the sensitivity, but this cannot be reduced too far as it makes the resonant frequency ω of the instrument too small. The value of β is usually chosen such that the damping ratio ξ is as close to 0.7 as possible.

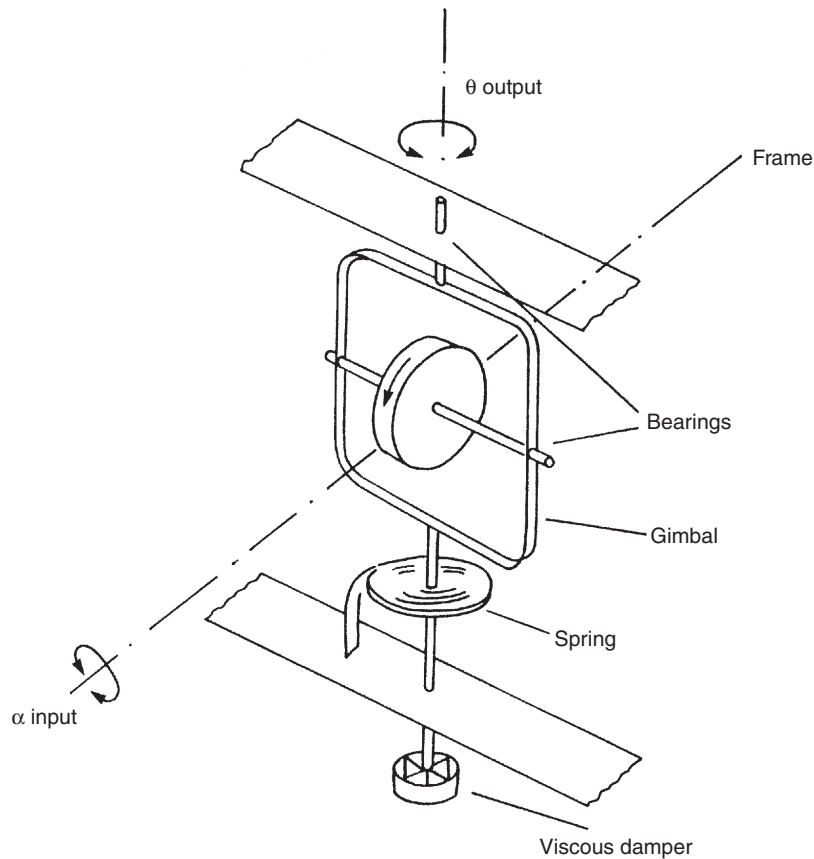


Figure 20.22
Rate gyroscope.

20.3.5 Fiber-optic gyroscope

This is a relatively new instrument that makes use of fiber-optic technology. Incident light from a source is separated by a beam splitter into a pair of beams a and b , as shown in Fig. 20.23. These travel in opposite directions around an optic-fiber coil (which may be several hundred meters long) and emerge from the coil as the beams marked a' and b' . The beams a' and b' are directed by the beam splitter into an interferometer. Any motion of the coil causes a phase shift between a' and b' , which is detected by the interferometer.

20.3.6 MEMS gyroscope

MEMS gyroscopes are now widely available for measuring rotational velocity. The typical structure of a MEMS gyroscope is shown in Fig. 20.24. This operates on a tuning fork principle, and consists of two equal masses M that oscillate and are always moving in

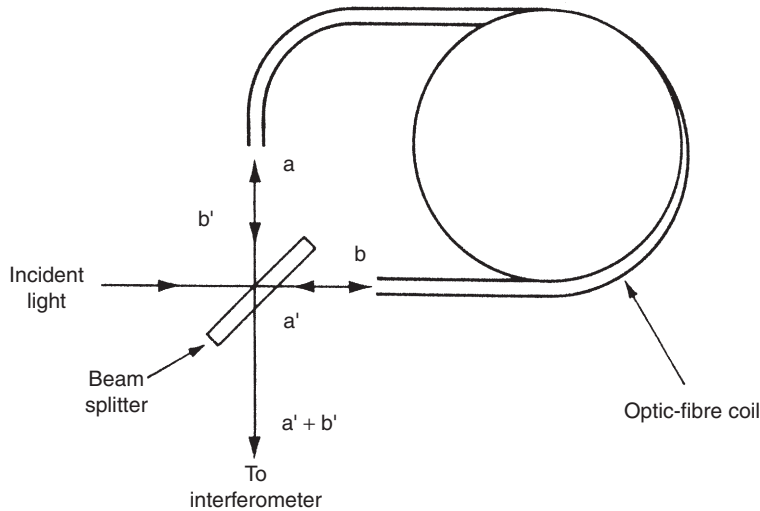


Figure 20.23
Fiber-optic gyroscope.

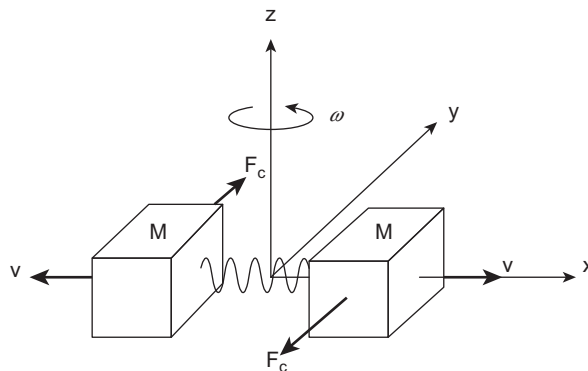


Figure 20.24
Typical structure of a MEMS gyroscope.

opposite directions to each other. When the gyroscope is subjected to an angular velocity ω , a Coriolis force F is generated on each mass given by: $F = -2M\omega \times v$, where v is the instantaneous velocity of the masses. The Coriolis forces on the masses act in opposite directions and cause a lateral displacement of the masses relative to each other. This displacement causes a change in capacitance between the two masses that is proportional to the magnitude of the angular velocity ω applied. This change in capacitance is either converted to a digital number in digital MEMS gyroscopes or into an analog voltage in the case of analog MEMS gyroscopes.

In common with other types of MEMS devices, MEMS gyroscopes are relatively cheap to produce and provide high performance with low power consumption. They are entirely insensitive to linear motion and only respond to angular motion. Any motion of the device in the x , y , or z direction (as shown on Fig. 20.24) causes both masses to move by the same amount in the direction of the motion. Hence, there is no relative displacement between the two masses and so there is no change in capacitance.

MEMS gyroscopes are now used in a wide range of applications for measuring rotational velocities up to 2700 degrees/s. They are used in digital cameras to provide image stabilization by detecting rotations due to camera shake. Within motor vehicles, they have several uses. One use is to measure changes in vehicle orientation during loss of a GPS signal and so allow satellite navigation systems to continue to function. Another use is to activate the electronic stability control system when the vehicle swerves to avoid a collision. Yet another use is to trigger the deployment of an airbag when a vehicle rollover condition is detected.

20.3.7 Differentiation of angular displacement measurements

Angular velocity measurements can be obtained by differentiating the output signal from angular displacement transducers. Unfortunately, the process of differentiation amplifies any noise in the measurement signal, and therefore this technique has been used only rarely in the past. However, the technique has become more feasible with the advent of intelligent instruments. For example, using an intelligent instrument to differentiate and process the output from a resolver can produce a velocity measurement with a maximum inaccuracy of $\pm 1\%$.

20.3.8 Integration of the output from an accelerometer

In measurement systems that already contain an angular acceleration transducer, it is possible to obtain a velocity measurement by integrating the acceleration measurement signal. This produces a signal of acceptable quality, as the process of integration attenuates any measurement noise. However, the method is of limited value in many measurement situations because the measurement obtained is the average velocity over a period of time, rather than a profile of the instantaneous velocities as motion takes place along a particular path.

20.3.9 Choice between rotational velocity transducers

Choice between different rotational velocity transducers is influenced strongly by whether an analog or digital form of output is required. Distinction also has to be made between devices that measure continuous velocity and those that measure short-term, transient

velocities. Digital output instruments are now widely used for the measurement of continuous velocities and choice has to be made between the variable reluctance transducer, devices using electronic light pulse counting methods, and the stroboscope. The first two of these are used to measure angular speeds up to about 10,000 r.p.m. and the last one can measure speeds up to 25,000 r.p.m.

Probably the most common form of analog output device used for measuring continuous velocities is the DC tachometer. This is a relatively simple device that measures speeds up to about 5000 r.p.m. with a maximum inaccuracy of $\pm 1\%$. Where better accuracy is required within a similar range of speed measurement, AC tachometers are used. The squirrel-cage rotor type has an inaccuracy of only $\pm 0.25\%$, and drag-cup rotor types can have inaccuracies as low as $\pm 0.05\%$.

The drag-cup tachometer also has an analog output but its typical inaccuracy is $\pm 5\%$. However, it is cheap and therefore suitable for use in vehicle speedometers where an inaccuracy of $\pm 5\%$ is normally acceptable.

The case of measuring short-term, transient velocities is the province of MEMS gyroscopes. Versions measuring rotation rates up to 2000 degrees/s are widely available and some manufacturers produce devices that can measure rates up to 2700 degrees/s.

20.3.10 Calibration of rotational velocity transducers

The main device used as a calibration standard for most forms of rotational velocity transducer is the stroboscope. Provided the flash frequency of the reference stroboscope is properly calibrated, it is possible to provide velocity measurements where the inaccuracy is less than $\pm 0.1\%$. In the case of MEMS gyroscopes, these are normally tested and calibrated during the manufacturing process. However, it should be noted that stresses applied to the device during assembly in the place of usage may cause small deviations from their published parameters that are difficult to measure. However, it is possible to calculate the variation in performance caused by temperature variations, using parameters given in the data sheet for each device.

20.4 Rotational acceleration

Rotational accelerometers work on very similar principles to translational motion accelerometers. They consist of a rotatable mass mounted inside a housing that is attached to the accelerating, rotating body. Rotation of the mass is opposed by a torsional spring and damping. Any acceleration of the housing causes a torque $J\ddot{\theta}$ on the mass. This torque is opposed by a backward torque due to the torsional spring and in equilibrium:

$$J\ddot{\theta} = K\theta \text{ and hence : } \ddot{\theta} = k\theta/J.$$

A damper is usually included in the system to avoid undying oscillations in the instrument. This adds an additional backward torque $B\dot{\theta}$ to the system and the equation of motion becomes:

$$J\ddot{\theta} = B\dot{\theta} + K\theta$$

Different manufacturers produce accelerometers that measure the angular displacement of the mass within the accelerometer in different ways. However, it should be noted that the number of manufacturers producing rotational accelerometers is substantially less than the number manufacturing translational motion accelerometers because the requirement to measure rotational acceleration occurs much less frequently than requirements to measure translational acceleration.

20.4.1 Calibration of rotational accelerometers

This is normally carried out by comparison with a reference standard rotational accelerometer. The task is usually delegated to specialist calibration companies or accelerometer manufacturers because of the relatively small number of applications for rotational accelerometers and the corresponding shortage of personnel having the necessary calibration skills.

20.5 Summary

Having discussed sensors for measuring translational motion in the previous chapter, this chapter has been concerned with the measurement of the three aspects of rotational motion, which are displacement, velocity, and acceleration. Starting with sensors for measuring rotation displacement, we first discussed circular and helical potentiometers. Next we considered the merits of the rotational differential transformer, incremental shaft encoder, coded-disk shaft encoder, resolver, synchro, rotary inductosyn, and both the free and rate-integrating gyroscope.

Moving on to the measurement of rotational velocity, we first explored the various forms of digital tachometer available. Discussion then moved on to stroboscopic methods, followed by a review of analog tachometers, which we noted were less accurate than digital tachometers but still remained in fairly widespread use. Next, we covered two forms of gyroscope that measure rotational velocity, the rate gyro and the fiber-optic gyro. Following this, we covered MEMS gyroscopes, which are a new class of device that is finding widespread application for measuring transient velocities. Finally, we mentioned that a velocity measurement could be obtained by differentiating an angular displacement measurement or by integrating an acceleration measurement. However, we noted that while the latter is acceptable because the process of integration attenuates any

measurement noise, the differentiation technique is not used unless carried out within an intelligent instrument that can deal with the noise amplification that is inherent when measurements are obtained via differentiation.

Our final subject in the chapter was the measurement of rotational acceleration. We noted that rotational accelerometers worked on very similar principles to their translational motion counterparts, while observing that the requirement of measure rotational acceleration did not commonly arise.

20.6 Problems

- 20.1 Explain the mode of operation and characteristics of the following devices, using simple sketches to support your explanation: circular potentiometer, helical potentiometer, rotary differential transformer.
- 20.2 Sketch an incremental shaft encoder. Explain what it measures and how it works. What special design features can be implemented to increase the measurement resolution of a disk of a given diameter?
- 20.3 What is a coded-disk shaft encoder? How does its output differ from that of an incremental shaft encoder? What are the main types of coded-disk shaft encoder?
- 20.4 Discuss the mode of operation of an optical coded-disk shaft encoder, illustrating your discussion by means of a sketch.
- 20.5 Explain the mode of operation and characteristics of the following: (a) contacting (electrical) digital shaft encoder, (b) magnetic digital shaft encoder.
- 20.6 What is the main consequence of any misalignment of the windows in an optical coded-disk shaft encoder? Describe two ways in which the problem caused by window misalignment can be overcome.
- 20.7 Explain what a resolver is in the context of rotational position measurement. Discuss the two alternative forms of resolver that exist.
- 20.8 How does a synchro work? Illustrate your explanation with a simple sketch.
- 20.9 What is a gyroscope? Discuss the characteristics and mode of operation of three kinds of gyroscope that can measure angular position.
- 20.10 Explain the mode of construction and characteristics of each of the following: digital tachometer, optical tachometer, Hall-effect tachometer.
- 20.11 Describe how an induction tachometer works and explain its characteristics.
- 20.12 Discuss the characteristics of stroboscopic methods for measuring rotational velocity.
- 20.13 What are the main types of analog tachometer available? Discuss the main characteristics of each.
- 20.14 How does a rate gyroscope work? What is its main application?

Summary of other measurements

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21.11 Summary 675**21.12 Problems 675****21.1 Introduction**

The previous chapters in this book reviewed measurement techniques available for measuring a range of commonly met physical variables. In this discussion, we explored the measurement of temperature, pressure, the flow rate of both solids and fluids, the level of liquids and powdered/granular solids in a container, mass (including weight), force, torque, and motion in both translational and rotation forms. However, we have certainly not covered an exhaustive list of all the different physical variables that exist in the world. Without doubt, it would be possible to produce an encyclopedia that covered the measurement of every physical variable that exists, but this would inevitably be expensive and it is certainly not appropriate for inclusion within a textbook such as this. Nevertheless, for the benefit of readers who may need some information on sensors to measure physical variables that have not been discussed so far, this chapter provides a brief summary of appropriate measurement techniques for a number of lesser-met measurement requirements. The following sections therefore cover the measurement of dimensions, angles, surface flatness, volume, viscosity, moisture (including humidity), sound, and pH (acidity/alkalinity), with a final section on gas sensing and analysis.

21.2 Dimension measurement

Dimension measurement includes measurement of the length, width, and height of components and also the depth of holes and slots within components. Tapes and rules are commonly used to give approximate measurements, and various forms of caliper and micrometer are employed where more accurate measurements are required. Gauge blocks and length bars are also used when high accuracy is required, although these are primarily intended for calibration duties.

A necessary component in most instances of dimension measurement is a flat and level *reference plane* on which the components being measured are placed. Such reference planes are available in a range of standard sizes, and a means of adjusting the feet is always provided to ensure that the surface is exactly level. Smaller sizes exist as a *surface plate* resting on a supporting table, whereas larger sizes take the form of freestanding

tables that usually have a projection at the edge to facilitate the clamping of components. They are normally used in conjunction with box cubes and vee blocks (Fig. 21.1) that locate components in a fixed position. In modern tables, granite has tended to supersede iron as the preferred material for the plate, although iron plates are available. Granite is ideal for this purpose because it does not corrode, is dimensionally very stable, and does not form burrs when damaged. Iron plates, on the other hand, are prone to rusting and susceptible to damage; this results in burrs on the surface that interfere with measurement procedures. Surface plates and tables are available in three grades, which are usually called calibration, inspection, and toolroom. These vary according to their flatness, defined as the distance between two parallel planes that just contain all points in the table surface. Standards of flatness vary according to the size of the table. For a large table of dimensions $2000 \times 1500 \times 300$ mm, the allowable maximum permitted deviations from flatness are 0.005 mm for the calibration grade, 0.010 mm for the inspection grade, and 0.020 mm for the toolroom grade.

As well as using accurate instruments on a flat surface, the quality of dimension measurements critically depends on the skill of the human operator using the instruments. Incorrect use can introduce measurement inaccuracy in various ways, and therefore the scope for human-induced errors is far greater than that which exists when measuring most other physical quantities. Hence, checks on the way that the human operator is using equipment are as important as calibration checks on the measuring instruments themselves. The golden rule in dimension measurement is that the line of measurement and the line of the dimension being measured should coincide. In the case of steel rules and tapes, the greatest potential source of user-induced error is failure to position the rule

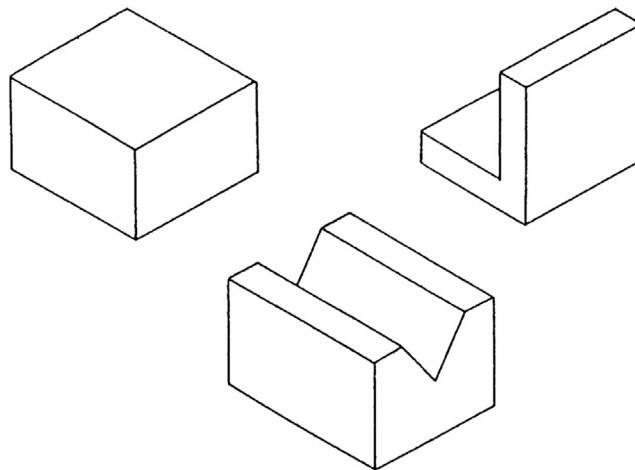


Figure 21.1
Box cubes and Vee blocks.

squarely across the dimension being measured. Parallax error is also possible if the user does not position the rule and read it from directly above. Calipers and micrometers are less susceptible to these types of error, but it still remains sensible to carry out periodic checks on the way in which these instruments are being used, verifying in particular that measurements are being made squarely.

21.2.1 Rules and tapes

Rules and tapes are the simplest way of measuring larger dimensions. Steel rules are generally available to measure dimensions only up to 1 m. Beyond this, steel tapes (measuring to 30 m) or an ultrasonic rule (measuring to 10 m) are used.

The **steel rule** is undoubtedly the simplest instrument available for measuring length. Measurement accuracy is only modest using standard rules, which typically have rulings at 0.5-mm intervals, but the best rules have rulings at 0.05-mm intervals and a measurement resolution of 0.02 mm. When used by placing the rule against an object, the measurement accuracy is much dependent on the skill of the human measurer and, at best, the inaccuracy is likely to be at least $\pm 0.5\%$.

The retractable **steel tape** is another well-known instrument. The end of the tape is usually provided with a flat hook that is loosely fitted so as to allow for automatic compensation of the hook thickness when the rule is used for internal measurements. Again, measurement accuracy is governed by human skill, but with care, the measurement inaccuracy can be made to be as low as $\pm 0.01\%$ of full-scale reading.

The **ultrasonic rule** consists of an ultrasonic energy source, an ultrasonic energy detector, and battery-powered, electronic circuitry housed within a handheld box, as shown in Fig. 21.2. Both the source and detector often consist of the same type of piezoelectric crystal excited at a typical frequency of 40 kHz. Energy travels from the source to a target

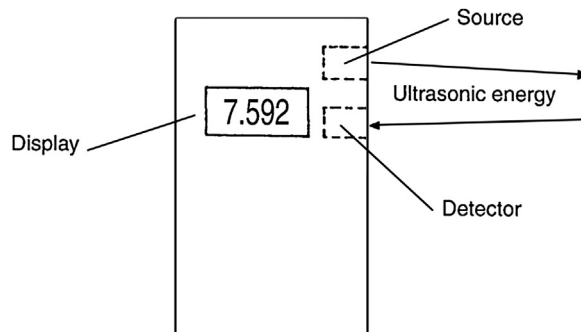


Figure 21.2
Ultrasonic rule.

object and is then reflected back into the detector. The time of flight of this energy is measured and this is converted into a distance reading by the enclosed electronics. A laser pointer is often incorporated into the device to ensure that the distance to the correct target is being measured. Maximum measurement inaccuracy of $\pm 1\%$ of the full-scale reading is claimed. This is only a modest level of accuracy, but it is sufficient for purposes such as measuring rooms by estate agents before producing sales literature, in which the ease and speed of making measurements is of great value.

A fundamental problem in the use of ultrasonic energy of this type is the limited measurement resolution (7 mm) imposed by the 7-mm wavelength of sound at this frequency. Further problems are caused by the variation in the speed of sound with humidity (variations of $\pm 0.5\%$ are possible) and the temperature-induced variation of 0.2% per degrees Celsius. Therefore, the conditions of use must be carefully controlled if the claimed accuracy value is to be met.

21.2.2 Calipers

Calipers are generally used in situations where measurement of dimensions with a rule or tape is not accurate enough. Two versions exist: the standard caliper and the vernier caliper.

Fig. 21.3 shows two alternative forms of the *standard caliper*. The range of measurement, according to the version used, is up to 600 mm. Calipers are used to transfer the measured dimension from the workpiece to a steel rule. This avoids the need to align the end of the rule exactly with the edge of the workpiece and reduces measurement inaccuracy by a

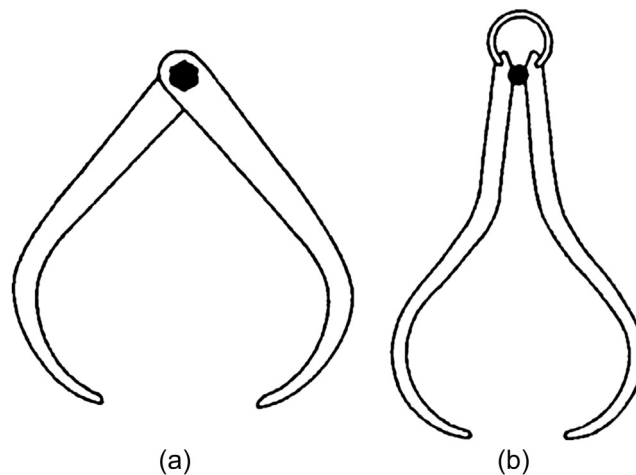


Figure 21.3
(a) Standard caliper; (b) spring-loaded caliper.

factor of two. In the basic caliper, careless use can allow the setting of the caliper to be changed during transfer from the workpiece to the rule. Hence, the spring-loaded type, which prevents this happening, is preferable.

The *vernier caliper*, shown in Fig. 21.4(a), is a combination of a standard caliper and a steel rule. The main body of the instrument includes a scale and a fixed anvil at one end. This carries a sliding anvil that is provided with a second, vernier scale. This second scale is shorter than the main scale and is divided into units that are slightly smaller than the main scale units but related to them by a fixed factor. Determination of the point where the two scales coincide enables very accurate measurements to be made, with typical inaccuracy levels down to $\pm 0.01\%$.

Fig. 21.4(b) shows details of a typical combination of main and vernier scales. The main scale is ruled in 1-mm units. The vernier scale is 49 mm long and divided into 50 units, which makes each unit 0.02 mm smaller than the main scale units. Each group of five units on the vernier scale thus differs from the main scale by 0.1 mm and the numbers marked on the scale thus refer to these larger units of 0.1 mm. In the particular position shown in the figure, the zero on the vernier scale indicates a measurement between 8 and 9 mm. Both scales coincide at a position of 6.2 (large units). This defines the interval between 8 and 9 mm to be $6.2 \times 0.1 = 0.62$ mm (i.e., the measurement is 8.62 mm).

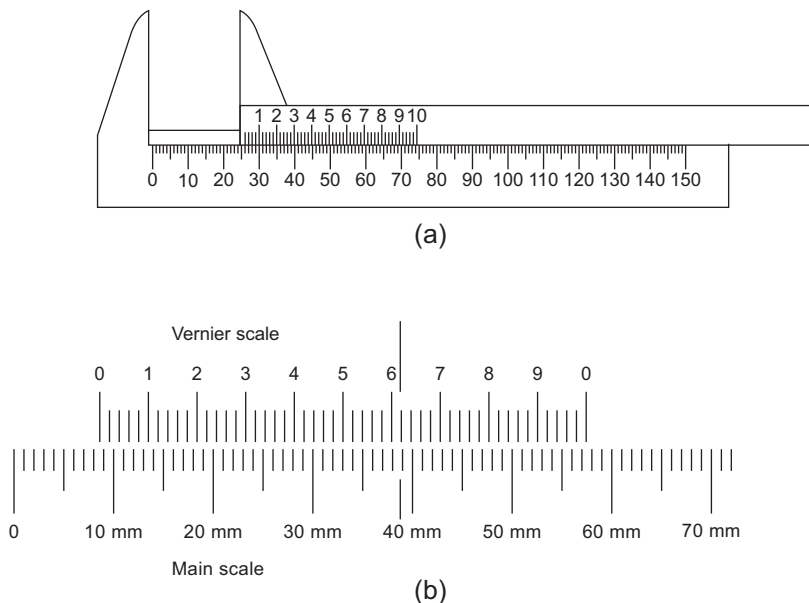


Figure 21.4

Vernier caliper: (a) Basic instrument; (b) details of scale.

Intelligent digital calipers are now available that give a measurement resolution of 0.01 mm and a low inaccuracy of ± 0.03 mm. These have automatic compensation for wear, and hence calibration checks have to be very infrequent. In some versions, the digital display can be directly interfaced to an external computer monitoring system.

21.2.3 Micrometers

Micrometers provide a means of measuring dimensions to high accuracy. Different forms provide measurement of both internal and external dimensions of components, and of holes, slots, and so on within components. In the *standard micrometer*, shown in Fig. 21.5(a), measurement is made between two anvils, one fixed and one that is moved along by the rotation of an accurately machined screw thread. One complete rotation of the screw typically moves the anvil by a distance of 0.5 mm. Such movements of the anvil are measured using a scale marked with divisions every 0.5 mm along the barrel of the instrument. A scale marked with 50 divisions is etched around the circumference of the spindle holder: each division therefore corresponds to an axial movement of 0.01 mm. Assuming that the user is able to judge the position of the spindle on this circular scale against the datum mark to within one-fifth of a division, a measurement resolution of 0.002 mm is possible.

The most common measurement ranges are either 0–25 mm or 25–50 mm, with inaccuracy levels down to $\pm 0.003\%$. However, a whole family of micrometers is available in which each has a measurement span of 25 mm, but with the minimum distance measured varying from 0 up to 575 mm. Thus, the last instrument in this family measures the range from 575 to 600 mm. Some manufacturers also provide micrometers with two or

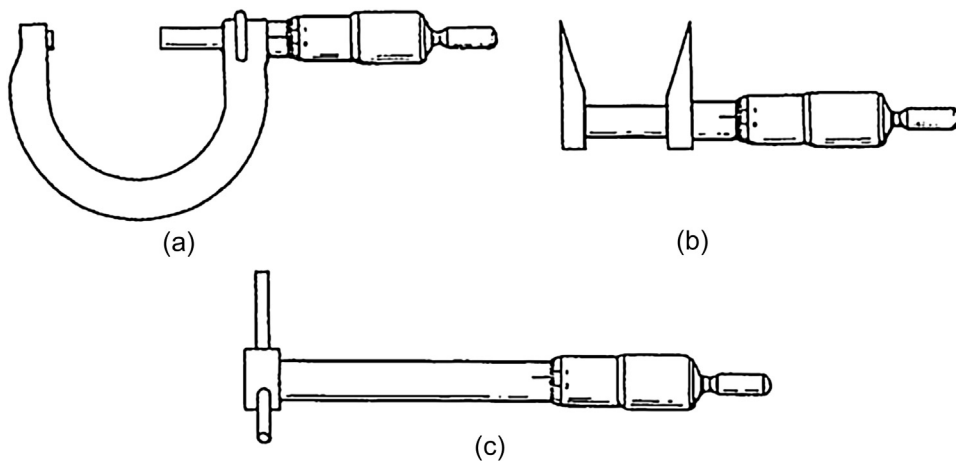


Figure 21.5

Micrometers: (a) Standard (external) micrometer; (b) Internal micrometer; (c) bore micrometer.

more interchangeable anvils, which extends the span measurable with one instrument to between 50 and 100 mm according to the number of anvils supplied. Therefore, an instrument with four anvils might, for instance, measure the range from 300 to 400 mm, by making appropriate changes to the anvils.

The *internal micrometer* (Fig. 21.5(b)) is able to measure internal dimensions such as the diameter of holes. In the case of measuring holes, micrometers are inaccurate if there is any ovality in the hole, unless the diameter is measured at several points. An alternative solution to this problem is to use a special type of instrument known as a *bore micrometer* (Fig. 21.5(c)). In this, three probes move out radially from the body of the instrument as the spindle is turned. These probes make contact with the sides of the hole at three equidistant points, thus averaging out any ovality.

Intelligent micrometers in the form of the electronic *digital micrometer* are now available. These have a self-calibration capability and a digital readout, with a measurement resolution of 0.001 mm (1 μm). They are available in the same forms as nondigital micrometers (i.e., standard, internal and bore types).

21.2.4 Gauge blocks (*slip gauges*) and length bars

Gauge blocks, also known as *slip gauges* (Fig. 21.6A), consist of rectangular blocks of hardened steel that have flat and parallel end faces. These faces are machined to very high standards of accuracy in terms of their surface finish and flatness. The purpose of gauge blocks is to provide a means of checking whether a particular dimension in a component is within the allowable tolerance rather than actually measuring what the dimension is. To do this, a number of gauge blocks are joined to make up the required dimension to be checked. Gauge blocks are available in four grades of accuracy according to the maximum

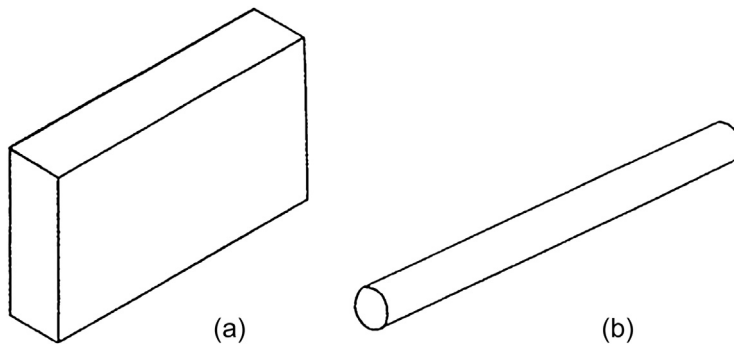


Figure 21.6
(a) Gauge block; (b) length bar.

tolerance allowed in the length dimension. Blocks of the grade with the largest tolerance are used for general workplace measurements and those of the grade with the next largest tolerance are used for inspection duties. Blocks of a grade with the next to smallest tolerance are used for workplace calibration. Finally, blocks of the grade with the smallest tolerance are used for higher-level calibration. Unfortunately, these different grades are coded differently according to different national and international standards, as summarized in [Table 21.1](#).

Gauge blocks are available in boxed sets containing a range of block sizes, which allows any dimension up to 200 mm to be constructed by joining an appropriate number of blocks. Although 200 mm is the maximum dimension that should be set up with gauge blocks alone, they can be used in conjunction with length bars to set up much greater standard dimensions. Blocks are joined by wringing, a procedure in which the two end faces are rotated slowly against each other. This removes the air film and allows adhesion to develop by intermolecular attraction. Adhesion is so good, in fact, that if groups of blocks were not separated within a few hours, the molecular diffusion process would continue to the point where the blocks would be permanently welded. The typical interblock gap resulting from wringing has been measured at 0.001 μm , which is effectively 0. Thus, any number of blocks can be joined without creating any significant measurement error.

It is fairly common practice with all block sets except those of the highest reference grade to include an extra pair of 2-mm-thick blocks in the set that are made from wear-resisting tungsten carbide. These are marked with a letter *P* and are designed to protect the other blocks from wear during use. When such protector blocks are used, due allowance has to be made for their thickness (4 mm) in calculating the sizes of block needed to make up the required length.

A necessary precaution when using gauge blocks is to avoid handling them more than necessary. The length of a block that was 100 mm long at 20°C would increase to

Table 21.1: Grade standards for gauge blocks.

	Japan: JIS B 7506 Germany: DIN 861 USA: ASME UK: BS 4311	U.S. Federal Standard Specification GGG-G-15C	American National Standard ANSI/ASME B 89.1.9M ISO 3650
Reference grade	Grade AAA	Grade 0.5	Grade 00
Calibration grade	Grade AA	Grade 1	Grade 0
Inspection grade	Grade A	Grade 2	Grade 1
Workplace grade	Grade B	Grade 3	Grade 2

100.02 mm at 37°C (body temperature). Hence, after wringing blocks together, they should be left to stabilize back to the ambient room temperature before use. This wait might need to be several hours if the blocks have been handled to a significant extent.

Where a greater dimension than 200 mm is required, gauge blocks are used in conjunction with **length bars** (Fig. 21.6B). Length bars consist of straight, hardened, high-quality steel bars of a uniform 22-mm diameter and in a range of lengths between 100 and 1200 mm. When using length bars, it is important that they are exactly horizontal and mounted so that they do not bend under their own weight. This is achieved by mounting the bars at their **airy** points. The airy points are at a distance of $0.2117L$ from the ends of the bar and are marked by circumferential lines (where L is the length of the bar).

Length bars are available in four grades of accuracy, reference, calibration, inspection, and workplace. As for gauge blocks, the codes for these grades vary according to different national and international standards. Reference and calibration grades have accurately flat end faces, which allows a number of bars to be wrung together to obtain the required standard length. Inspection grade and workplace grade blocks have threaded ends that allow them to be screwed together. By combining length bars with gauge blocks, any dimension up to about 2 m can be set up with a resolution of 0.0005 mm.

21.2.5 Height and depth measurement

The height of objects and the depth of holes, slots, and so on are measured by the height gauge and depth gauge, respectively. A dial gauge is often used in conjunction with these instruments to improve measurement accuracy. The **height gauge**, shown in Fig. 21.7(a), effectively consists of a vernier caliper mounted on a flat base. Measurement inaccuracy levels down to $\pm 0.015\%$ are possible. The **depth gauge** (Fig. 21.7(b)) is a further variation on the standard vernier caliper principle that has the same measurement accuracy capabilities as the height gauge.

In practice, certain difficulties can arise in the use of these instruments when either the base of the instrument is not properly located on the measuring table or the point of contact between the moving anvil and the workpiece is unclear. In such cases, a dial gauge, which has a clearly defined point of contact with the measured object, is used in conjunction with the height or depth gauge to avoid these possible sources of error. These instruments can also be obtained in intelligent versions that give a digital display and have self-calibration capabilities.

The **dial gauge**, shown in Fig. 21.8(a), consists of a spring-loaded probe that drives a pointer around a circular scale via rack and pinion gearing. Typical measurement

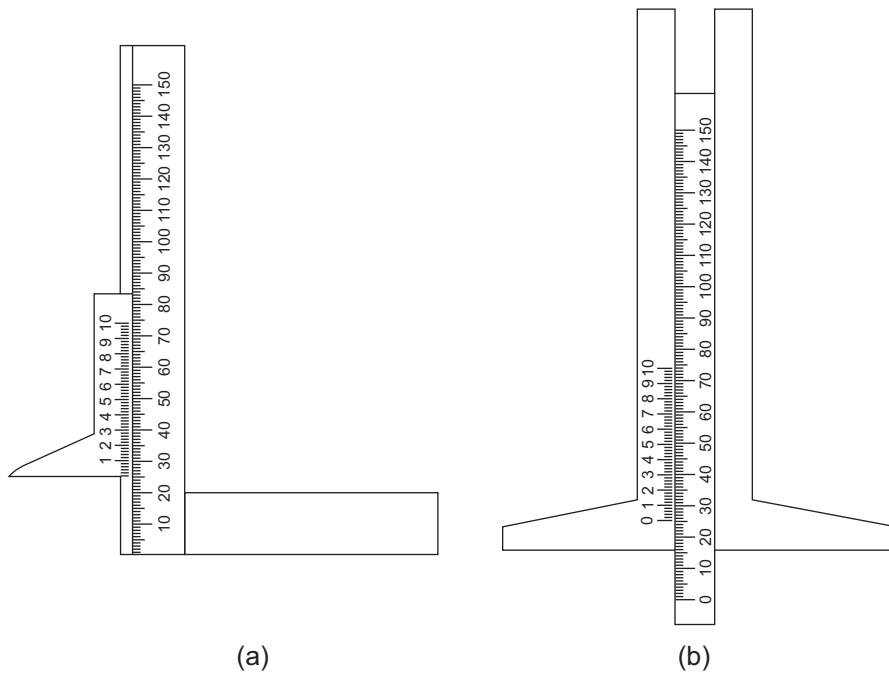


Figure 21.7
(a) Height gauge; (b) depth gauge.

resolution is 0.01 mm. When used to measure the height of objects, it is clamped in a retort stand and a measurement taken of the height of the unknown component. Then, it is put in contact with a height gauge (Fig. 21.8(b)) that is adjusted until the reading on the dial gauge is the same. At this stage, the height gauge is set to the height of the object. The dial gauge is also used in conjunction with the depth gauge in an identical manner. (Gauge blocks can be used instead of height and depth gauges in such measurement procedures if greater accuracy is required.)

21.2.6 Calibration of dimension measurements

Instruments measuring dimensions up to 200 mm are calibrated in the workplace by comparing their reading with standard dimensions set up with calibration grade gauge blocks used on an inspection grade surface table that is itself calibrated for surface flatness and horizontal alignment. For higher-level calibration, reference-grade gauge blocks on a calibration-grade surface table are used. In both cases, a high-magnification comparator is required to test the alignment between the scale markings of the instruments being calibrated and the edge of the gauge blocks. Special end fittings are also used with gauge

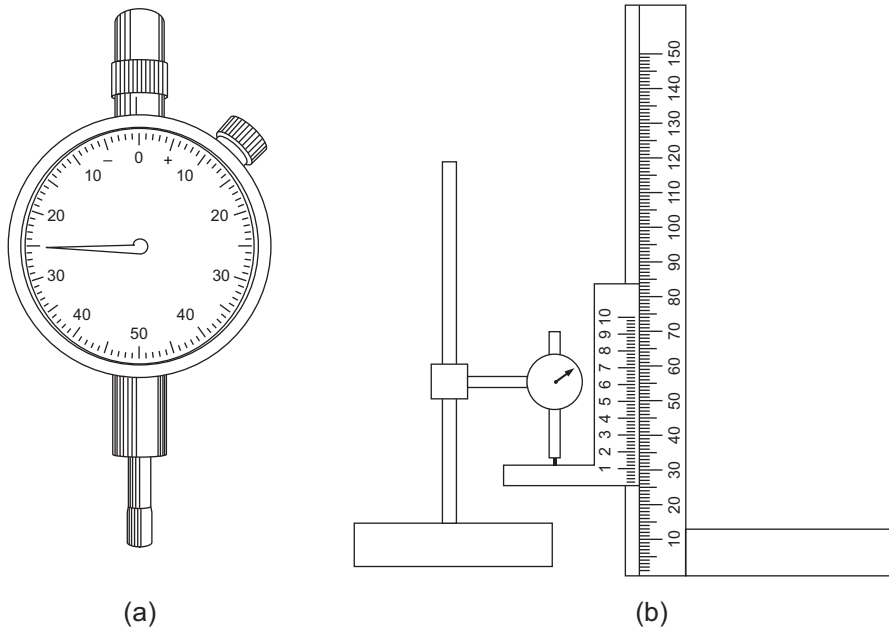


Figure 21.8

Dial gauge: (a) Basic instrument; (b) use in conjunction with height gauge.

blocks when they are used for calibrating micrometers and calipers. Gauge blocks used as a calibration reference must themselves be calibrated against gauge blocks of the next higher grade. This calibration must include a check on their flatness and the degree of parallelism of the opposing end faces, because these latter two parameters are as important as length in determining possible error in total length when a number of blocks are wrung together. At the top of the calibration chain, reference grade gauge blocks are calibrated by national standards organizations using interferometric methods.

Mention should also be made of the *setting gauge*, which is used to calibrate internal micrometers. For testing instruments measuring up to 50 mm, the setting gauge consists of a 25-mm-diameter steel disk (with ± 0.001 -mm tolerance on its diameter). For testing larger range instruments, the setting gauge consists of either a flat-ended or spherical-ended steel rod, with a length tolerance that varies from ± 0.002 mm on a 125-mm-long gauge to ± 0.006 mm on a 575-mm-long gauge.

Extra comments are also necessary about the calibration of external micrometers, for which it is important to check measurement accuracy at intermediate positions of the thimble as well as for full revolutions of the thimble. Recommended procedure for a 0- to

25-mm micrometer is to set up gauge blocks to give the following dimensions in turn: 2.5, 5.1, 7.7, 12.9, 15.0, 17.6, 20.2, 22.8, and 25.0 mm. This tests the thimble at equidistant positions during its rotation. Calibration checks should include testing the flatness and parallelism of the anvil faces. Flatness is checked by bringing the anvil into contact with an optically flat glass plate, in which the degree of nonflatness is indicated by the color and number of interference bands on the surface. Parallelism of the measuring faces is tested by moving an optical flat about between them and observing the changes in the number of interference bands. If calibration checks show errors in length measurement, micrometers are provided with a means of adjustment to take up small amounts of wear in the screw thread. They also usually have some means of rotating the body scale so that the zero mark can be reset.

Some further comment is also needed about the calibration of height gauges, depth gauges and dial gauges. These are calibrated against reference piles of gauge blocks using a special form of dial gauge that has a measurement resolution of 1 μm .

For instruments measuring between 200 mm and 2 m, standard dimensions for calibration are set up by using gauge blocks in conjunction with length bars. Again, calibration grade length bars are used for workplace calibration and reference grade for higher-level calibration. As in the case of gauge blocks, checking the parallelism of their end faces is important when calibrating the length bars themselves.

Specialized equipment is needed to calibrate steel tapes, which are routinely used to measure dimensions greater than 2 m. This equipment is composed of a flat, horizontal stainless-steel bench up to 60 m long that has a measuring carriage consisting of a cube corner reflector and an attached microscope. The tape is stretched along the table and the scale graduations are located through the microscope. The position of each graduation is determined by a laser interferometer attached to the carriage. This equipment is very expensive and therefore tape calibration is normally devolved to specialist calibration companies or tape manufacturers. Ultrasonic rules are also used to measure distances greater than 2 m, but because these give only approximate measurements anyway, calibration using a steel tape is adequate.

The necessary frequency of calibration, as with so many other types of measuring instrument, depends largely on the rate and conditions of use of dimension-measuring instruments. Use records and the results of calibration checks are the only means of setting a sensible calibration frequency. However, whatever recalibration period is defined, vigilant watch must be kept for signs of physical damage to the instrument at intermediate times. Instruments are often abused and mistreated, such as by using the end of rulers to open tins and using vernier calipers as a spanner! Whenever such misuse is suspected, the instrument must be withdrawn immediately for calibration checks to be carried out.

21.3 Angle measurement

Measurement of angles is one of the less common measurement requirements that instrumentation technologists are likely to meet. However, angle measurement is required under some circumstances, such as when the angle between adjoining faces on a component must be checked. The main instruments used are protractors and a form of angle-measuring spirit level.

Under some circumstances, a simple protractor of the sort used in school for geometry exercises can be used. However, the more sophisticated form of *angle protractor* shown in Fig. 21.9 provides better measurement accuracy. This consists of two straight edges, one of which is able to rotate with respect to the other. Referring to Fig. 21.9, graduated circular scale *A* attached to straight edge *C* rotates inside a fixed circular housing attached to the other straight edge *B*. The relative angle between the two straight edges in contact with the component being measured is determined by the position of the moving scale with respect to a reference mark (shown as the etched line in Figure 21.9) on fixed housing *B*. With this type of instrument, measurement inaccuracy is at least $\pm 1\%$. An alternative form, the *bevel protractor* is similar to this form of angle protractor, but it has a vernier scale on the fixed housing. This allows the inaccuracy level to be reduced to ± 10 minutes of arc.

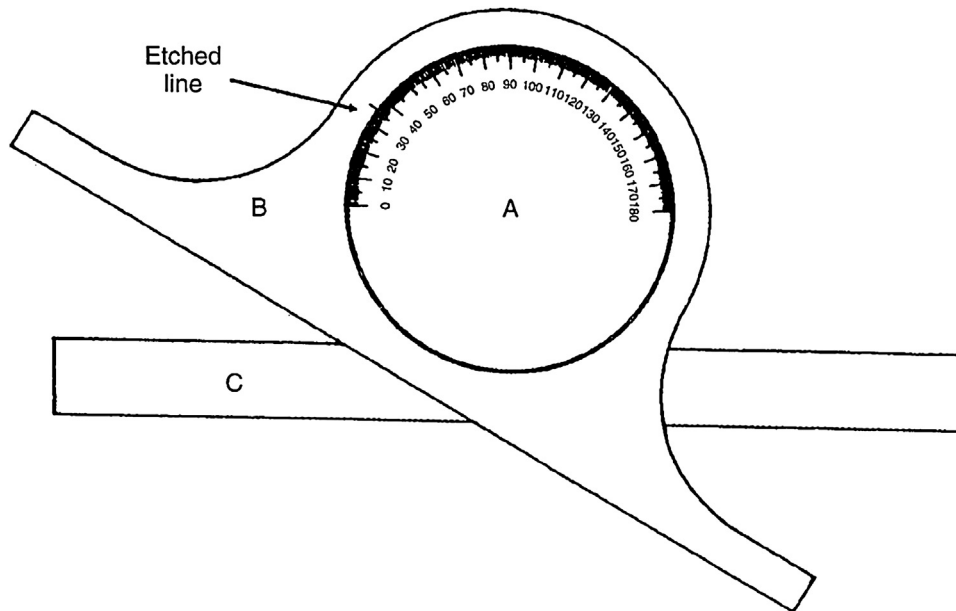


Figure 21.9
Angle protractor.

The form of *spirit level* shown in Fig. 21.10 is an alternative angle-measuring instrument. It consists of a standard spirit level attached to a rotatable circular scale mounted inside an accurately machined square frame. When placed on the sloping surfaces of components, rotation of the scale to centralize the bubble in the spirit level allows the angle of slope to be measured. Again, measuring inaccuracies down to ± 10 minutes of arc are possible if a vernier scale is incorporated in the instrument. A number of different forms exist for this type of angle-measuring spirit level.

The *electronic spirit level* contains a pendulum whose position is sensed electrically. Measurement resolution as good as 0.2 seconds of arc is possible.

21.3.1 Calibration

The simplest piece of equipment available for calibrating angle-measuring instruments is the *sine bar*, which is illustrated in Fig. 21.11. This is used in conjunction with gauge blocks on a surface table of inspection grade or better. The sine bar consists of a rectangular-section piece of steel into which a pair of rollers are located. The distance between the centers of the rollers is known accurately and is commonly set to be 250 mm

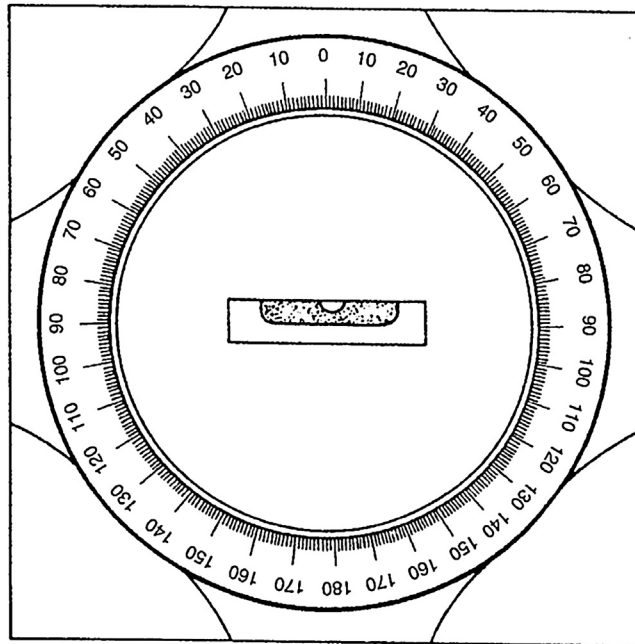


Figure 21.10
Angle-measuring spirit level.

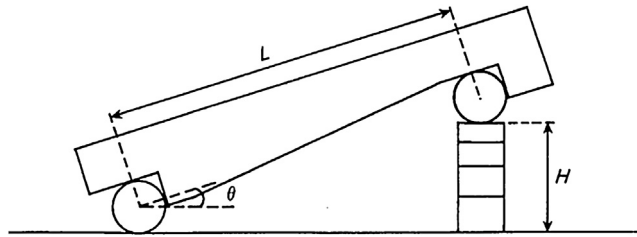


Figure 21.11
Sine bar.

or some multiple thereof. By setting the bar up with one roller on a reference table and the other resting on a pile of gauge blocks, any angle can be set up, given by (Fig. 21.11):

$$\theta = \sin^{-1}(H / L)$$

Maximum measurement inaccuracy is 30 seconds of arc (1/120th of a degree or 145 μ radian).

An alternative way of calibrating angle-measuring instruments is to use **angle blocks**. These consist of a set of steel blocks, with each block having some specific (but different) angle between its faces. A typical angle block is shown in Fig. 21.12. The blocks can be stacked together in various combinations on a surface table to provide a range of different reference angles. Each block is calibrated to a very high standard of accuracy, with a typical measurement uncertainty of 0.1 second of arc (0.5 μ radian). In use, the total measurement uncertainty is normally some multiple of 0.1 second of arc according to how many blocks are stacked together to provide the reference angle required. However, this is clearly a substantially higher standard of accuracy than that provided by a sine bar.

21.4 Surface flatness measurement

The only dimensional parameter not yet discussed for which a measurement requirement may exist is the flatness of the surface of a component. This is measured by a **variation gauge**. As shown in Fig. 21.13, this has four feet, three of which are fixed and one of which floats in a vertical direction. Motion of the floating foot is measured by a dial gauge that is calibrated such that its reading is 0 when the floating foot is exactly level with the fixed feet. Thus, any nonzero reading on the dial gauge indicates nonflatness at the point of contact of the floating foot. By moving the variation gauge over the surface of a component and taking readings at various points, a contour map of the flatness of the surface can be obtained.

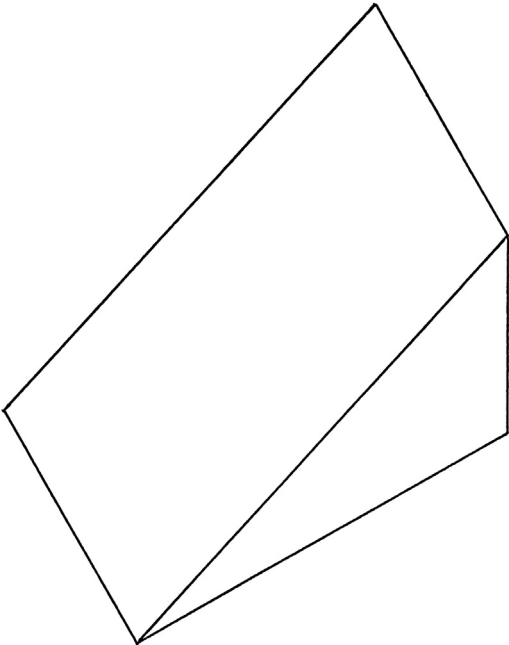


Figure 21.12
Angle block.

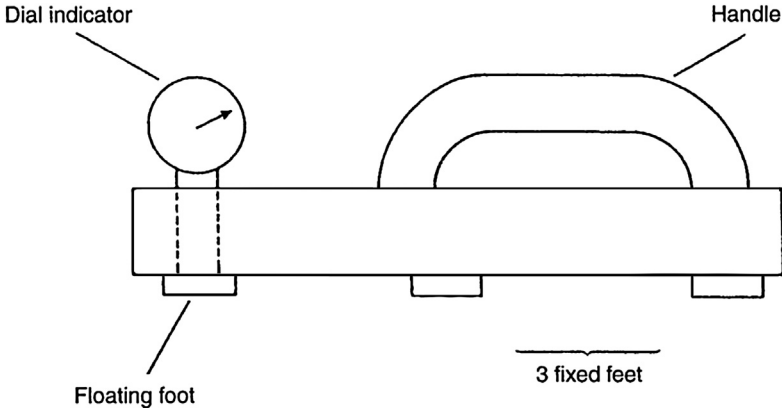


Figure 21.13
Variation gauge.

21.4.1 Calibration of variation gauge

The accuracy of flatness measurements made by a variation gauge depends first on the calibration of the dial gauge within the variation gauge, and second, on the repeatability of the measurement obtained when the dial gauge is brought down onto a horizontal surface. Calibration of the dial gauge itself is obtained by moving it down to gauge blocks set to known heights. The measurement repeatability is tested by moving the dial gauge down to a reference surface table a number of times and recording the variation in the readings obtained.

21.5 Volume measurement

Volume measurement is required in its own right as well as a necessary component in some techniques for the measurement and calibration of other quantities such as volume flow rate and viscosity. The volume of vessels of a regular shape, in which the cross-section is circular or oblong, can be readily calculated from the dimensions of the vessel. Otherwise, for vessels of irregular shape, it is necessary to use either gravimetric techniques or a set of calibrated volumetric measures.

In the gravimetric technique, the dry vessel is weighed and then completely filled with water and weighed again. The volume is then simply calculated from this weight difference and the density of water. Measurement uncertainty is low, because it is possible to measure the two weights to a high degree of accuracy. However, because the arithmetic operation of subtracting the two weights amplifies errors, it is sensible to arrange for the difference between the two weights to be as large as possible.

The alternative technique involves transferring the liquid from the vessel into an appropriate number of volumetric measures taken from a standard-capacity, calibrated set. Each vessel in the set has a mark that shows the volume of liquid contained when the vessel is filled to the mark. Special care is needed to ensure that the meniscus of the water is in the correct position with respect to the reference mark on the vessel when it is deemed to be full. Normal practice is to set the water level such that the reference mark forms a smooth tangent with the convex side of the meniscus. This is made easier to achieve if the meniscus is viewed against a white background and the vessel is shaded from stray illumination.

The measurement uncertainty using calibrated volumetric measures depends on the number of measures used for any particular measurement. The total error is a multiple of the individual error of each measure, typical values of which are shown in [Table 21.2](#).

Table 21.2: Typical measurement uncertainty of calibrated volumetric measures.

Capacity	Volumetric uncertainty
1 ml	±4%
10 ml	±0.8%
100 ml	±0.2%
1 l	±0.1%
10 l	±0.05%
100 l	±0.02%
1000 l	±0.02%

21.5.1 Calibration of volume measurements

Calibration of volume measurements is normally achieved with the gravimetric technique, as described earlier, but with certain special precautions applied. First, the water used to fill the vessel is either distilled or deionized. (When very large vessels are being calibrated and it is impractical to use anything other than tap water, the difference in water density compared with pure water gives a typical error of 0.02%.) Second, the calibration is carried out under specified temperature and pressure conditions, and sufficient time must be allowed for the vessel and water to become in close thermal equilibrium. This latter condition is most easily achieved by maintaining a store of distilled or deionized water in the calibration laboratory, which ensures that the water is automatically at the correct temperature. Finally, correction may have to be made for changes in air buoyancy, which become significant if calibration is carried out at altitudes greater than 150 m.

21.6 Viscosity measurement

Viscosity measurement is important in many process industries. In the food industry, the viscosity of raw materials such as dough, batter, and ice cream has a direct effect on the quality of the product. Similarly, in other industries such as ceramic industry, the quality of raw materials affects the final product quality. Viscosity control is also very important in assembly operations that involve the application of mastics and glue flowing through tubes. Clearly, successful assembly requires such materials to flow through tubes at the correct rate, and therefore it is essential that their viscosity is correct.

Viscosity describes the way in which a fluid flows when it is subject to an applied force. Consider an elemental cubic volume of fluid and a shear force F applied to one of its faces of area A . If this face moves a distance L and at a velocity V relative to the opposite face of the cube under the action of F , shear stress (s) and shear rate (r) are given by:

$$s = F/A ; r = V/L$$

The *coefficient of viscosity* (C_V) is the ratio of shear stress to shear rate; i.e., $C_V = s/r$.

C_V is often described simply as the viscosity. A further term, *kinematic viscosity*, is also sometimes used, given by: $K_V = C_V / \rho$, where K_V is the kinematic viscosity and ρ is the fluid density. To avoid confusion, C_V is often known as the *dynamic viscosity*, to distinguish it from K_V . C_V is measured in units of poise or Ns/m^2 and K_V is measured in units of Stokes or m^2/s .

Viscosity was originally defined by Newton, who assumed that it was constant with respect to shear rate. However, it has since been shown that the viscosity of many fluids varies significantly at high shear rates and the viscosity of some even varies at low shear rates. The worst non-Newtonian characteristics tend to occur with emulsions, pastes and slurries. For non-Newtonian fluids, subdivision into further classes can also be made according to the manner in which the viscosity varies with shear rate, as shown in Fig. 21.14.

The relationship between the input variables and output measurement for instruments that measure viscosity normally assumes that the measured fluid has Newtonian characteristics. For non-Newtonian fluids, a correction must be made for shear rate variations. If such a correction is not made, the measurement obtained is known as the **apparent viscosity**, and this can differ from the true viscosity by a large factor. The true viscosity is often called the **absolute viscosity** to avoid ambiguity. Viscosity also varies with fluid temperature and density.

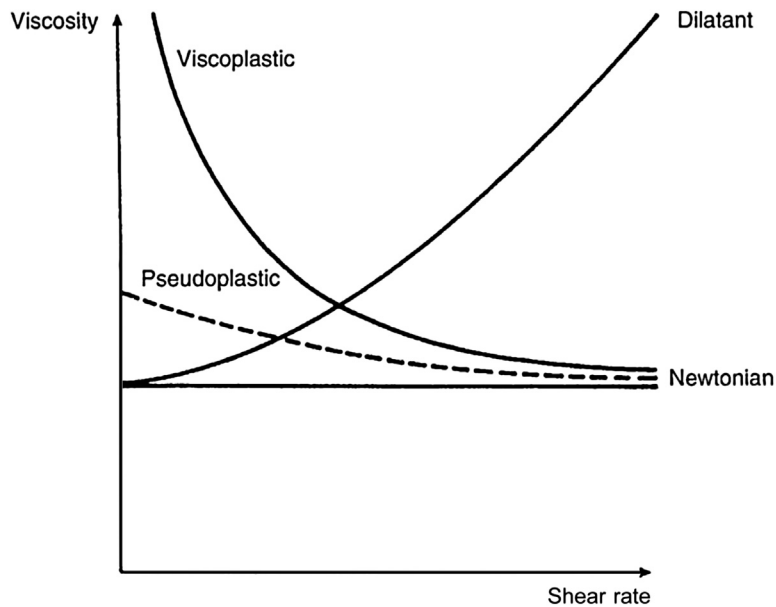


Figure 21.14
Different viscosity—shear rate relationships.

Instruments for measuring viscosity work on one of three physical principles:

- Rate of flow of the liquid through a tube
- Rate of fall of a body through the liquid
- Viscous friction force exerted on a rotating body

Capillary viscometers

These are the most accurate types of viscometer, with typical measurement inaccuracy levels down to $\pm 0.3\%$. Liquid is allowed to flow from a reservoir, under gravity, through a capillary tube of a known cross-section. The pressure difference across the ends of the tube and the time for a given quantity of liquid to flow are measured, and then the liquid viscosity for Newtonian fluids can be calculated as (in units of poise):

$$C_V = \frac{1.25\pi R^4 PT}{LV}$$

where R is the radius (m) of the tube, L is its length (m), P is the pressure difference (N/m^2) across the ends, and V is the volume of liquid flowing in time T (m^3/s).

For non-Newtonian fluids, corrections must be made for shear rate variations. For any given viscometer, R , L , and V are constant and Eq. (21.1) can be written as:

$$C_V = KPT$$

where K is known as the viscometer constant.

Falling body viscometer

The falling body viscometer is particularly recommended for measuring high-viscosity fluids. It can give measurement uncertainty levels down to $\pm 1\%$. It involves measuring the time taken for a spherical body to fall a given distance through the liquid. The viscosity for Newtonian fluids is then given by Stokes formula as (in units of poise):

$$C_V = \frac{R^2 g (\rho_s - \rho_l)}{450V}$$

where R is the radius (m) of the sphere, g is the acceleration due to gravity (m/s^2), ρ_s and ρ_l are the specific gravities (g/m^3) of the sphere and liquid, respectively, and V is the velocity (m/s) of the sphere.

Unfortunately, the falling body viscometer is accurate only for Newtonian fluids, because correction for the variation in shear rate that occurs in non-Newtonian fluids is difficult.

Rotational viscometers

Rotational viscometers are relatively easy to use but their measurement inaccuracy is typically $\pm 5\%$. All types have some form of element rotating inside the liquid at a

constant rate. One common version has two coaxial cylinders with the fluid to be measured contained between them. One cylinder is driven at a constant angular velocity by a motor and the other is suspended by torsion wire. After the driven cylinder starts from rest, the suspended cylinder rotates until an equilibrium position is reached where the force due to the torsion wire is just balanced by the viscous force transmitted through the liquid. The viscosity (in poise) for Newtonian fluids is then given by:

$$C_V = 2.5G \left(\frac{1/R_1^2 - 1/R_2^2}{\pi h \omega} \right)$$

where G is the couple (Nm) formed by the force exerted by the torsion wire and its deflection, R_1 and R_2 , are the radii (m) of the inner and outer cylinders, h is the length of the cylinder (m), and ω is the angular velocity (rad/s) of the rotating cylinder. Again, corrections have to be made for non-Newtonian fluids.

21.6.1 Viscosity calibration

Viscosity standards are defined only for Newtonian fluids. No such standards exist for non-Newtonian fluids.

Viscosity calibration for Newtonian liquids is carried out using a set of glass capillary tubes with accurately known viscometer constants. These provide reference standards over a range of viscosities. The set is calibrated by a step-up procedure from a calibration on distilled water, using a series of standard liquids. The steps in this procedure are as follows:

- a) A master viscometer is calibrated first using distilled water as the flowing fluid.
- b) The master is then used to measure the viscosity of the first standard viscosity liquid.
- c) This liquid is then used to calibrate the first standard viscometer.
- d) After this, the viscosity of the second standard liquid is measured by the first standard viscometer and then the second standard viscometer is calibrated using the second standard liquid.
- e) This procedure is repeated to calibrate the other standard viscometers in the set using the other standard liquids.

It is common practice to maintain two sets of such standard viscometers: one as a working set for routine calibrations and one as a reference set for calibration checks on the working set. Higher-level calibration is provided by comparison with a set of standard glass-capillary viscometers maintained by national standards organizations. Such organizations also commonly supply a range of standard viscosity liquids as an alternative calibration source. Where standard liquids are kept for calibration, they must be checked periodically to ensure their continuing stability as they age.

Because there are no absolute physical quantities against which the calibration of viscosity values can be checked, the various national standards organizations carry out cross-checks between each other's laboratories. These checks take two forms. One is to transport a reference set of viscometers between laboratories and to compare the viscometer constants measured, with appropriate correction for differences in the value of g (acceleration due to gravity). The other form of check is to compare measurements made on standard liquids at a specified temperature.

In the case of non-Newtonian fluids, as has already been stated, there are no formal viscosity standards. Therefore, the only course of action available for checking viscometers intended to measure non-Newtonian fluids is to calibrate them against standard-viscosity Newtonian fluids.

21.7 Moisture measurement

There are many industrial requirements for the measurement of the moisture content in materials. The material containing moisture may be in solid, liquid, or gaseous form. There are several common reasons for needing to know the moisture of materials in each form.

The physical properties and storage stability of most solid materials are affected by their water content. There is also a statutory requirement to limit the moisture content in the case of many materials sold by weight. In consequence, the requirement for moisture measurement pervades a large number of industries involved in the manufacture of foodstuffs, pharmaceuticals, cement, plastics, textiles, and paper.

Measurement of the water content in liquids is commonly needed for fiscal purposes, but it is also often necessary to satisfy statutory requirements. The petrochemical industry has wide-ranging needs for moisture measurement in oil, and so on. The food industry also needs to measure the water content of products such as beer and milk.

In the case of moisture in gases, the most common measurement is the amount of moisture in air. This is usually known as the *humidity* level. Humidity measurement and control are an essential requirement in many buildings, greenhouses, and vehicles.

21.7.1 Industrial moisture measurement techniques

Industrial methods for measuring moisture are based on the variation of some physical property of the material with moisture content in it. Many different properties can be used, and therefore the range of available techniques, as listed below, is large.

Electrical methods

The most common technique for determining moisture content is to measure the amount of **absorption of microwave energy** beamed through the material. This is the difference between the amount of energy passing into the material and the amount of energy detected by a sensor when the energy emerges from the material. This uses the principle that microwaves at wavelengths between 1 mm and 1 m are absorbed to a much greater extent by water than by most other materials. Wavelengths of 30 or 100 mm are commonly used because off-the-shelf equipment to produce these is readily available from instrument suppliers. The technique is suitable for measuring the moisture content of solids, liquids, and gases at moisture-content levels up to 45% and measurement uncertainties down to $\pm 0.3\%$ are possible.

The **capacitance moisture meter** uses the principle that the dielectric constant of materials varies according to their water content. The moisture content of materials is therefore related to the capacitance. The instrument is useful for measuring moisture-content levels up to 30% in both solids and liquids, and measurement uncertainty down to $\pm 0.3\%$ are possible. Drawbacks of the technique include (1) limited measurement resolution owing to the difficulty in measuring small changes in a relatively large standing capacitance value, and (2) difficulty when the sample has a high electrical conductivity. An alternative capacitance charge transfer technique overcomes these problems by measuring the charge-carrying capacity of the material. In this technique, wet and dry samples of the material are charged to a fixed voltage and then simultaneously discharged into charge-measuring circuits.

The **electrical conductivity** of most materials varies with moisture content and this therefore provides another means of measurement. Techniques using electrical conductivity variation are cheap and can measure moisture levels up to 25%. However, the presence of other conductive substances in the material such as salts or acids affects the measurement accuracy.

A further electrical method is to measure the frequency change in a **quartz crystal** that occurs as it takes in moisture.

Neutron moderation

Neutron moderation measures moisture content using a radioactive source and a neutron counter. Fast neutrons emitted from the source are slowed down by hydrogen nuclei in the water, forming a cloud whose density is related to the moisture content. Measurements take a long time because the output density reading may take up to a few minutes to reach steady-state, according to the nature of the materials involved. Also, the method cannot be used with materials that contain hydrogen molecules, such as oils and fats, because these slow down neutrons as well. Specific humidities up to 15% can be measured, with a

measurement uncertainty of $\pm 1\%$. The main application is measuring the moisture content of soil or rock.

Low-resolution nuclear magnetic resonance

Low-resolution nuclear magnetic resonance (NMR) has not yet achieved widespread use but studies have shown good performance in measuring the moisture content in materials such as wood and sludge. The technique involves subjecting the sample to both an unidirectional and an alternating radiofrequency (RF) magnetic field. The amplitude of the unidirectional field is varied cyclically, which causes resonance once per cycle in the coil producing the RF field. Under resonance conditions, protons are released from the hydrogen content of the water in the sample. These protons cause a measurable moderation of the amplitude of the RF oscillator waveform that is related to the moisture content of the sample.

Materials that naturally have a hydrogen content cannot normally be measured. However, pulsed NMR techniques have been developed that overcome this problem by taking advantage of the different relaxation times of hydrogen nuclei in water and oil. In such pulsed techniques, dependence on the relaxation time limits the maximum fluid flow rate for which moisture can be measured.

Optical methods

The *refractometer* is a well-established instrument used for measuring the water content of liquids. It measures the refractive index of the liquid, which changes according to the moisture content.

Moisture-related *energy absorption* of near-infrared light can be used for measuring the moisture content of solids, liquids, and gases. At a wavelength of 1.94 μm , energy absorption due to moisture is high, whereas at 1.7 μm , absorption due to moisture is 0. Therefore, measuring absorption at both 1.94 and 1.7 μm allows absorption due to components in the material other than water to be compensated for, and the resulting measurement is directly related to moisture content. The latest instruments use multiple-frequency infrared energy and have an even greater capability for eliminating the effect of components in the material other than water that absorb energy. Such multifrequency instruments also cope much better with variations in particle size in the measured material.

In alternative versions of this technique, energy is either transmitted through the material or reflected from its surface. In either case, materials that are either very dark or highly reflective give poor results. The technique is particularly attractive when applicable, because it is a noncontact method that can be used to monitor moisture content continuously at moisture levels up to 50%, with inaccuracy as low as $\pm 0.1\%$ in the measured moisture level.

Ultrasonic methods

The presence of water changes the speed of propagation of ultrasonic waves through liquids. The moisture content of liquids can therefore be determined by measuring the transmission speed of ultrasound. This has the inherent advantage of being a noninvasive technique, but temperature compensation is essential because the velocity of ultrasound is particularly affected by temperature changes. The method is best suited to measurement of high moisture levels in liquids that are neither aerated nor of high viscosity. Typical measurement uncertainty is $\pm 1\%$ but measurement resolution is very high, with changes in moisture level as small as 0.05% being detectable.

Change in mechanical properties

Density changes in many liquids and slurries can be measured and related to moisture content, with good measurement resolution up to 0.2% moisture. Moisture content can also be estimated by measuring the moisture level–dependent viscosity of liquids, pastes, and slurries.

21.7.2 Laboratory techniques for moisture measurement

Various laboratory techniques for measuring moisture content are available, as described below. Although these generally take much longer to produce a measurement than the industrial techniques described above, the measurement accuracy obtained is usually much better.

Water separation

A number of laboratory techniques are available that enable the moisture content of liquids to be measured accurately by separating the water from a sample of the host liquid. Separation is effected by titration (Karl Fischer technique), distillation (Dean and Stark technique), or a centrifuge. Any of these methods can measure water content in a liquid with measurement uncertainty levels down to $\pm 0.03\%$.

Gravimetric methods

Moisture content in solids can be measured accurately by weighing the moist sample, drying it, and then weighing again. Great care must be taken in applying this procedure, because many samples rapidly take up moisture again if they are removed from the drier and exposed to the atmosphere before being weighed. The normal procedure is to put the sample in an open container, dry it in an oven, and then screw an airtight top onto the container before it is removed from the oven.

Phase-change methods

The boiling and freezing points of materials are altered by the presence of moisture, and therefore the moisture level can be determined by measuring the phase-change temperature. This technique is used for measuring the moisture content in many food products and also in some oil and alcohol products.

Equilibrium relative humidity measurement

This technique involves placing a humidity sensor in close proximity to the sample in an airtight container. The water vapor pressure close to the sample is related to the moisture content of the sample. The moisture level can therefore be determined from the humidity measurement.

21.7.3 Humidity measurement

The term *humidity* describes the moisture content of air. This moisture content can be expressed in three alternative ways: absolute humidity, specific humidity, and relative humidity.

Absolute humidity is the mass of water in a unit volume of moist air.

Specific humidity is the mass of water in a unit mass of moist air.

Relative humidity is the ratio of the actual water vapor density in air to the saturation vapor density, usually expressed as a percentage. The saturation vapor density varies with temperature, and so the relative humidity also varies with temperature for any given measured value of actual vapor density.

The three major instruments used for measuring humidity in industry are the electrical hygrometer, psychrometer, and dew point meter. The dew point meter is the most accurate of these and is commonly used as a calibration standard.

The electrical hygrometer

The electrical hygrometer measures the change in capacitance or conductivity of a hygroscopic material as its moisture level changes. Instruments measuring conductivity change use two noble metal electrodes on either side of an insulator coated in a hygroscopic salt such as calcium chloride. Capacitance types have two plates either side of a hygroscopic dielectric such as aluminum oxide.

These instruments are suitable for measuring moisture levels between 15% and 95%, with a typical measurement uncertainty of $\pm 3\%$. Atmospheric contaminants and operation in

saturation conditions both cause drift in the characteristics of the instrument, and therefore the recalibration frequency has to be determined according to the conditions of use.

The psychrometer (wet and dry bulb hygrometer)

The psychrometer, also known as the wet and dry bulb hygrometer, has two temperature sensors, one exposed to the atmosphere and one enclosed in a wet wick. Air is blown across the sensors, which causes evaporation and a reduction in temperature in the wet sensor. The temperature difference between the sensors is related to the humidity level. The lowest measurement uncertainty attainable is $\pm 4\%$.

Dew point meter

The elements of the dew point meter, also known as the dew point hygrometer, are shown in Fig. 21.15. The sample is introduced into a vessel with an electrically cooled mirror surface. The mirror surface is cooled until a light source—light detector system detects the formation of dew on the mirror, and the condensation temperature is measured by a sensor bonded to the mirror surface. The dew point is the temperature at which the sample becomes saturated with water. Therefore, this temperature is related to the moisture level in the sample. A microscope is also provided in the instrument so that the thickness and nature of the condensate can be observed.

Even small levels of contaminants on the mirror surface can cause large changes in the dew point, and therefore the instrument must be kept clean. When necessary, the mirror should be cleaned with deionized or distilled water applied with a lint-free swab. Any contamination can be detected by a skilled operator, because this makes the

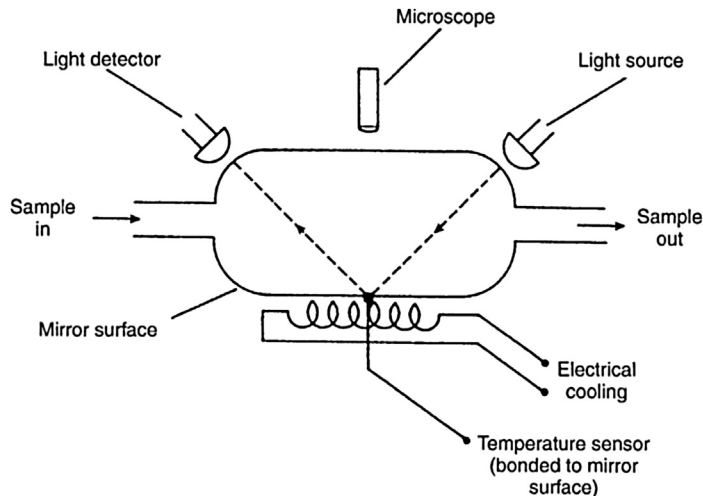


Figure 21.15
Dew point meter.

condensate look blotchy when viewed through the microscope. The microscope also shows up other potential problems such as large ice crystals in the condensate that cause temperature gradients between the condensate and the temperature sensor. When used carefully, the instrument is accurate and is often used as a reference standard.

Microelectromechanical system (MEMS) relative humidity sensor

Microelectromechanical system relative humidity sensors usually incorporate two sensing elements: one to measure humidity and one to measure temperature (because the relative humidity for any given value of measured water vapor density varies with temperature). The humidity sensor is a differential capacitance type that consists of a layer sensitive to water vapor that is sandwiched between two electrodes that act as capacitor plates. The upper electrode consists of a grid that allows water vapor to pass into the sensitive layer, thus altering the capacitance between the two electrodes. On-chip circuits carry out automatic calibration and signal processing to produce a relative humidity measurement.

21.7.4 Calibration of moisture and humidity measurements

Microwave techniques as described at the start of [Section 21.7.1](#) are usually accurate enough for working standard calibration. The electrical hygrometer can also be used for calibration at this level provided it is not subjected to near-saturation conditions and it is stored carefully away from possible contaminants. The dew point meter is also suitable for workplace calibration. For calibration to a higher standard of accuracy, gravimetric or water-separation laboratory techniques (see [Section 21.7.2](#)) are used. In the case of calibration even further up the calibration chain to secondary and primary reference standards, the method used depends on whether the instrument being calibrated is designed to measure a fluid (liquid or gas) or a solid.

In the case of calibration of instruments measuring the moisture content of gases and liquids to secondary and primary reference standards, test samples are used. These are created by making up mixtures of a gas or liquid with water in known proportions, such that the water content is accurately known. The instrument being calibrated is applied to these samples to see whether its output reading agrees with the known water content. Test samples created in a company's own calibration laboratory can be used as a secondary reference standard. However, for calibration to primary reference standards, test samples provided by national standards organizations must be used.

The calibration of instruments measuring the moisture content of solids to secondary and primary reference standards is harder. It is often difficult to make up a test sample with an even distribution of water through it. Therefore, gravimetric techniques of calibration (see [Section 21.7.2](#)) are normally used instead.

21.8 Sound measurement

Noise can arise from many sources in both industrial and nonindustrial environments. Even low levels of noise can cause great annoyance to the people subjected to it and high levels of noise can actually cause hearing damage. Apart from annoyance and possible hearing loss, noise in the workplace causes loss of output when persons subjected to it are involved in tasks requiring high concentration. Extreme noise can even cause material failures through fatigue stresses set up by noise-induced vibration.

Most countries have legislation to control the creation of noise. Court orders can be made against houses or factories in a neighborhood that create noise exceeding a certain acceptable level. In extreme cases, when hearing damage may be possible, health and safety legislation comes into effect. Such legislation clearly requires the existence of accurate methods of quantifying sound levels. Sound is measured in terms of the *sound pressure level*, S_p , defined as:

$$S_p = 20 \log_{10} \left(\frac{P}{0.0002} \right) \text{decibels (dB)}$$

where P is the root-mean-square (rms) sound pressure in μbar .

The quietest sound that the average human ear can detect is a tone at a frequency of 1 kHz and sound pressure level of 0 dB ($2 \times 10^{-4} \mu\text{bar}$). At the upper end, sound pressure levels of 144 dB (3.45 mbar) cause physical pain.

Sound is usually measured with a sound meter. This essentially processes the signal collected by a microphone, as shown in Fig. 21.16. The microphone is a diaphragm-type

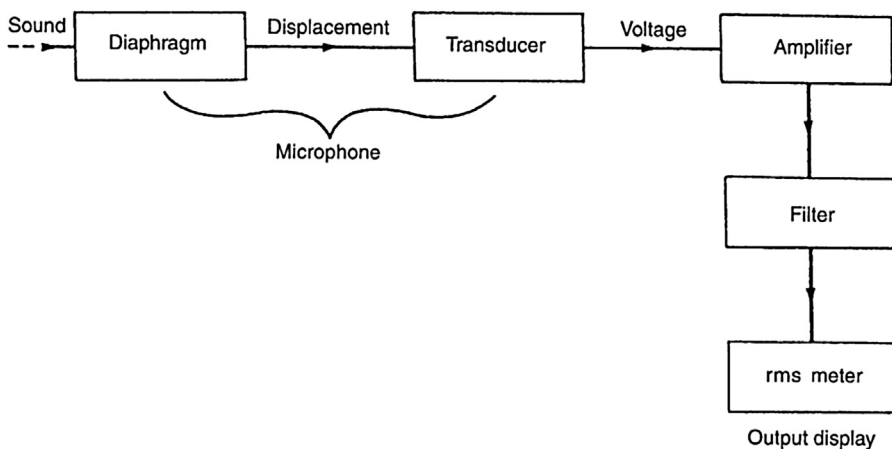


Figure 21.16
Components in a sound meter.

pressure-measuring device that converts sound pressure into a displacement. The displacement is applied to a displacement transducer (normally capacitive, inductive, or piezoelectric type) that produces a low-magnitude voltage output. This is amplified and filtered and finally gives an output display on an rms meter. The filtering process has a frequency response approximating that of the human ear so that the sound meter hears sounds in the same way as a human ear. In other words, the meter selectively attenuates frequencies according to the sensitivity of the human ear at each frequency, so that the sound level measurement output accurately reflects the sound level heard by humans. However, if sound level meters are used to measure sound to predict vibration levels in machinery, they are used without filters so that the actual rather than human-perceived sound level is measured.

MEMS sound sensors are also now available in which the varying-capacitance sensing element consists of a fixed back plate and a movable membrane plate. Acoustic holes in the membrane plate allow sound to pass through, which modulates the air gap and hence capacitance between the plates. Different forms of sensor offer both analog and digital forms of output. The analog type converts the capacitance change into a varying voltage output whereas the digital type converts the capacitance change into a form of pulse density modulation. Such MEMS devices are useful in applications such as cellphones, for which their small size and weight are extremely important.

21.8.1 Calibration of sound meters

Calibration of sound meters requires the use of a sound-calibrating instrument that generates standard sound pressure levels at several defined frequencies. The reading of the sound meter is then compared with the magnitude of the sound pressure level emitted by the calibration instrument at each defined sound frequency. In practice, great skill is needed to perform the calibration to ensure that the results are accurate, because there are several potential difficulties. Appropriate instructions are provided with proprietary calibration instruments, but it is generally safer to give the task of sound meter calibration to specialist calibration companies or the sound meter manufacturer.

21.9 pH measurement

pH is a parameter that quantifies the level of acidity or alkalinity in a chemical solution. It defines the concentration of hydrogen atoms in the solution in grams per liter and is expressed as:

$$pH = \log_{10} [1 / H^+]$$

where H^+ is the hydrogen ion concentration in the solution.

The value of pH can range from 0, which describes extreme acidity, to 14, which describes extreme alkalinity. Pure water has a pH of 7. pH measurement is required in many process industries, especially those involving food and drink production. The most universally known method of measuring pH is to use *litmus paper* or some similar *chemical indicator* that changes color according to the pH value. Unfortunately, this method gives only an approximate indication of pH unless used under highly controlled laboratory conditions. The most common alternative measurement device is the glass electrode.

The glass electrode

The glass electrode consists of a glass probe containing two electrodes, a measuring one and a reference one, separated by a solid glass partition. Neither of the electrodes is in fact glass. The reference electrode is a screened electrode immersed in a buffer solution, which provides a stable reference electromotive force (emf) that is usually 0 V. The tip of the measuring electrode is surrounded by a pH-sensitive glass membrane at the end of the probe, which permits the diffusion of ions according to the hydrogen ion concentration in the fluid outside the probe. The measuring electrode therefore generates an emf proportional to pH that is amplified and fed to a display meter. The combination of a glass electrode and its display meter is commonly known as a *pH meter*. These are available in both benchtop and handheld versions.

Although it is theoretically capable of measuring the full range of pH values between 0 and 14, the upper measurement limit is generally a pH value of about 12 in practice because electrode contamination at very high alkaline concentrations becomes a serious problem and glass starts to dissolve at such high pH values. However, measurement of any pH value requires careful use of a glass electrode type of pH probe. First, the measuring probe has a very high resistance (typically 10⁸ Ω) and a very low output. Hence, the output signal from the probes must be electrically screened to prevent any stray pickup, and electrical insulation of the assembly must be very high. The assembly must also be very efficiently sealed to prevent the ingress of moisture.

A second problem with the glass electrode is the deterioration in accuracy that occurs as the glass membrane becomes coated with various substances to which it is exposed in the measured solution. Cleaning at prescribed intervals is therefore necessary, and this must be carried out carefully using the correct procedures to avoid damaging the delicate glass membrane at the end of the probe. The best cleaning procedure varies according to the nature of the contamination. In some cases, careful brushing or wiping is adequate, whereas in other cases spraying with chemical solvents is necessary. Ultrasonic cleaning is often a useful technique, although it tends to be expensive. Steam cleaning should not be attempted, because this damages the pH-sensitive membrane. Mention must also be made about storage. The glass electrode must not be allowed to dry out during storage, because this would cause serious damage to the pH-sensitive layer, and the normal procedure is to keep

it in a potassium chloride solution if it is normally used for measuring alkaline pH levels (pH above 7) or in weak sulfuric acid if it is used to measure acidic pH levels (pH of 0–7). The response time of a glass electrode also requires careful attention. The instrument has a relatively large time constant of 1–2 minutes, and so it must be left to settle for a long time before the reading is taken.

Various forms of glass electrode are available to maximize performance under various measurement conditions. High-temperature versions made from special glass are available for measurement at high temperatures. These are designed to minimize premature aging that occurs when ordinary electrodes are used at high temperature, which reduces both their accuracy and their service life. Special low-temperature versions are available made from low-resistivity glass that maximizes their speed of response when used at low temperatures. Flat-profile electrodes are available for use in situations in which high levels of particles are present in the measured liquid, such as occurs in paper-making applications. The flat profile makes them largely self-cleaning and avoids the rapid degradation in performance that would occur with standard glass electrodes. Special glass is also needed for pH measurement in acid solutions containing fluoride, because standard glass dissolves in such an environment.

Other methods of pH measurement

The ***antimony electrode*** has a construction similar to that of the glass electrode but uses antimony instead of glass. The device is more robust than the glass electrode and can be cleaned by rubbing it with emery cloth. However, its time constant is very large and its output response is grossly nonlinear, which limits its application to environments where the glass electrode is unsuitable. Such applications include acidic environments containing fluoride and environments containing abrasive particles. The normal measurement range is pH 1–11.

Solid-state pH electrodes are another relatively recent development. These are based on an ion-selective field effect transistor in which the current flowing between two semiconductor elements in the device varies with pH in an analogous way to the variation in potential difference within a glass electrode. Such solid-state sensors are physically much smaller than glass electrodes and are also much more robust, both of which are significant advantages in some applications. However, the fact that their output is in the form of a varying current means they cannot be used within a standard pH meter and their accuracy is inferior to that of glass electrodes.

A ***fiber-optic pH*** sensor is another available device, as described earlier in Chapter 13, in which the pH level is indicated by the intensity of light reflected from the tip of a probe coated in a chemical indicator whose color changes with pH. Unfortunately, this device has the capability to measure over only a small range of pH (typically 2 pH) and it has a short life.

21.9.1 pH calibration

The characteristics of the glass electrode are dependent on ambient temperature, with both zero drift and sensitivity drift occurring. Thus, temperature compensation is essential and it is normally necessary to recalibrate the measurement system every time there is a change in temperature.

Calibration is performed by immersing the probe in buffer solutions at reference pH values. These solutions are available commercially and can be bought as either a ready-made solution or a solid form. In the case of the solid form, the buffer solution is made by dissolving the solid in deionized water. Whatever the source of the calibration buffer solution, great care must be taken to control the temperature at which the calibration is performed and ensure that it is the same as that specified with the buffer solution for its pH value to be valid.

21.10 Gas sensing and analysis

Gas sensing and analysis are required in many applications. A primary role of gas sensing is in hazard monitoring to predict the onset of conditions in which flammable gases are reaching dangerous concentrations. Danger is quantified in terms of the **lower explosive level**, which is usually reached when the concentration of a flammable gas in air is in the range of 1–5%.

Gas sensing also provides a fire detection and prevention function. When materials burn, a variety of gaseous products result. Most sensors used for fire detection measure carbon monoxide concentration, because this is the most common combustion product. Early fire detection enables fire extinguishing systems to be triggered, preventing serious damage from occurring in most cases. However, fire prevention is even better than early fire detection, and solid-state sensors, based on a sintered mass of polycrystalline tin oxide, can now detect gaseous products (generally various types of hydrocarbon) generated when materials become hot but before they actually burn.

Health and safety legislation creates a further requirement for gas sensors. Certain gases, such as carbon monoxide, hydrogen sulfide, chlorine, and nitrous oxide, cause fatalities above a certain concentration, and sensors must provide warning of impending danger. For other gases, health problems are caused by prolonged exposure, and so the sensors in this case must integrate gas concentration over time to determine whether the allowable exposure limit over a given time has been exceeded. Solid-state sensors are now available to fulfill this function.

Concern about general environmental pollution is also making the development of gas sensors necessary in many areas. Legislation is growing rapidly around the world to control the emission of everything that is proven or suspected to cause health problems or environmental damage. The current list of controlled emissions includes nitrous oxide, oxides of sulfur, carbon monoxide and dioxide, chlorofluorocarbons (CFCs), ammonia, and hydrocarbons. Sensors are required both at the source of these pollutants, where concentrations are high, and to monitor the much lower concentrations in the general environment. Oxygen concentration measurement is also often greatly important in pollution control, because the products of combustion processes are highly affected by the air–fuel ratio.

Sensors associated with pollution monitoring and control usually have to satisfy stringent specifications, particularly when the sensors are located at the pollutant source. Robustness is usually essential, because such sensors are subjected to bombardment from a variety of particulate matters and must endure conditions of high humidity and temperature. They are also frequently located in inaccessible locations, such as in chimneys and flues, which means that they must have stable characteristics over long periods without calibration checks being necessary. The need for such high-specification sensors makes such pollutant-monitoring potentially very expensive if there are several problem gases involved. However, because the concentration of all output gases tends to vary to a similar extent according to the condition of filters, and so on, it is frequently necessary to measure the concentration of only one gas from which the concentration of other gases can be predicted reliably. This greatly reduces the cost involved in such monitoring.

Various devices are currently available that detect, quantify the concentration and analyze gases. These are discussed in the following paragraphs. However, it should be noted that research into gas sensors is an active field and new developments are being made all the time.

Catalytic bead (calorimetric) sensors

Catalytic sensors, also known as calorimetric sensors, have widespread use for measuring the concentration of combustible hydrocarbon (CHC) gases. They consist of a coil of wire coated first with glass or a ceramic material and then with a catalyst on the outside. The coil is electrically heated to a temperature at which the monitored gas starts to burn. The burning process liberates heat proportional to the gas concentration. The consequent increase in the temperature of the wire coil increases its resistance, and the gas concentration can then be calculated by measuring the resistance change.

This type of sensor is cheap, small, and robust. Unfortunately, the catalysts normally used in these devices are adversely affected by many common industrial substances such as lead, phosphorous, silicon, and sulfur. Also, devices with a standard catalyst cannot be

used to measure high gas concentrations because the corresponding high temperature at which they have to operate adversely affects them. Fortunately, new types of poison-resistant and heat-resistant catalyst are now available and used in sensors available from several manufacturers.

Paper tape sensors

By moving a paper tape impregnated with a reagent sensitive to a specific gas (e.g., lead acetate tape to detect hydrogen sulfide) through an air stream, the time history of the concentration of gas is indicated by the degree of color change in the tape. This technique is now old technology, but it is still widely used as a low-accuracy but reliable and cheap means of detecting the presence of many gases, including hydrogen sulfide, isocyanates, ammonia and others.

Color change badges

The principle of paper tape sensors is used in special badges worn by personnel to indicate the presence of toxic gases. Badges are available for detecting hydrogen sulfide, chlorine, hydrazine, carbon monoxide, ozone, nitrogen dioxide, ammonia, and other gases.

Liquid electrolyte electrochemical cells

These consist of two electrodes separated by electrolyte, to which the measured air supply is directed through a permeable membrane, as shown in Fig. 21.17. The gas in the air to which the cell is sensitive reacts at the electrodes to form ions in the solution. This produces a voltage output from the cell.

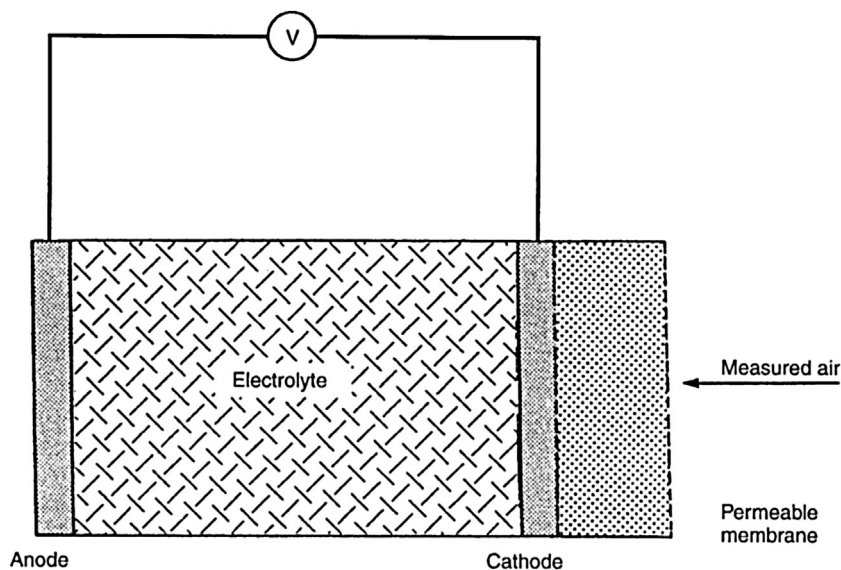


Figure 21.17

Liquid electrolyte electrochemical cell.

Electrochemical cells have stable characteristics and give good measurement sensitivity and accuracy even when monitoring low concentrations of gas. However, they are expensive and their durability is relatively poor, with life being generally limited to about 1–2 years at most. A further restriction is that they cannot be used above temperatures of about 50°C, because their performance deteriorates rapidly at high temperatures because of interference from other atmospheric substances.

The main use of such cells is in measuring toxic gases in satisfaction of health and safety legislation. Versions of the cell for this purpose are currently available to measure carbon monoxide, chlorine, nitrous oxide, hydrogen sulfide, and ammonia. Cells to measure other gases are under development.

In addition, electrochemical cells are used to a limited extent to monitor carbon monoxide emissions in flue gases for environmental control purposes. Precooling of the emitted gases is a necessary condition for this application.

Solid-state electrochemical cell (zirconia sensor)

This type of cell, commonly called the zirconia sensor, is primarily used for measuring oxygen concentration. The oxygen-measurement cell consists of two chambers separated by a zirconia wall. One chamber contains gas with a known oxygen concentration and the other contains the air being measured. Ions are conducted across the zirconia wall according to the difference in oxygen concentration across it, and this produces an output emf. The device is rugged but requires high temperatures to operate efficiently. However, it is well-proven and a standard choice for oxygen measurement. In industrial uses, it is often located in chimney stacks, where expensive mounting and protection systems are needed. However, low-cost versions are used in some vehicle exhaust systems as part of the engine management system.

Catalytic gate field effect transistors

These consist of field effect transistors with a catalytic palladium gate that is sensitive to hydrogen ions in the environment. The gate voltage and hence the characteristics of the device change according to the hydrogen concentration. They can be made sensitive to gases such as hydrogen sulfide, ammonia, and hydrocarbons as well as hydrogen. They are cheap and find applications in workplace monitoring, in satisfaction of health and safety legislation, and in fire detection (mainly detecting hydrocarbon products).

Semiconductor (metal oxide) sensors

In these devices, use is made of the fact that the surface conductivity of semiconductor metal oxides changes according to the concentration of certain gases with which they are

in contact. Gas concentration is therefore indicated by measuring the change in resistance. This type of sensor is used to detect hydrogen sulfide and some hydrocarbons. It is regarded as the best device available for measuring low concentrations of hydrogen sulfide. Particular advantages of this type of sensor are that they are cheap and robust.

Infrared sensors

This technique uses infrared light at a particular wavelength that is directed across a chamber between a source and detector. The amount of light absorption is a function of the unknown gas concentration in the chamber. The instrument normally has a second chamber containing gas at a known concentration across which infrared light at the same wavelength is directed to provide a reference. Sensitivity to carbon monoxide, carbon dioxide, chlorine, ammonia, hydrocarbons, and some other gases can be provided according to the wavelength used. Microcomputers are routinely incorporated in the instrument to reduce its sensitivity to gases other than the one being sensed, and so improve measurement accuracy. These sensors tend to be relatively expensive and can be adversely affected by moisture in the monitored gas. Nevertheless, the instrument requires little maintenance and finds widespread use in chimney/flue emission monitoring and in general process measurements.

Mass spectrometers

The mass spectrometer is a laboratory device for analyzing gases. It first reduces a gas sample to a very low pressure. The sample is then ionized, accelerated, and separated into its constituent components according to the respective charge-to-mass ratios. Almost any mixture of gases can be analyzed and the individual components quantified, but the instrument is expensive and requires a skilled user. Mass spectrometers have existed for over half a century but recent advances, but advances in electronic data processing techniques have greatly improved their performance.

Gas chromatography

This is also a laboratory instrument in which a gaseous sample is passed down a packed column. This separates the gas into its components, which are washed out of the column in turn and measured by a detector. Like the mass spectrometer, the instrument is versatile but expensive and it requires skilled use.

21.10.1 Calibration of gas sensors

Gas detectors are particularly susceptible to drift in their characteristics, which may result in either overestimation or underestimation of the true gas concentration. The rate and

extent to which characteristics change depend entirely on the nature of the sensor, the identity of the gas involved, and the temperature of the environment being sensed. Hence, the necessary recalibration frequency can be determined only by practical experience of using the sensor involved measuring a particular gas under the normal environmental conditions of measurement. However, the advice of the sensor manufacturer should be sought in the first instance.

The only way to calibrate sensors is to test them on standard gas mixtures that contain a certified concentration of the relevant gas. Such mixtures are available from many suppliers and consist of the sensed gas mixed with either air or nitrogen. Such calibration gas mixtures can be obtained in concentrations as low as a few parts per billion. As well as mixtures of a single gas with air or nitrogen, mixtures of several gases at given concentrations can be obtained, such as a mixture of carbon monoxide, methane, pentane, and oxygen.

21.11 Summary

This final chapter has been concerned with looking at techniques for measuring a number of physical variables that were not covered earlier in the book. In this coverage, we looked in turn at the measurement of dimensions, angles, surface flatness, volume, viscosity, moisture (including humidity), sound, and pH (acidity/alkalinity), with a final section on gas sensing and analysis. This is certainly not a full list of physical variables that were not covered earlier because, as noted in the introduction to this chapter, proper coverage of techniques available to measure every physical variable that exists would be prohibitive in terms of both the size of book needed and the cost of producing it. However, the material in this final chapter provides some reference material for readers with respect to some of the variables that they may need to measure at some point in their career.

21.12 Problems

- 21.1 What is a reference plane? How is it used in dimension measurement?
- 21.2 Discuss the various forms of rules and tapes that are used in dimension measurement.
- 21.3 Explain, with the aid of a sketch, what the following are, how they work, and what they measure: standard caliper, vernier caliper.
- 21.4 Name three different types of micrometer. What is each used for?
- 21.5 What are gauge blocks and how are they used?
- 21.6 Draw a sketch of (a) a height gauge and (b) a depth gauge. What practical difficulties may be met in using these instruments?

- 21.7 Discuss the main procedures involved in calibrating dimension-measuring instruments.
- 21.8 What instruments are available for measuring angles?
- 21.9 How are instruments used for measuring angles calibrated?
- 21.10 (a) Explain how surface flatness is measured, and sketch the usual instrument used.
(b) Explain how would you calibrate an instrument used to measure surface flatness.
- 21.11 What are the main ways of measuring the volume of vessels? How are measurements of volume calibrated?
- 21.12 How is viscosity defined? Under what circumstances is viscosity difficult to measure? Name some industrial situations in which viscosity measurement is needed and briefly explain why measurement is necessary.
- 21.13 What are the three main principles on which viscosity measurement is based? Describe one instrument that works on each of these principles.
- 21.14 How are viscosity measurements in Newtonian liquids calibrated?
- 21.15 Name some industrial situations in which measurement of the moisture content in materials is needed and briefly explain why measurement is necessary.
- 21.16 Discuss the main electrical methods of measuring the moisture content in materials in an industrial environment.
- 21.17 Discuss the methods other than electrical ones that exist to measure the moisture content in materials in an industrial environment.
- 21.18 What are the main laboratory techniques for measuring the moisture content in materials? Briefly explain each technique.
- 21.19 What are the three alternative ways in which humidity can be defined? What are the three main instruments used to measure humidity? Briefly discuss how each of these works, using simple sketches to illustrate your discussion as appropriate.
- 21.20 Discuss the calibration of instruments used to measure humidity and moisture.
- 21.21 How is sound defined and measured? Under what circumstances is sound measurement required?
- 21.22 What are the main components in a sound meter and how are these combined to produce a sound magnitude measurement? How are sound meters calibrated?
- 21.23 Carry out an Internet search for the use of microelectromechanical system (MEMS) and nanoelectromechanical system (NEMS) sensors in miniature microphones.
- 21.24 What is pH and how is it defined? What components does a pH meter have and how do these produce a pH measurement?
- 21.25 Discuss the characteristics of a glass electrode.
- 21.26 What alternatives to the glass electrode exist for measuring pH? Briefly describe each of these alternatives.
- 21.27 What is the need for gas sensing? What functions do gas sensors perform?

- 21.28 Discuss the mode of operation and characteristics of each of the following types of gas sensor: (a) calorimetric sensor, (b) paper tape sensor, (c) color change personnel badge.
- 21.29 What are the two main types of electrochemical cell used as gas sensors? Discuss the main features of each type.
- 21.30 Briefly explain how each of the following work and what they are typically used for: (a) catalytic gate field effect transistor gas sensor, (b) metal oxide gas sensor, (c) infrared gas sensor, (d) mass spectrometer.

Imperial—metric—SI conversion tables

Length

SI units: mm, m, km

Imperial units: in, ft, mile

	mm	m	km	in	ft	mile
mm	1	10^{-3}	10^{-6}	0.039 3701	3.281×10^{-3}	—
m	1000	1	10^{-3}	39.3701	3.280 84	6.214×10^{-4}
km	10^6	10^3	1	39 370.1	3280.84	0.621 371
in	25.4	0.0254	—	1	0.083 333	—
ft	304.8	0.3048	3.048×10^{-4}	12	1	1.894×10^{-4}
mile	—	1609.34	1.609 34	63 360	5280	1

Area

SI units: mm^2 , m^2 , km^2

Imperial units: in^2 , ft^2 , mile^2

	mm^2	m^2	km^2	in^2	ft^2	mile^2
mm^2	1	10^{-6}	—	1.550×10^{-3}	1.076×10^{-5}	—
m^2	10^6	1	10^{-6}	1550	10.764	—
km^2	—	10^6	1	—	1076×10^7	0.3861
in^2	645.16	6.452×10^{-4}	—	1	6.944×10^{-3}	—
ft^2	92 903	0.092 90	—	144	1	—
mile^2	—	2.590×10^6	2.590	—	2.788×10^7	1

Second moment of areaSI units: mm⁴, m⁴Imperial units: in⁴, ft⁴

	mm ⁴	m ⁴	in ⁴	ft ⁴
mm ⁴	1	10 ⁻¹²	2.4025 × 10 ⁻⁶	1.159 × 10 ⁻¹⁰
m ⁴	10 ¹²	1	2.4025 × 10 ⁶	115.86
in ⁴	416 231	4.1623 × 10 ⁻⁷	1	4.8225 × 10 ⁻⁵
ft ⁴	8.631 × 10 ⁹	8.631 × 10 ⁻³	20 736	1

VolumeSI units: mm³, m³

Metric units: ml, l

Imperial units: in³, ft³, UK gallon

	mm ³	ml	l	m ³	in ³	ft ³	UK Gallon
mm ³	1	10 ⁻³	10 ⁻⁶	10 ⁻⁹	6.10 × 10 ⁻⁵	—	—
ml	10 ³	1	10 ⁻³	10 ⁻⁶	0.061 024	3.53 × 10 ⁻⁵	2.2 × 10 ⁻⁴
l	10 ⁶	10 ³	1	10 ⁻³	61.024	0.035 32	0.22
m ³	10 ⁹	10 ⁶	10 ³	1	61 024	35.31	220
in ³	16 387	16.39	0.0164	1.64 × 10 ⁻⁵	1	5.79 × 10 ⁻⁴	3.61 × 10 ⁻³
ft ³	—	2.83 × 10 ⁴	28.32	0.028 32	1728	1	6.229
UK gallon	—	4546	4.546	4.55 × 10 ⁻³	277.4	0.1605	1

Additional unit: 1 US gallon = 0.8327 UK gallon.

DensitySI unit: kg/m³Metric unit: g/cm³Imperial units: lb/ft³, lb/in³

	kg/m ³	g/cm ³	lb/ft ³	lb/in ³
kg/m ³	1	10 ⁻³	0.062 428	3.605 × 10 ⁻⁵
g/cm ³	1000	1	62.428	0.036 127
lb/ft ³	16.019	0.016 019	1	5.787 × 10 ⁻⁴
lb/in ³	27 680	27.680	1728	1

Mass

SI units: g, kg, t

Imperial units: lb, cwt, ton

	g	kg	t	lb	cwt	ton
g	1	10^{-3}	10^{-6}	2.205×10^{-3}	1.968×10^{-5}	9.842×10^{-7}
kg	10^3	1	10^{-3}	2.204 62	0.019 684	9.842×10^{-4}
t	10^6	10^3	1	2204.62	19.6841	0.984 207
lb	453.592	0.453 59	4.536×10^{-4}	1	8.929×10^{-3}	4.464×10^{-4}
cwt	50 802.3	50.8023	0.050 802	112	1	0.05
ton	1.016×10^6	1016.05	1.01605	2240	20	1

Force

SI units: N, kN

Metric unit: kg_f Imperial units: pdl (poundal), lb_f , UK ton_f

	N	kg_f	kN	pdl	lb_f	UK ton_f
N	1	0.1020	10^{-3}	7.233	0.2248	1.004×10^{-4}
kg_f	9.807	1	9.807×10^{-3}	70.93	2.2046	9.842×10^{-4}
kN	1000	102.0	1	7233	224.8	0.1004
pdl	0.1383	0.0141	1.383×10^{-4}	1	0.0311	1.388×10^{-5}
lb_f	4.448	0.4536	4.448×10^{-3}	32.174	1	4.464×10^{-4}
UK ton_f	9964	1016	9.964	72 070	2240	1

Additional unit: 1 dyne = 10^{-5} N = 7.233×10^{-5} pdl.**Torque (moment of force)**

SI unit: N m

Metric unit: kg_f mImperial units: pdl ft, lb_f ft

	N m	kg_f m	pdl ft	lb_f ft
N m	1	0.1020	23.73	0.7376
kg_f m	9.807	1	232.7	7.233
pdl ft	0.042 14	4.297×10^{-3}	1	0.031 08
lb_f ft	1.356	0.1383	32.17	1

InertiaSI unit: N m^2 Imperial unit: $\text{lb}_f \text{ft}^2$

$$1 \text{ lb}_f \text{ft}^2 = 0.4132 \text{ N m}^2$$

$$1 \text{ N m}^2 = 2.420 \text{ lb}_f \text{ft}^2$$

PressureSI units: mbar, bar, N/m^2 (pascal)Imperial units: lb/in^2 , in Hg, atm

	mbar	bar	N/m^2	lb/in^2	in Hg	atm
mbar	1	10^{-3}	100	0.014 50	0.029 53	9.869×10^{-4}
bar	1000	1	10^5	14.50	29.53	0.9869
N/m^2	0.01	10^{-5}	1	1.450×10^{-4}	2.953×10^{-4}	9.869×10^{-6}
lb/in^2	68.95	0.068 95	6895	1	2.036	0.068 05
in Hg	33.86	0.033 86	3386	0.4912	1	0.033 42
atm	1013	1.013	1.013×10^5	14.70	29.92	1

Additional conversion factors

$$1 \text{ inch water} = 0.073 56 \text{ in Hg} = 2.491 \text{ mbar}$$

$$1 \text{ torr} = 1.333 \text{ mbar}$$

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

Energy, work, heat

SI unit: J

Metric units: $\text{kg}_f \text{m}$, kW hImperial units: ft lb_f , cal, Btu

	J	$\text{kg}_f \text{m}$	kW h	ft lb_f	cal	Btu
J	1	0.1020	2.778×10^{-7}	0.7376	0.2388	9.478×10^{-4}
$\text{kg}_f \text{m}$	9.8066	1	2.724×10^{-6}	7.233	2.342	9.294×10^{-3}
kW h	3.600×10^6	367 098	1	2.655×10^6	859 845	3412.1
ft lb_f	1.3558	0.1383	3.766×10^{-7}	1	0.3238	1.285×10^{-3}
cal	4.1868	0.4270	1.163×10^{-6}	3.0880	1	3.968×10^{-3}
Btu	1055.1	107.59	2.931×10^{-4}	778.17	252.00	1

Additional conversion factors

$$1 \text{ therm} = 10^5 \text{ Btu} = 1.0551 \times 10^8 \text{ J}$$

$$1 \text{ thermie} = 4.186 \times 10^6 \text{ J}$$

$$1 \text{ hp h} = 0.7457 \text{ kW h} = 2.6845 \times 10^6 \text{ J}$$

$$1 \text{ ft pdl} = 0.042 14 \text{ J}$$

$$1 \text{ erg} = 10^{-7} \text{ J}$$

Power

SI units: W, kW

Imperial units: HP, ft lb_f/s

	W	kW	HP	ft lb _f /s
W	1	10^{-3}	1.341×10^{-3}	0.735 64
kW	10^3	1	1.341 02	735.64
HP	745.7	0.7457	1	548.57
ft lb _f /s	1.359 35	1.359×10^{-3}	1.823×10^{-3}	1

Velocity

SI units: mm/s, m/s

Metric unit: km/h

Imperial units: ft/s, mile/h

	mm/s	m/s	km/h	ft/s	mile/h
mm/s	1	10^{-3}	3.6×10^{-3}	3.281×10^{-3}	2.237×10^{-3}
m/s	1000	1	3.6	3.280 84	2.236 94
km/h	277.778	0.277 778	1	0.911 344	0.621 371
ft/s	304.8	0.3048	1.097 28	1	0.681 818
mile/h	447.04	0.447 04	1.609 344	1.466 67	1

AccelerationSI unit: m/s^2 Other metric unit: cm/s^2 Imperial unit: ft/s^2 Other unit: g

	m/s^2	cm/s^2	ft/s^2	g
m/s^2	1	100	3.281	0.102
cm/s^2	0.01	1	0.0328	0.001 02
ft/s^2	0.3048	30.48	1	1
g	9.81	981	32.2	1

Mass flow rateSI unit: g/s Metric units: kg/h , $tonne/d$ Imperial units: lb/s , lb/h , ton/d

	g/s	kg/h	$tonne/d$	lb/s	lb/h	ton/d
g/s	1	3.6	0.086 40	2.205×10^{-3}	7.937	0.085 03
kg/h	0.2778	1	0.024 00	6.124×10^{-4}	2.205	0.023 62
$tonne/d$	11.57	41.67	1	0.025 51	91.86	0.9842
lb/s	453.6	1633	39.19	1	3600	38.57
lb/h	0.1260	0.4536	0.010 89	2.788×10^{-4}	1	0.010 71
ton/d	11.76	42.34	1.016	0.025 93	93.33	1

Volume flow rateSI unit: m^3/s Metric units: l/h , ml/s Imperial units: gal/h , ft^3/s , ft^3/h

	l/h	ml/s	m^3/s	gal/h	ft^3/s	ft^3/h
l/h	1	0.2778	2.778×10^{-7}	0.2200	9.810×10^{-6}	0.035 316
ml/s	3.6	1	10^{-6}	0.7919	3.532×10^{-5}	0.127 14
m^3/s	3.6×10^6	10^6	1	7.919×10^5	35.31	1.271×10^5
gal/h	4.546	1.263	1.263×10^{-6}	1	4.460×10^{-5}	0.160 56
ft^3/s	1.019×10^5	2.832×10^4	0.028 32	2.242×10^4	1	3600
ft^3/h	28.316	7.8653	7.865×10^{-6}	6.2282	2.778×10^{-4}	1

Specific energy (heat per unit volume)SI units: J/m³, kJ/m³, MJ/m³Imperial units: kcal/m³, Btu/ft³, therm/UK gal

	J/m ³	kJ/m ³	MJ/m ³	kcal/m ³	Btu/ft ³	therm/UK gal
J/m ³	1	10 ⁻³	10 ⁻⁶	1.388 × 10 ⁻⁴	2.684 × 10 ⁻⁵	—
kJ/m ³	1000	1	10 ⁻³	0.2388	0.02684	—
MJ/m ³	10 ⁶	1000	1	238.8	26.84	4.309 × 10 ⁻⁵
kcal/m ³	4187	4.187	4.187 × 10 ⁻³	1	0.1124	1.804 × 10 ⁻⁷
Btu/ft ³	3.726 × 10 ⁴	37.26	0.03726	8.899	1	1.605 × 10 ⁻⁶
therm/UK gal	—	—	2.321 × 10 ⁴	5.543 × 10 ⁶	6.229 × 10 ⁵	1

Dynamic viscositySI unit: N s/m²

Metric unit: cP (centipoise), P (poise) (1 P = 100 g/m s)

Imperial unit: lb_m/ft h

	lb _m /ft h	P	cP	N s/m ²
lb _m /ft h	1	4.133 × 10 ⁻³	0.4134	4.134 × 10 ⁻⁴
P	241.9	1	100	0.1
cP	2.419	0.01	1	10 ⁻³
N s/m ²	2419	10	1000	1

Additional unit: 1 Pa s = 1 N s/m².**Kinematic viscosity**SI unit: m²/s

Metric unit: cSt (centistokes), St (stokes)

Imperial unit: ft²/s

	ft ² /s	m ² /s	cSt	St
ft ² /s	1	0.0929	9.29 × 10 ⁴	929
m ² /s	10.764	1	10 ⁶	10 ⁴
cSt	1.0764 × 10 ⁻⁵	10 ⁻⁶	1	0.01
St	1.0764 × 10 ⁻³	10 ⁻⁴	100	1

Thévenin's theorem

Thévenin's theorem is extremely useful in the analysis of complex electrical circuits. It states that any network that has two accessible terminals, A and B, can be replaced, as far as its external behavior is concerned, by a single electromotive force (emf) acting in series with a single resistance between A and B. The single equivalent emf is that emf measured across A and B when the circuit external to the network is disconnected. The single equivalent resistance is the resistance of the network when all current and voltage sources within it are reduced to zero. To calculate this internal resistance of the network, all current sources within it are treated as open circuits and all voltage sources as short circuits. The proof of Thévenin's theorem can be found in [Skilling \(1967\)](#).

[Fig. A2.1](#) shows part of a network consisting of a voltage source and four resistances. As far as its behavior external to terminals A and B is concerned, this can be regarded as a single voltage source, V_t , and a single resistance, R_t . Applying Thévenin's theorem, R_t is found first of all by treating V_1 as a short circuit, as shown in [Fig. A2.2](#). There are simply two resistances, R_1 and $(R_2 + R_4 + R_5)$, in parallel. The equivalent resistance, R_t , is thus given by

$$R_t = \frac{R_1(R_2 + R_4 + R_5)}{R_1 + R_2 + R_4 + R_5},$$

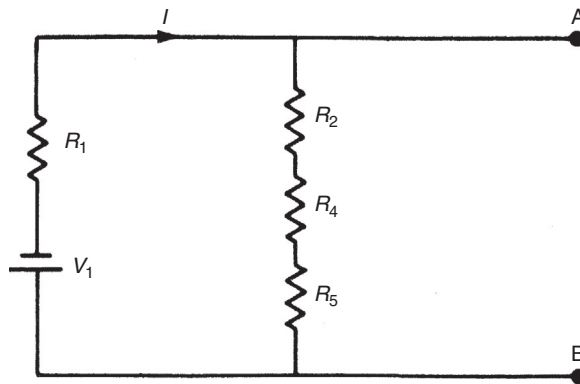


Figure A2.1

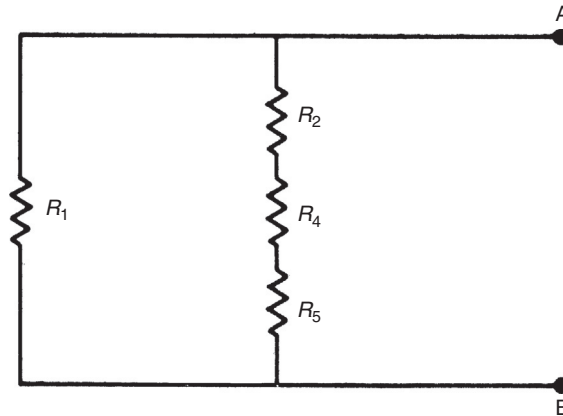


Figure A2.2

where V_t is the voltage drop across AB. To calculate this, it is necessary to carry out an intermediate step of working out the current flowing, I . Referring to Fig. A2.1, this is given by

$$I = \frac{V_1}{R_1 + R_2 + R_4 + R_5}.$$

Now, V_t can be calculated from

$$\begin{aligned} V_t &= I(R_2 + R_4 + R_5) \\ &= \frac{V_1(R_2 + R_4 + R_5)}{R_1 + R_2 + R_4 + R_5}. \end{aligned}$$

The network of Fig. A2.1 has thus been reduced to the simpler network shown in Fig. A2.3.

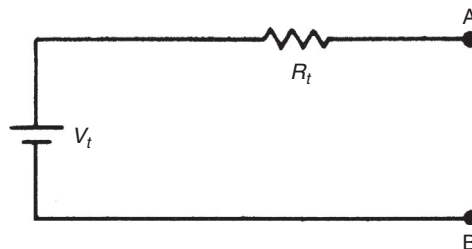


Figure A2.3

Let us now proceed to the typical network problem of calculating the current flowing in the resistor R_3 of Fig. A2.4. R_3 can be regarded as an external circuit or load on the rest of the network consisting of V_1 , R_1 , R_2 , R_4 , and R_5 , as shown in Fig. A2.5. This network of V_1 , R_1 , R_2 , R_4 , and R_5 is that shown in Fig. A2.6. This can be rearranged to the network shown in Fig. A2.1, which is equivalent to the single voltage source and resistance, V_t and R_t , calculated earlier. The whole circuit is then equivalent to that shown in Fig. A2.7, and the current flowing through R_3 can be written as

$$I_{AB} = \frac{V_t}{R_t + R_3}.$$

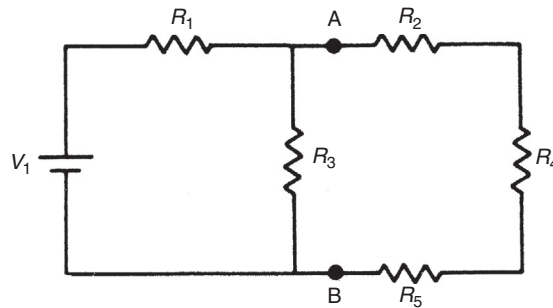


Figure A2.4

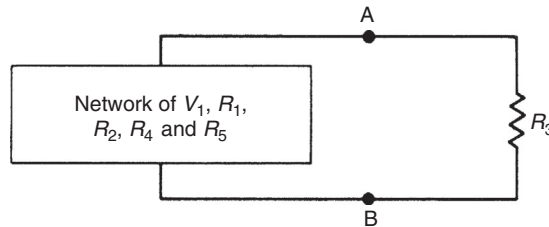


Figure A2.5

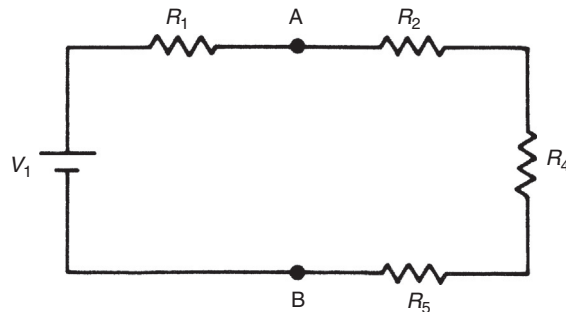


Figure A2.6

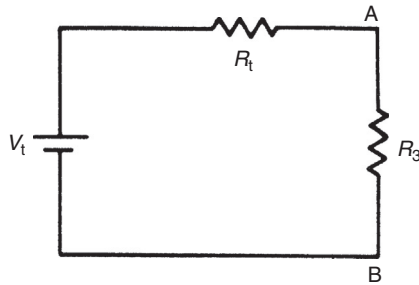


Figure A2.7

Thévenin's theorem can be applied successively to solve ladder networks of the form shown in Fig. A2.8. Suppose, in this network, that it is required to calculate the current flowing in branch XY.

The first step is to imagine two terminals, A and B, in the circuit and regard the network to the right of AB as a load on the circuit to the left of AB. The circuit to the left of AB can be reduced to a single equivalent voltage source, E_{AB} , and resistance, R_{AB} , by Thévenin's theorem. If the 50-V source is replaced by its zero internal resistance (i.e., by a short circuit), then R_{AB} is given by

$$\frac{1}{R_{AB}} = \frac{1}{100} + \frac{1}{2000} = \frac{2000 + 100}{200,000}.$$

Hence,

$$R_{AB} = 95.24 \, \Omega.$$

When AB is an open circuit, the current flowing around the loop to the left of AB is given by

$$I = \frac{50}{100 + 2000}.$$

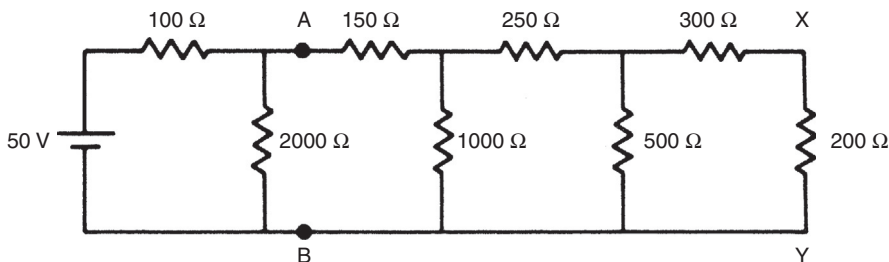


Figure A2.8

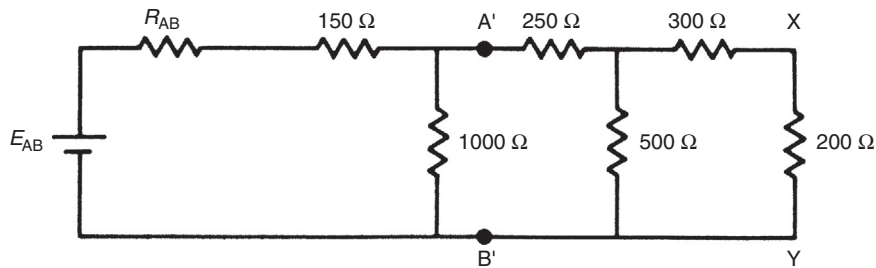


Figure A2.9

Hence, E_{AB} , the open circuit voltage across AB, is given by

$$E_{AB} = I \times 2000 = 47.62 \text{ volts.}$$

We can now replace the circuit shown in Fig. A2.8 by the simpler equivalent circuit shown in Fig. A2.9.

The next stage is to apply an identical procedure to find an equivalent circuit consisting of voltage source $E_{A'B'}$ and resistance $R_{A'B'}$ for the network to the left of points A' and B' in Fig. A2.9:

$$\frac{1}{R_{A'B'}} = \frac{1}{R_{AB} + 150} + \frac{1}{1000} = \frac{1}{245.24} + \frac{1}{1000} = \frac{1245.24}{245,240}.$$

Hence,

$$R_{A'B'} = 196.94 \text{ } \Omega$$

$$E_{A'B'} = \frac{1000}{R_{AB} + 150 + 1000} \times E_{AB} = 38.24 \text{ volts.}$$

The circuit can now be represented in the yet simpler form shown in Fig. A2.10.

Proceeding as before to find an equivalent voltage source and resistance, $E_{A''B''}$ and $R_{A''B''}$, for the circuit to the left of A'' and B'' in Fig. A2.10:

$$\frac{1}{R_{A''B''}} = \frac{1}{R_{A'B'} + 250} + \frac{1}{500} = \frac{500 + 446.94}{223,470}.$$

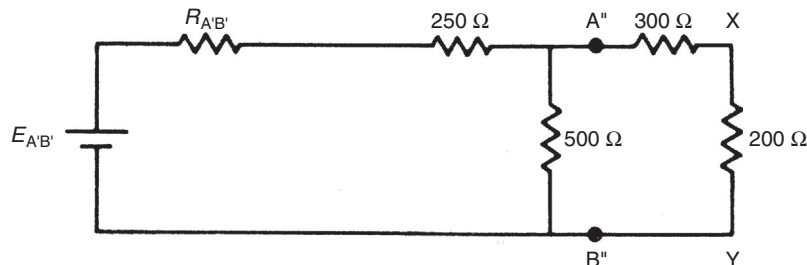


Figure A2.10

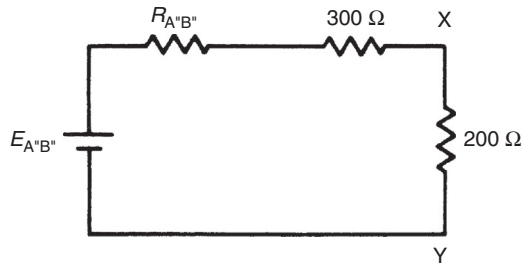


Figure A2.11

Hence,

$$R_{A''B''} = 235.99 \Omega$$

$$E_{A''B''} = \frac{500}{R_{A'B'} + 250 + 500} E_{A'B'} = 20.19 \text{ volts.}$$

The circuit has now been reduced to the form shown in Fig. A2.11, which the current through branch XY can be calculated simply as

$$I_{XY} = \frac{E_{A''B''}}{R_{A''B''} + 300 + 200} = \frac{20.19}{735.99} = 27.43 \text{ mA.}$$

References

Skilling, H. H., 1967. Electrical Engineering Circuits. Wiley, New York.

Thermocouple tables

Type E: chromel—constantan

Type J: iron—constantan

Type K: chromel—alumel

Type N: nicrosil—nihil

Type S: platinum/10% rhodium—platinum

Type T: copper—constantan

Temperature (°C)	Type E	Type J	Type K	Type N	Type S	Type T
-270	-9.834		-6.458	-4.345		
-260	-9.795		-6.441	-4.336		
-250	-9.719		-6.404	-4.313		
-240	-9.604		-6.344	-4.277		-6.105
-230	-9.456		-6.262	-4.227		-6.003
-220	-9.274		-6.158	-4.162		-5.891
-210	-9.063	-8.096	-6.035	-4.083		-5.753
-200	-8.824	-7.890	-5.891	-3.990		-5.603
-190	-8.561	-7.659	-5.730	-3.884		-5.438
-180	-8.273	-7.402	-5.550	-3.766		-5.261
-170	-7.963	-7.122	-5.354	-3.634		-5.070
-160	-7.631	-6.821	-5.141	-3.491		-4.865
-150	-7.279	-6.499	-4.912	-3.336		-4.648
-140	-6.907	-6.159	-4.669	-3.170		-4.419
-130	-6.516	-5.801	-4.410	-2.994		-4.177
-120	-6.107	-5.426	-4.138	-2.807		-3.923
-110	-5.680	-5.036	-3.852	-2.612		-3.656
-100	-5.237	-4.632	-3.553	-2.407		-3.378
-90	-4.777	-4.215	-3.242	-2.193		-3.089
-80	-4.301	-3.785	-2.920	-1.972		-2.788
-70	-3.811	-3.344	-2.586	-1.744		-2.475
-60	-3.306	-2.892	-2.243	-1.509		-2.152
-50	-2.787	-2.431	-1.889	-1.268	-0.236	-1.819
-40	-2.254	-1.960	-1.527	-1.023	-0.194	-1.475
-30	-1.709	-1.481	-1.156	-0.772	-0.150	-1.121
-20	-1.151	-0.995	-0.777	-0.518	-0.103	-0.757
-10	-0.581	-0.501	-0.392	-0.260	-0.053	-0.383
0	0.000	0.000	0.000	0.000	0.000	0.000

Continued

cont'd

Temperature (°C)	Type E	Type J	Type K	Type N	Type S	Type T
10	0.591	0.507	0.397	0.261	0.055	0.391
20	1.192	1.019	0.798	0.525	0.113	0.789
30	1.801	1.536	1.203	0.793	0.173	1.196
40	2.419	2.058	1.611	1.064	0.235	1.611
50	3.047	2.585	2.022	1.339	0.299	2.035
60	3.683	3.115	2.436	1.619	0.365	2.467
70	4.329	3.649	2.850	1.902	0.432	2.908
80	4.983	4.186	3.266	2.188	0.502	3.357
90	5.646	4.725	3.681	2.479	0.573	3.813
100	6.317	5.268	4.095	2.774	0.645	4.277
110	6.996	5.812	4.508	3.072	0.719	4.749
120	7.683	6.359	4.919	3.374	0.795	5.227
130	8.377	6.907	5.327	3.679	0.872	5.712
140	9.078	7.457	5.733	3.988	0.950	6.204
150	9.787	8.008	6.137	4.301	1.029	6.702
160	10.501	8.560	6.539	4.617	1.109	7.207
170	11.222	9.113	6.939	4.936	1.190	7.718
180	11.949	9.667	7.338	5.258	1.273	8.235
190	12.681	10.222	7.737	5.584	1.356	8.757
200	13.419	10.777	8.137	5.912	1.440	9.286
210	14.161	11.332	8.537	6.243	1.525	9.820
220	14.909	11.887	8.938	6.577	1.611	10.360
230	15.661	12.442	9.341	6.914	1.698	10.905
240	16.417	12.998	9.745	7.254	1.785	11.456
250	17.178	13.553	10.151	7.596	1.873	12.011
260	17.942	14.108	10.560	7.940	1.962	12.572
270	18.710	14.663	10.969	8.287	2.051	13.137
280	19.481	15.217	11.381	8.636	2.141	13.707
290	20.256	15.771	11.793	8.987	2.232	14.281
300	21.033	16.325	12.207	9.340	2.323	14.860
310	21.814	16.879	12.623	9.695	2.414	15.443
320	22.597	17.432	13.039	10.053	2.506	16.030
330	23.383	17.984	13.456	10.412	2.599	16.621
340	24.171	18.537	13.874	10.772	2.692	17.217
350	24.961	19.089	14.292	11.135	2.786	17.816
360	25.754	19.640	14.712	11.499	2.880	18.420
370	26.549	20.192	15.132	11.865	2.974	19.027
380	27.345	20.743	15.552	12.233	3.069	19.638
390	28.143	21.295	15.974	12.602	3.164	20.252
400	28.943	21.846	16.395	12.972	3.260	20.869
410	29.744	22.397	16.818	13.344	3.356	
420	30.546	22.949	17.241	13.717	3.452	
430	31.350	23.501	17.664	14.091	3.549	
440	32.155	24.054	18.088	14.467	3.645	
450	32.960	24.607	18.513	14.844	3.743	
460	33.767	25.161	18.938	15.222	3.840	

—cont'd

Temperature (°C)	Type E	Type J	Type K	Type N	Type S	Type T
470	34.574	25.716	19.363	15.601	3.938	
480	35.382	26.272	19.788	15.981	4.036	
490	36.190	26.829	20.214	16.362	4.135	
500	36.999	27.388	20.640	16.744	4.234	
510	37.808	27.949	21.066	17.127	4.333	
520	38.617	28.511	21.493	17.511	4.432	
530	39.426	29.075	21.919	17.896	4.532	
540	40.236	29.642	22.346	18.282	4.632	
550	41.045	30.210	22.772	18.668	4.732	
560	41.853	30.782	23.198	19.055	4.832	
570	42.662	31.356	23.624	19.443	4.933	
580	43.470	31.933	24.050	19.831	5.034	
590	44.278	32.513	24.476	20.220	5.136	
600	45.085	33.096	24.902	20.609	5.237	
610	45.891	33.683	25.327	20.999	5.339	
620	46.697	34.273	25.751	21.390	5.442	
630	47.502	34.867	26.176	21.781	5.544	
640	48.306	35.464	26.599	22.172	5.648	
650	49.109	36.066	27.022	22.564	5.751	
660	49.911	36.671	27.445	22.956	5.855	
670	50.713	37.280	27.867	23.348	5.960	
680	51.513	37.893	28.288	23.740	6.064	
690	52.312	38.510	28.709	24.133	6.169	
700	53.110	39.130	29.128	24.526	6.274	
710	53.907	39.754	29.547	24.919	6.380	
720	54.703	40.382	29.965	25.312	6.486	
730	55.498	41.013	30.383	25.705	6.592	
740	56.291	41.647	30.799	26.098	6.699	
750	57.083	42.283	31.214	26.491	6.805	
760	57.873	42.922	31.629	26.885	6.913	
770	58.663	43.563	32.042	27.278	7.020	
780	59.451	44.207	32.455	27.671	7.128	
790	60.237	44.852	32.866	28.063	7.236	
800	61.022	45.498	33.277	28.456	7.345	
810	61.806	46.144	33.686	28.849	7.454	
820	62.588	46.790	34.095	29.241	7.563	
830	63.368	47.434	34.502	29.633	7.672	
840	64.147	48.076	34.908	30.025	7.782	
850	64.924	48.717	35.314	30.417	7.892	
860	65.700	49.354	35.718	30.808	8.003	
870	66.473	49.989	36.121	31.199	8.114	
880	67.245	50.621	36.524	31.590	8.225	
890	68.015	51.249	36.925	31.980	8.336	
900	68.783	51.875	37.325	32.370	8.448	
910	69.549	52.496	37.724	32.760	8.560	
920	70.313	53.115	38.122	33.149	8.673	

Continued

—cont'd

Temperature (°C)	Type E	Type J	Type K	Type N	Type S	Type T
930	71.075	53.729	38.519	33.538	8.786	
940	71.835	54.341	38.915	33.926	8.899	
950	72.593	54.949	39.310	34.315	9.012	
960	73.350	55.553	39.703	34.702	9.126	
970	74.104	56.154	40.096	35.089	9.240	
980	74.857	56.753	40.488	35.476	9.355	
990	75.608	57.349	40.879	35.862	9.470	
1000	76.357	57.942	41.269	36.248	9.585	
1010		58.533	41.657	36.633	9.700	
1020		59.121	42.045	37.018	9.816	
1030		59.708	42.432	37.402	9.932	
1040		60.293	42.817	37.786	10.048	
1050		60.877	43.202	38.169	10.165	
1060		61.458	43.585	38.552	10.282	
1070		62.040	43.968	38.934	10.400	
1080		62.619	44.349	39.315	10.517	
1090		63.199	44.729	39.696	10.635	
1100		63.777	45.108	40.076	10.754	
1110		64.355	45.486	40.456	10.872	
1120		64.933	45.863	40.835	10.991	
1130		65.510	46.238	41.213	11.110	
1140		66.087	46.612	41.590	11.229	
1150		66.664	46.985	41.966	11.348	
1160		67.240	47.356	42.342	11.467	
1170		67.815	47.726	42.717	11.587	
1180		68.389	48.095	43.091	11.707	
1190		68.963	48.462	43.464	11.827	
1200		69.536	48.828	43.836	11.947	
1210			49.192	44.207	12.067	
1220			49.555	44.577	12.188	
1230			49.916	44.947	12.308	
1240			50.276	45.315	12.429	
1250			50.633	45.682	12.550	
1260			50.990	46.048	12.671	
1270			51.344	46.413	12.792	
1280			51.697	46.777	12.913	
1290			52.049	47.140	13.034	
1300			52.398	47.502	13.155	
1310			52.747		13.276	
1320			53.093		13.397	
1330			53.438		13.519	
1340			53.782		13.640	
1350			54.125		13.761	
1360			54.467		13.883	
1370			54.807		14.004	
1380					14.125	

—cont'd

Temperature (°C)	Type E	Type J	Type K	Type N	Type S	Type T
1390					14.247	
1400					14.368	
1410					14.489	
1420					14.610	
1430					14.731	
1440					14.852	
1450					14.973	
1460					15.094	
1470					15.215	
1480					15.336	
1490					15.456	
1500					15.576	
1510					15.697	
1520					15.817	
1530					15.937	
1540					16.057	
1550					16.176	
1560					16.296	
1570					16.415	
1580					16.534	
1590					16.653	
1600					16.771	
1610					16.890	
1620					17.008	
1630					17.125	
1640					17.243	
1650					17.360	
1660					17.477	
1670					17.594	
1680					17.711	
1690					17.826	
1700					17.942	
1710					18.056	
1720					18.170	
1730					18.282	
1740					18.394	
1750					18.504	
1760					18.612	

Using mathematical tables

A mathematical table serves to provide the value of some function $F(x)$ of a variable x , for various values of x . Mathematical tables are published both within textbooks such as this one, and also as collections of tables in stand-alone publications with titles like “Mathematical Tables” or “Statistical Tables.” The resolution provided by tables, in terms of the number of different values of x covered, affects both their size and cost. Increasing the number of x values inevitably requires more pages for a given table, and hence increases both size and publication costs.

Fig. A4.1 shows part of the error function table that is presented in complete form as Table 4.1 in Chapter 4. This provides values of the function $F(z)$ for various values of z , where $F(z) = \int_{-\infty}^z \frac{1}{\sigma\sqrt{2\pi}} e^{(-z^2/2)} dz$.

In the table in Fig. A4.1, the left-hand column gives various values of z with one figure after the decimal point from 1.0 to 1.9. The number at the top of the columns to the right gives the second figure after the decimal point for the value of z , from 0.00 to 0.09. The numbers in the rest of the table give the value of $F(z)$ corresponding to the value of z given at the left-hand end of each row and the “second figure after the decimal point” shown at the top. This will become clearer with some examples.

	$F(z)$									
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8906	0.8925	0.8943	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9648	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

Figure A4.1
Part of an error function table.

If we want the value of $F(z)$ corresponding to $z = 1.40$, we look down the left-hand column until we find “1.4” and we then look along this row to the column that has “0.00” at the top (column 2). This gives a value of $F(z) = 0.9192$.

If we want the value of $F(z)$ corresponding to $z = 1.78$, we look down the left-hand column until we find “1.7” and we then look along this row to the column that has “0.08” at the top (column 10). This gives a value of $F(z) = 0.9625$.

The relevant numbers for both these examples are highlighted by shading in [Figure A4.1](#).

Interpolation

It frequently happens that the value of a parameter is expressed to a greater number of figures after the decimal point than is provided for in a mathematical table. For example, suppose we want to find the value of $F(z)$ for $z = 1.783$. This value of z has three figures after the decimal point, but the table only provides for values of z with two figures after the decimal point. We can find a value of $F(z) = 0.9625$ corresponding to $z = 1.78$ from the table and we can also find a value of $F(z) = 0.9633$ corresponding to $z = 1.79$ from the table. The required value of $F(z)$ corresponding to $z = 1.783$ lies between these two values of $F(z)$. We need to use *interpolation* to determine the exact value of $F(z)$. The process of interpolation is illustrated in [Figure A4.2](#).

If we had a graphical representation of the relationship between $F(z)$ and z , it would be a curve, and we could show the part of the curve corresponding to values of z between 1.78 and 1.79 as in [Figure A4.2](#). To find the value of $F(z)$ corresponding to $z = 1.783$, we would measure $3/10$ of the distance along the z -axis between $z = 1.78$ and $z = 1.79$ and then draw a vertical line up to the curve of $F(z)$ against z (shown as a vertical dashed line in the figure). Then, drawing a horizontal line from the point of intersection with the curve

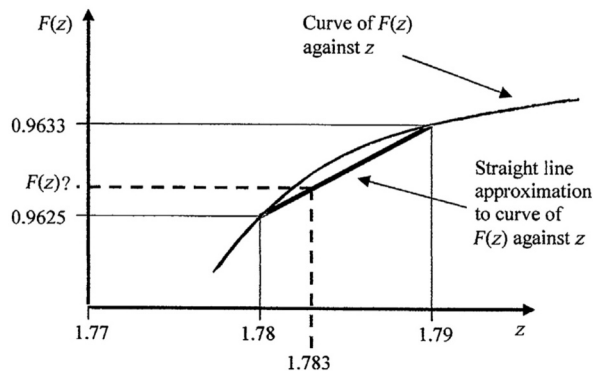


Figure A4.2

Illustrating the process of interpolation in a data table.

along to the $F(z)$ -axis would give the correct value of $F(z)$ for $z = 1.783$ at the point where this horizontal line cuts the $F(z)$ axis.

However, in practice, we do not have a graph of $F(z)$ against z available. All we have are the $F(z)$ data points for $z = 1.78$ and for $z = 1.79$ given in a table. What interpolation does is to effectively draw an imaginary straight line between the data points at $z = 1.78$ and $z = 1.79$ (which is an approximation to the actual curve between these two data points). The vertical line up from the value of $z = 1.783$ is drawn as far as this straight line, and then the horizontal line from the point of intersection to the $F(z)$ axis (shown as a horizontal dashed line in the figure) defines the value of $F(z)$ corresponding to $z = 1.783$. As can be seen in this example, this is an approximation to the correct value of $F(z)$, but it is the best we can get when we have a value of z that is between the values given in the table.

The process of calculating the value of $F(z)$ for a value $z = 1.783$ can therefore be summarized as follows.

Since the z value of 1.783 is 3/10 of the distance between $z = 1.78$ and $z = 1.79$, the value of $F(z)$ for $z = 1.783$ is 3/10 of the distance between the values of $F(z)$ corresponding to $z = 1.78$ and $z = 1.79$. Hence, $F(1.783)$ is calculated as:

$$F(1.783) = F(1.780) + 0.3[F(1.79) - F(1.78)] = 0.9625 + 0.3[0.9633 - 0.9625] = 0.9627$$

The interpolation procedure is exactly the same if we have four or more figures after the decimal point for the value of z (although the change in the value of $F(z)$ will often be negligible for such a very small change in the value of z). The correct value of $F(z)$ for $z = 1.7836$ is 36/100 of the distance between the values of $F(z)$ corresponding to $z = 1.78$ and $z = 1.79$.

$$\begin{aligned} \text{Hence, } F(1.7836) &= F(1.780) + 0.36[F(1.79) - F(1.78)] \\ &= 0.9625 + 0.36[0.9633 - 0.9625] = 0.9628. \end{aligned}$$

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‘Note: Page numbers followed by “f” indicate figures and “t” indicate tables.’

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